No. 1,481.—“On a Current Meter, a Deep-sea Current Indicator, and an Improved Ship's Log.” By BENJAMIN THEOPHILUS MOORE, M.A., C.E.

THIS Paper contains an account of three new instruments, a Current Meter, a Deep-sea Current Indicator, and an Improved Ship's Log. They are all constructed upon the same mechanical principle, and have many features in common. The Current Meter was designed to measure the velocity of running water, with a degree of accuracy not attainable by any other known means, and at the same time to do this rapidly. The Current Indicator is introduced in the belief that it will prove a useful instrument in ascertaining the direction of marine currents, especially of deep-sea currents; and, in combination with the Deep-sea Current Meter, of determining their velocity as well as their direction. The Improved Ship's Log, by its peculiar construction, travels at a uniform depth below the surface of the water, and with remarkable steadiness, keeping a true dead reckoning.

THE CURRENT METER.

The two main features of this instrument are the frame, which is so constructed as to secure the required position of the instrument in running water; and the rotating cylinder, containing the internal mechanism for recording the number of revolutions made in a given time. The instrument can be lowered into water to the required depth by a light chain or cord from a boat or other platform, and the rotating cylinder can be set in action or stopped at any instant while under water. When the current meter is lowered into running water it takes up a definite position with respect to the direction of the stream, and steadily maintains that position, no apparatus being required to fix it. In Plate 8, Fig. 1, is a side elevation of the instrument, and Fig. 2 a front elevation. Fig. 3 is a vertical longitudinal section, and Fig. 4 a plan of the frame with a section of the revolving cylinder, showing the mechanism within it. Figs. 5 and 6 are transverse sections of the revolving cylinder, and Fig. 7 shows the suspended frame which contains the mechanism. The frame consists mainly of three flat thin bars of brass, united to the solid ogival head, or cutwater,

which forms the front of the instrument, and terminating in a long double tail or rudder, the section of which is a cross with equal arms. They are bound together, to form a stiff and strong frame, but so as to offer the least possible resistance to running water.

The frame is suspended from a stirrup by two bearings, the geometrical axis of which is perpendicular to the longitudinal axis of the frame, viz., that which passes through the point of the ogival head and the line of intersection of the plates of the double tail; and the sides of this stirrup are extended downwards, and pierced with two holes for the purpose of attaching a lead weight to keep the instrument in place when used in a rapid stream.

The rotating cylinder is immediately behind the ogival head, having its axis coincident with the longitudinal axis of the frame. The centre of gravity of the whole instrument is in the geometrical axis of the bearings by which the instrument is suspended from the stirrup, and midway between them.

The instrument is symmetrical about a vertical plane through the longitudinal axis, and (with the exception of the keel plate) about a horizontal plane through the same axis. Thus, when the current meter is lowered into running water by a cord attached to the swivel at the top of the stirrup, the stream, acting upon the double rudder, will bring the longitudinal axis of the instrument in the direction of the current, with the ogival head pointing accurately up the stream, and this position of the instrument will hold good however great may be the velocity of the stream; for the vertical line through the centre of gravity, the direction of the pull upon the stirrup, and the resultant pressure of the running water upon the instrument, all pass through one point, and are in equilibrium, and consequently no couple is brought into action to turn the instrument out of the required position.

This does not take into account the action of the water on the keel plate, which produces a couple of small moment tending to raise the tail of the instrument; but this tendency is easily corrected by giving a slight inclination upwards to the horizontal plate of the tail, which calls into action a couple of equal moment in the opposite direction, and thus the axis of the instrument is kept in the direction of the stream.

The cylinder is set in rotation, or stopped, by operating on the spring which is fixed to the frame and partly embraces the cylinder. A light cord attached to the spring passes side by side with the cord which suspends the instrument to the hand of the operator;

by raising the cord a few inches, until it is tight, the spring is lifted and the cylinder is released, and when the cord is let go the spring is set free and again engages the cylinder. It is convenient to pass this cord through small rings upon the main cord at intervals of 1 foot; thus all chance of the stream lifting the spring is avoided, and the rings serve to measure the depth to which the instrument is lowered. The cylinder is set in motion, when free, by the action of the running water on the screw blades fixed to it.

