On the Design and Winding of Alternate-Current Electromagnets

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XIV. *On the Design and Winding of Alternate-Current Electromagnets.* By SILVANUS P. THOMPSON, D.Sc., F.R.S., and MILES WALKER*.

1. In designing electromagnets for use with alternate currents various considerations enter which do not enter into the design of electromagnets for use with continuous currents. Chief of these considerations are the self-inductive action of the windings, and the frequency of the alternation of the current.

As with continuous-current electromagnets, so with alternate-current magnets, the degree to which magnetization is carried in the magnetic circuit of given configuration depends upon the number of ampere-turns of the excitation. The methods of predetermining the number needed to produce any desired degree of magnetization are too well known to need any reference. Our present aim is to show how to determine the winding which will, under given circumstances as to frequency, voltage, &c., produce any desired number of ampere-turns.

2. As a preliminary, however, it is needful to touch upon the point whether an electromagnet (properly laminated, it is understood) will give an equal pull, when excited with an equal number of ampere-turns of alternating current.

* Read April 27, 1894.

Since the pull of an electromagnet upon its armature depends upon the normal tension along the magnetic field integrated over the area of the pole-face, and this tension is proportional to the square of the intensity of the magnetic flux, then, if this flux were proportional to the number of ampere-turns of excitation (as would be the case if the permeability were constant), the pull would be proportional to the square of the number of ampere-turns. Now the ammeters for alternate currents operate upon principles that cause them to indicate the square root of the mean square values of the current. Hence it follows that, if permeability were constant, the pull should be proportional to the square of the current, whether the current be continuous or alternating. But since, with an alternating current of the same nominal value as a continuous current, the magnetism is carried at each period to a point considerably higher than with the continuous current, and since also at the higher degrees of magnetization the permeability is lower, it follows that the pull of the electromagnet when excited by alternate currents ought to be slightly less if the excitation is so great as to carry the magnetization into the regions commonly spoken of as approaching saturation. The pull ought to be independent of the frequency of the alternation provided this exceeds a certain minimum.

Experiments made show that these views are justified in practice.

An electromagnet of the form shown in fig. 1 was made up of iron stampings about 0·5 millim. thick.

The principal dimensions were as follows :—

Mean length of limbs and yoke	38	cms.
Breadth of limb	3·1	cms.
Gross thickness of limb	2·54	cms.
Nett thickness of limb	2·1	cms.
Nett cross section of limb	6·5	sq. cms.
Width between limbs	6·3	cms.
Total length of armature	12·7	cms.

The stampings of the armature were of the same thickness, number, and breadth as those of the electromagnet. In the succeeding experiments the electromagnet was wound with 163 turns of copper wire 2.03 millim. in diameter, and the armature was separated from the pole-faces by a distance-piece of wood 0.952 centim. thick.

TABLE showing Pull and Different Frequencies with various Currents.

Exciting Current. Amperes.	Mean lines per sq. centimetre.	Pull, in grammes, with Continuous Current.	Pull, in grammes, with Alternate Current.
<i>A. At a frequency of 30 periods per second.</i>			
8	2300	280	280
12.5	3600	820	820
<i>B. At a frequency of 60 periods per second.</i>			
8	2300	280	280
10.2	2900	530	550
<i>C. At a frequency of 90 periods per second.</i>			
4	800	60	60
4.2	1100	100	100
10.2	2900	530	530
21	5600	2400	2200
<i>D. At a frequency of 120 periods per second.</i>			
13.2	3900	900	900
21	5600	2400	2200

These experiments show that the pull was exactly the same when the intensity of the flux did not exceed 4000 lines per square centimetre. At higher degrees of magnetization the pull with alternate currents is slightly less.

It may be remarked that at higher intensities and with higher frequencies, eddy-currents, if any are present, will produce, for an equal flow of exciting current, a greater demagnetizing action, and therefore lessen the pull. On the other hand, if the current is supplied at a constant (alternating) voltage, eddy currents do not decrease the magnetization but cause more current to flow in proportion.