In the instrument shown in Plate 8, the recording mechanism is placed inside the rotating cylinder, which is water-tight and contains a strip of glass so placed that the mechanism can be clearly seen through it. In other instruments the mechanism is enclosed within a water-tight tube of glass which slides in and out of the rotating cylinder; this construction is shown in Plate 9, Figs. 1, 2, and 3, illustrative of "Ships' Logs." The tube is closed by two brass discs connected together by a thin steel spindle, the axis of which coincides with that of the tube and cylinder; this spindle is screwed at its ends into the discs, by which means they are drawn close to the ends of the glass tube, and the joints are made water-tight by thin rings of leather. When the cylinder containing the glass tube is set in rotation, the small steel spindle also revolves about its own axis. From this spindle is suspended a small rectangular brass frame having holes at the ends through which the spindle passes. The frame contains a simple train of wheels connected by a worm-wheel with an endless screw upon the spindle, the centre of gravity of the frame and wheels being below the axis of the spindle; consequently, when the spindle revolves the frame remains at rest, or oscillates very slightly, and motion is set up in the train of wheels, and the revolutions of the cylinder are recorded by graduated dials revolving with the wheels. There is no stuffing-box, stiff joint, or bearing, in any part of the instrument. The resistances of the working parts are extremely small, and the cylinder may be made to revolve with a velocity much greater than that due to any running water without fear of the suspended frame tripping, or turning over. The rotating cylinder being hollow and water-tight in some instruments, and containing a water-tight glass tube in others, is supported by the water to the extent of the weight of the water displaced, the difference between its weight in water and in air being the weight upon the bearings. This resulting weight is less than 1 oz. in some of the lighter instruments, and in the heavier instruments, containing the glass tube, it gives an average pressure

on each bearing of about 5 oz.; and as the bearings are less than $\frac{1}{16}$ inch in diameter, the frictional resistance which they offer to the motion of the cylinder is extremely small. Moreover, the instrument is far more sensitive to the action of running water than would at first appear, because the weight of the rotating cylinder and its contents is twice as great in air as in water, being on an average 10 oz. in water and 21 oz. in air.

The friction of the internal mechanism within the glass cylinder is constant and wholly unaffected by the velocity of the stream. This mechanism is lubricated by oil, which seldom requires to be renewed.

The friction on the front external bearing is also constant, as this bearing only supports one end of the cylinder and sustains no thrust; the weight upon it is about 6 oz. The friction on the back bearing, which also acts as a pivot, is the only variable friction in any working part. This pivot has to sustain the thrust caused by the action of the running water on the screw blades only, there being no thrust due to the running water on the front end of the cylinder, because that is protected by the solid ogival head which forms the front of the frame. Thus the only variable friction is that due to the pressure on the back pivot. The moment of the resistance due to this friction is proportional to the product of the pressure and the radius of the pivot. This radius being less than $\frac{1}{3}\frac{1}{2}$ inch, while the effective radius of the screw blades is not far short of 2 inches, the ratio which this moment of resistance bears to the driving moment caused by the action of the running water on the screw blades is extremely small, and consequently the variable resistance of this pivot will not be sensible in practice; thus the friction of the working parts is for all practical purposes a constant quantity. The bearings upon which the cylinder rotates are the only bearings exposed to water. They are made of steel, nickel-plated, and are lubricated by the water in which the instrument works.

The rate of the instrument was obtained by drawing it several times through still water, for a known distance, at different velocities, for which purpose it was attached to, and underneath, a float about 6 feet in length. By comparing a number of results obtained in this way the following formula was arrived at, viz.:—

$$V = 1.2 R + Q,$$

where V is the velocity of the instrument through the water, in feet per minute, or, which is the same thing, the velocity of the water with respect to the instrument; R the number of revolutions of the

cylinder per minute, and Q a quantity which vanishes when R is equal to or greater than 60, and increases, as R diminishes, in the ratio of 1 to 5, the general value of Q being $\frac{60 - R}{5}$,

$$\therefore V = 1.2 R + \frac{60 - R}{5} = 1.2 R + 12 - .2 R = R + 12,$$

for values of R less than 60, and

$$V = 1.2 R$$

for values of R greater than 60.

From this it follows that when the velocity of the stream is 12 feet per minute, the rotating cylinder is bordering upon motion, its moment of inertia being just balanced by the moment of the force due to the action of the running water on the screw blades.

The instrument will not measure a smaller velocity than 12 feet per minute when it is suspended from a boat at anchor, or from a fixed platform; but smaller velocities may be measured by it if drawn, with a known velocity, against the current. Thus if the instrument be drawn with a velocity of 20 feet per minute against a stream whose velocity is V , and the number of revolutions be 14,

$$20 + V = 14 + 12 = 26, \text{ and } \therefore V = 6.$$

In this way very small velocities may be measured.

The current meter when in use is always drawn back by the stream through a small distance, from the vertical line passing through the point of suspension, the amount increasing with the velocity of the stream and the depth to which the instrument is immersed. This distance is greatest when the rotating cylinder is at rest, because the screw blades, being then fixed, offer greater resistance to the water. When the cylinder is set free, by raising the spring, the resistance is diminished, and the instrument advances through a small distance to meet the stream. This advance will tend to increase the number of revolutions of the cylinder in a given time, but, on the other hand, it is equivalent to a momentary quickening of the stream, which helps to overcome the inertia of the cylinder and to set it in full rotation rather more quickly than would otherwise be the case. But the greatest error which could arise from this advance of the instrument is extremely small, compared with the whole number of revolutions made in a given time; by allowing the instrument to remain some minutes under water this small error is distributed over the whole number of turns, and becomes practically inappreciable. The

vertical depth of the instrument below the surface is also affected by the distance through which it is drawn back by the stream, but this error in the depth would be only a small fraction of the horizontal distance, and consequently it would not be necessary, even if it were possible, to take it into account.

To use the instrument, put back the stirrup upon the frame and raise the frame to the level of the eyes, holding the tail with the right hand and the pointed head with the left. The dials will then be seen through the glass, and their reading must be taken down. The instrument is next to be lowered into the water, started and stopped at known instants by the spring, then drawn up out of the water, and the reading again taken. The difference between this and the former reading will give the number of revolutions in the time observed. In Plate 8, Fig. 7, the frame which contains the mechanism is shown detached from the cylinder, the reading of the dials being 32,705.

The dials will record 100,000 revolutions of the cylinder, after which the same reading will recur. This number of revolutions is equivalent to 120,000 feet, or more than 22 miles. Thus the instrument might be left under water for eleven hours, in a stream running 2 miles an hour, before the dials would go through a complete period; and, even if the period were overrun, a comparison of the readings, with an approximate estimate of the velocity of the stream, would immediately reveal the fact.

The glass cylinders are constructed to bear a greater pressure of water than is to be found in any river, or other fresh water. They have been tested under a head of about 300 feet without injury, or leakage at the joints.

For deep-sea work the water is allowed to enter the cylinder, but it is intended to construct the instrument in a somewhat different manner, which will be described under the head of "Ship's Logs." It has been found by experiment with a cylinder filled with gun oil, that 400 revolutions per minute can be made by the cylinder without the resistance of the oil causing the suspended frame of wheels to turn over. This corresponds to a velocity of about $5\frac{1}{2}$ miles an hour, which is probably much greater than that of any deep-sea current.

An important characteristic of this current meter is the rapidity with which it can be used. A velocity at any depth down to 20 feet can be taken with ease in five minutes, allowing the instrument a run of four minutes under water; in other words, only 20 per cent. of the whole time is required for observing and recording the instrumental readings. By using two instruments together

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the time can be still further economised, and each instrument will remain a longer time under water.

FORM of FIELD BOOK for CURRENT METER.

$$\text{Rate of instrument, } 1 \cdot 2 = \frac{\text{velocity in feet per minute}}{\text{number of revolutions per minute}}$$

No. of Observation.	Times of Starting and Stopping.	Interval (min.)	Instrument Readings.	Differences.	Revolutions per minute.	Velocity. Feet per minute.	Mean adopted Velocity.	Remarks.
1	H. M. S.							
	3 2 15		23,714					
	3 5 15	3	23,927	213	71	85.2		
2	3 6 10		23,927					
	3 9 10	3	24,141	214	71.33	85.596	85.40	Station A.
3	3 11 20		24,141					
	3 15 20	4	24,513	372	93	111.6		
4	3 16 10		24,513					
	3 19 10	3	24,794	281	93.66	112.39	112	Station B.

To illustrate one of the uses of the current meter, the river Thames was gauged on the 1st of January, 1876; a section of the river was taken about $\frac{1}{4}$ mile below the railway bridge at Staines, and about fifty velocities in all were measured. The whole work was done between 9.30 A.M. and 2.30 P.M. During the first half of the time a strong breeze was blowing across the river, and during the latter heavy rain fell, both of which considerably impeded the work. Figs. 8 to 15, Plate 8, represent the plan of the site, a transverse section of the river, and six parallel sections of the mass water which passed through the transverse section in twelve seconds, the planes of these sections being perpendicular to that of the transverse section, and parallel to the direction of the stream. From the dimensions of the transverse section and the measured velocities, the quantity passing through this section was estimated to be 156,799 cubic feet per minute. The calculations by which this result was obtained are given in Appendix I.

On the plan are shown three irregular curved lines stretching across the river from end to end of the transverse section. These lines are the curves of flow, or velocity curves, at the surface, the mean depth, and the bottom, respectively, in twelve seconds from the section line. The ends of these curves are drawn with a free hand from experience in similar cases; they can, of course, be laid down with greater precision by taking more velocities at stations closer together, but the quantity which passes through the end portions of the section is very small compared with the whole flow, and it is therefore not of so much importance to

measure the velocities near shore with great accuracy. Moreover, the gauging in question is given rather as an illustration of the use of the current meter than as an exact determination of the flow of the river at a particular time; indeed, the unfavourable weather probably affected the results, and it is not unlikely that the curves of flow would have been more regular had the measurement been taken in calm weather.