Another experiment was made with a solenoidal electromagnet with a laminated U-form plunger belonging to a Brush alternate-current arc-lamp. This plunger with attached mechanism weighed about 820 grammes, and was arranged to be drawn up into position by a current of 10 amperes. This plunger was loaded with various weights and the current required to raise it was measured, with the following results:—

Total weight. Grammes.	Continuous Current. Amperes.	Alternate Current. Virtual Amperes.
820	10	10
933	17.2	18
1039	18.2	19.5
1276	25	26.5
1601	31	33
1728	36	40

3. When alternate currents are supplied at a given voltage to an electromagnet having a given configuration of the magnetic circuit, and it is desired to produce by a winding any given number of ampere-turns of excitation, it is obvious that there are many ways of attaining the result with coils of various sizes and numbers of turns. If the resistance of the coil is so small relatively to the inductance as to be negligible, then there will only be one possible solution as to the number of turns. If, however, the resistance of the coil is of a magnitude comparable with that of its inductance, then there are various possible solutions, for an equal number of ampere-turns may be produced by using currents of different strengths and coils of different numbers of turns. In the cases of

different windings the lag of phase of the currents will be different. If the resistance is large relatively to the inductance, the difference of phase between the current and the impressed volts will be small, and the wasted watts large. If the inductance is large relatively to the resistance, the difference of phase will be nearly 90° ; and the current will be nearly wattless, the efficiency being relatively high.

It is in accordance with good constructive practice to use such frequencies in the supply, and coils of such relatively high coefficients of self-induction as to justify the assumption, for the purpose of design, that the inductance is the important, and the resistance of the coil the unimportant matter. However, to bring the calculations to bear upon all cases that may occur, both resistance and inductance have been taken into consideration.

Let the impressed volts be called V , the current i (both being square-root of mean square values), the resistance of the coil R , its coefficient of self-induction L , the frequency of the period of alternation n , and the number of windings W . The inductance will then be $2\pi nL$, which for brevity may be written pL . Then the impedance will be $\sqrt{R^2 + p^2 L^2}$. The relation between volts and amperes in the coil will be expressed as

$$V = i \sqrt{R^2 + p^2 L^2}.$$

The angle of lag being called ϕ , its sine will have the value

$$pL \div \sqrt{R^2 + p^2 L^2}.$$

Hence we have the following relation :—

$$V \cdot \sin \phi = i p L ; \quad . \quad . \quad . \quad . \quad . \quad (1)$$

these being equivalent expressions for the effective part of the volts that is employed to balance the counter-electromotive force of self-induction in the coils. Now L is proportional to the square of the number of windings ; and may be written $= kW^2$, where k is the coefficient of self-induction of a single turn, and W the number of windings.

Now, let the prescribed number of ampere-turns that are to be produced be called $Z = iW$. Inserting these values in (1) we obtain the expression

$$V \sin \phi = k p W Z ; \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and finally

$$W = \frac{V \sin \phi}{kpZ}; \quad \dots \dots \dots (3)$$

which is the relation sought.

4. The quantity k denoted above, the coefficient of self-induction of a single turn, depends upon the dimensions of the iron parts of the magnetic circuit and upon those of the air-gap, as well as upon the permeability of the iron. If the magnetization is not carried beyond 5000 lines per square centimetre, the permeability may be considered as constant. But k will decrease with any increase in the reluctance of the magnetic circuit, such as the lengthening of the air-gap. On the contrary, as the armature is attracted up toward the poles k increases. The quantity k represents the number of magnetic lines ($\times 10^{-9}$) which permeate the magnetic circuit when one ampere circulates in a single turn of wire around the core. We have only to multiply k by the square of the number of windings to obtain the coefficient of self-induction of any coil subsequently to be wound upon this magnetic circuit. The quantity k must itself be experimentally determined. This can be done by winding upon the magnetic circuit an experimental coil having a known number of turns w , and a known resistance r . Using this coil, measure the virtual current c which passes when it is supplied at a voltage v of frequency n . During the experiment the armature should be fixed, say by wooden distance-pieces, in the position which it will occupy in use. Then the coefficient of self-induction l of the temporary coil will be

$$l = \sqrt{v^2 - c^2 r^2} \div \sqrt{4\pi^2 n^2 c^2}.$$

Then l being thus experimentally determined, it follows that $k = l/w^2$.

From the equation (3) we see that V , p , and k being constant, there is a certain maximum number of windings which must not be exceeded if we are to get the prescribed number of ampere-turns of excitation; for $\sin \phi$ cannot be greater than unity. If this maximum number of windings is exceeded, the result will be to give fewer ampere-turns. This is easily seen if equation (3) be put into the form

$$Z = \frac{V \sin \phi}{kpW}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Other things being equal, *the ampere-turns are inversely proportional to the number of windings*. This, at first sight, seems an anomaly; but is easily understood when we remember that the self-induction, and therefore the lessening of the current, is proportional to the square of the number of turns; so that whilst we are increasing our turns we are reducing our amperes in a much greater ratio.

Having ascertained k , the only other factor to be determined is $\sin \phi$. This in most practical cases can be assumed to be unity, since with the usual frequencies of supply and using magnets with iron cores, there is no difficulty in keeping the resistance negligibly small even when the number of windings is at the required maximum, so that ϕ is very nearly 90° . This will be seen from the results given below obtained from an actual magnet. In most ordinary cases, then, we merely write

$$W = V/kpZ; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and, having calculated W , we find what current the wire will have to carry from the equation $i = ZW$. If there is room, and in most cases there will be ample, we may wind with thicker wire than is necessary to carry the current; but there is no advantage in using thicker wire than such as makes R practically negligible.

5. The following results are from actual experiment. The core and armature were those already described in relation to fig. 1. The iron core was wound with 140 turns of copper wire, 1.22 millim. in diameter, for the purpose of first determining k when the armature was at the distances of 9.52, 6.35, 3.17, and 0 millim., respectively, from the poles. The resistance of the wire when heated by carrying 12 amperes was 0.3 ohm, and the frequency 93 periods per second. The fourth and fifth columns are worked out from the formulæ given above; for instance, in the first case

$$l = \frac{\sqrt{(31)^2 - (12 \times 0.3)^2}}{2\pi \times 93 \times 12} = 0.00434,$$

$$k = \frac{0.00434}{(140)^2} = 0.221 \times 10^{-6}.$$

Air-gap (millimetres).	Virtual Volts at terminals of coil.	Virtual Amperes.	Coefficient of Self-induction <i>l</i> .	Self-induction of one turn. <i>k</i> .
9.52	31	12	0.00434	0.221×10^{-6}
6.35	35.6	12	0.005	0.225×10^{-6}
3.17	46.8	12	0.00658	0.336×10^{-6}
0	51	2.76	0.023	1.68×10^{-6}

Suppose it were now required to wind the iron core so as to obtain 2400 ampere-turns when the armature was distant 9.52 millim. from the poles, using 50 volts and the same frequency as before.

Taking the equation (5) we obtain :

$$W = \frac{50}{0.221 \times 10^{-6} \times 2 \times 3.1416 \times 93 \times 2400} = 163 \text{ (approx.)}.$$

Accordingly 163 turns of wire 2.03 millimetres in diameter were wound upon the core. The current found, under the above conditions, to flow through the coil was 14.9 amperes, as measured by a Siemens' electro-dynamometer. This shows that the coil produced 2430 ampere-turns, the error being well within what may be expected when using commercial measuring-instruments.

Some other results obtained with this magnet are given below (table, p. 141).

6. An advantage of the alternate-current electromagnet is the greater constancy in the pull which it exerts over a given range. If supplied at constant voltage a fairly constant pull on the armature can be obtained over a considerable range. The reaction set up by the core tends to set up a back-electromotive force the limiting value of which is equal to the impressed voltage. If the resistance of the coil were actually zero and the magnetic leakage also negligible, this limiting value would be attained. Now suppose that when acting upon its armature at a distance the self-inductive reaction chokes the current down to a certain value, the magnetization of the core going round cycles between definite maxima.

<i>Coil 163 turns ; resistance (hot) 0.15 ohm.</i>				
Air-gap (millimetres).	Virtual Volts.	Virtual Amperes.	Coefficient of self-induction.	Self-induction of one turn.
9.52	50	14.9	0.00574	0.216×10^{-6}
6.35	50	12	0.00714	0.268×10^{-6}
<i>Coil 274 turns ; resistance (hot) 0.6 ohm.</i>				
9.52	50	5	0.01712	0.228×10^{-6}
6.35	50	4.4	0.0194	0.259×10^{-6}
3.17	50	3.5	0.0244	0.326×10^{-6}

Suppose the core to be drawn in nearer to the pole, the reluctance of the magnetic circuit is lessened, and fewer ampere-turns will suffice to produce an equal magnetization ; but because the reluctance is lessened the coefficient of self-induction is correspondingly increased, and the current choked in corresponding proportion. These two effects counter-balance one another, the magnetization going round cycles of practically the same amplitude as before. Hence the pull will be practically unchanged, save in so far as any change in the magnetic leakage at the different positions of the armature comes in to affect the question by weakening the field between the pole-faces and the armature. Were the pole-faces made relatively large so as to obviate the tendency to leakage and ensure the field being nearly uniform in the gap-spaces, the pull should be also very nearly constant throughout a range equal to at least one third of the breadth of the pole-face.

7. It is useful also to know the ratio between the different voltages that are needed, for a given electromagnet, to produce an equal number of ampere-turns with alternating and continuous electromotive forces.

Let V_a stand for the alternating volts and V_c for the

continuous volts which must be applied to the terminals of the coil in order to produce equal virtual amperes. Then

$$\frac{V_a}{V_c} = \frac{\sqrt{R^2 + p^2 L^2}}{R}. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

When, as is generally the case, R is small compared with pL , we may take as a sufficient approximation

$$\frac{V_a}{V_c} = \frac{pL}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The alternate voltage ratio is proportional to the frequency and to the time-constant of the electromagnet.--For this ratio we may, on certain assumptions, find an expression in terms of the dimensions of the core and coil. Confining ourselves to the case where the magnetic circuit is closed, we may find the value of L as follows:—

Let l_1 be the length (in centimetres) of the iron magnetic circuit, A_1 its cross section, v_1 its volume, m_1 its mass in grammes, μ its permeability, and S the number of windings in the coil. Now $m_1 = 7.79v_1$. Then L (in henries) will be

$$L = 4\pi A_1 \mu S^2 / 10^9 l_1 = 4\pi v_1 \mu S^2 / 10^9 l_1^2 = 4\pi m_1 \mu S^2 / 7.79 \times 10^9 l_1^2.$$

Assuming that B does not exceed 6000 lines per square centimetre, we may take for ordinary wrought-iron sheet $\mu = 2000$, and inserting this we obtain

$$L = S^2 m_1 / 310,000 l_1^2.$$

Further, let l_2 be the mean length of one turn of the copper winding, A_2 the cross section of the copper wire, k its conductivity (mhos per centimetre cube $= 10^6 \div 1.63$), v_2 the volume of copper ($= SA_2 l_2$), and m_2 the mass of copper ($= 8.8v_2$). Then we have for the resistance of the coil

$$R = l_2 S / A_2 k = l_2^2 S^2 / v_2 k = 8.8 l_2^2 S^2 / m_2 k.$$

Inserting these values for L and S we obtain:—

$$\begin{aligned} \frac{V_a}{V_c} &= \frac{pL}{R} = \frac{2\pi n S^2 m_1}{310000 l_1^2} \times \frac{m_2 \cdot 10^6}{8.8 l_2^2 S^2} \\ \frac{V_a}{V_c} &= 1.41 n \frac{m_1 m_2}{l_1^2 l_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8) \end{aligned}$$

It is, however, doubtful, since the magnetic circuit contains two joints, whether so high a value as 2000 can be expected

in ordinary work for μ . In that case the coefficient 1.41 will be lower. With the electromagnet described above, at a frequency $n=93$, and with the armature in contact, the coefficient was 0.6 instead of 1.41. This is tantamount to saying that the working permeability was only 850. As so arranged, L being 0.044, the inductance pL was 25.26 ohms, and R was 0.15 ohm. The alternate voltage ratio was therefore about 170.

As the armature was removed from proximity to the poles the self-induction, and therefore the impedance, fell, making the voltage ratio for equal currents lower. When the armature was at 3.17 millim. the voltage ratio fell to 34, and at 9.52 millim. to 21.5.

It was stated above that in ordinary cases the ratio pL/R might be taken instead of the more complete expression of equation (6). If it is desired to obtain a nearer approximation to $\sqrt{R^2 + p^2 L^2}/R$ than is afforded by simply neglecting the R under the square-root sign, we may take the value

$$\frac{V_a}{V_c} = \frac{pL}{R} + \frac{R}{2pL},$$

which, by reference to the preceding numerical instances, shows that the correcting term is really negligible, being in the first case, where pL/R was 170, only $\frac{1}{340}$, and in the last case, where pL/R was 21.5, only $\frac{1}{43}$, less than $\frac{1}{8}$ of 1 per cent.

8. Lastly, a very simple expression can be found for the number of windings, in terms of any desired mean value of the magnetic flux N in the iron. For if the iron of cross-section A is subjected to cycles of magnetization in which the mean value of the permeation is B , the mean flux is $N=AB$, and if the frequency be n periods per second, the self-induced electromotive force in W windings surrounding the iron will be equal to $2\pi nWN \div 10^8$ volts. If, then, the resistance is negligibly small, this may be equated to V , the mean volts of supply, whence we obtain at once

$$W = \frac{V \times 10^8}{pN} \dots \dots \dots (9)$$

For example :—Suppose that in the above magnet it were desired to obtain a permeation of 4000 lines to the square

centimetre, the total number of lines $\mathbf{N}$ would equal the area $\mathbf{A}$ (6.5 square centim.) multiplied by 4000, or $\mathbf{N}=26,000$. Taking the volts as 50, and the periods per second as 93, we find by formula (9), $\mathbf{W}=329$.

DISCUSSION.

Prof. PERRY pointed out that the constant pull of the alternate-current magnet followed immediately from the fundamental equation $e=rc+n\dot{\mathbf{I}}$, where rc is small, for if e be constant then $\dot{\mathbf{I}}$ and therefore $\mathbf{I}$ and $\mathbf{I}^2$ are constant. He was interested to see that the pulls for equal ampere-turns were the same, and indicated how the problem could be worked out mathematically when hysteresis was taken into account.

Mr. BLAKESLEY thought it better to fill the space on a magnet full of copper rather than use wire only just large enough to carry the current, for the loss of energy would be reduced. Some of the formulæ given might be put in simpler form.

Mr. SWINBURNE said an alternate-current magnet would only give constant pull in special cases. As another rule for winding alternate-current magnets, he said, wind the magnets as for continuous currents, and put a condenser in to take the same current, thereby reducing the voltage required.

Mr. BLAKESLEY said it would require a capacity of 600 microfarads to suit the magnet mentioned in the paper.

Dr. THOMPSON, in reply, agreed with Mr. Blakesley that the condenser was impracticable in many cases.