SHIP'S LOG AND DEEP-SEA CURRENT METER.

This instrument is constructed on the same principle as the Current Meter, but with a different arrangement of the parts. Plate 9, Fig. 1, shows a vertical longitudinal section of the log, and the glass cylinder and mechanism are seen detached in Figs. 2 and 3. The light frame and tail being dispensed with, the rotating cylinder is furnished with large screw blades, and contains within it a glass tube and mechanism similar to that of the current meter. The cylinder is attached to a strong spindle which rotates in bearings contained within another tube of the same diameter, terminating in a solid ogival head. This log is not attached in the usual way to the log-line by a ring at its extremity, but the line is connected with the tube near to, and a little in front of, the centre of gravity of the whole instrument. By this means the log travels below the surface of the water, even when drawn up near the ship. The usual method of attaching the log-line to a ring at the front of the log causes a couple to come into action, which elevates the point of the log, giving it an upward direction, which quickly brings it to the surface. This couple disappears altogether when the log-line is attached at the centre of gravity.

A log so constructed will not only travel steadily below the surface of the water, but may be run alongside of the ship if drawn from the end of a spar projecting a few feet from the side. A log running constantly below the surface, is not so liable to foul weed or other floating matter, and when such fouling does happen, the point of the log dips and throws off the floating substance, and the log rights itself again.

The recording mechanism is not exposed to the action of the sea-water: it works smoothly inside the water-tight glass tube, which can be detached from the log. The reading of the dials is to be entered in the log-book, and the dial pointers are not set to zero each time the log is put overboard. This prevents errors arising, either from omission to set the pointers, or from their working loose from too frequent handling.

The reading of the pointers in Fig. 2 is 137·540. Suppose the log to be put overboard, and that, after one hour, the reading is found to be 149·790; the difference, 12·25 miles, is the run through the water in that time.

Another and more delicate form of log, intended to be used also as a deep-sea current meter, is shown in Plate 9, Figs. 4, 5, and 6. This instrument has an outer cylinder surrounding the screw blades, and the rotating cylinder turns upon two end pivots as in the current meter. The water which acts on the blades passes through this outer cylinder, and a protecting conical grating of copper throws off any floating substance which might impinge upon the log, while it allows sufficient water to pass to drive the blades. This instrument is much more sensitive to running water than the log previously described, and is intended chiefly as a deep-sea current meter. When used for this purpose water is allowed to enter the cylinder containing the mechanism, to protect it from being crushed by the pressure of the water at great depths.

As it would be impracticable to start and stop the rotation of this cylinder at great depths by a spring, as in the current meter, a self-acting starting and stopping gear is provided, as shown in Plate 9, Fig. 6. This consists of a small disc, cut away on one side, and terminating in a heavy sphere on the other, which swings freely on a pivot attached to the frame containing the wheel-work. On the spindle which supports this frame a pinion of four teeth is fixed, in such a position as to allow the teeth to pass freely through the opening on the disc, when the axis of the whole instrument is horizontal, or nearly so. But when the axis is much inclined from the horizontal, the disc passes between two consecutive teeth of the pinion and is caught by it, and the suspended frame then revolves with its supporting spindle, and consequently the wheel-work does not register the rotations.

This is the case when the instrument is descending in water, and also when it is being drawn up. Thus the rotations of the cylinder are registered only while the instrument is suspended at the depth at which it is desired to measure the velocity.

DEEP-SEA CURRENT INDICATOR.

The object of this instrument is the determination of the direction of submarine currents by referring them to the magnetic meridian of the place. The instrument consists of a strong, heavy spherical shell of gun-metal, formed of two parts, which can be screwed together so as to form a water-tight case, terminating in

one direction in a solid cone, or cone-like protuberance, and in the opposite direction in a long cross tail or rudder, the axis of the cone passing through the centre of the spherical shell, and coinciding with the intersection of the blades of the cross tail. This axis may be conveniently called the principal axis of the instrument.

The shell is suspended from a strong stirrup, which strides across it, by two pivots whose axis is a diameter of the shell, and is perpendicular to the principal axis, and the whole instrument is balanced, or nearly so, upon its pivots when suspended by the stirrup. The stirrup is provided with a swivel at the top, to which a line is attached for the purpose of lowering the instrument into deep water. Accordingly, when the instrument is lowered into running water from a stationary platform, the water, acting on the rudder-like tail, causes the instrument to take up a position with its principal axis in the direction of the stream, and the point of the cone facing it. Inside the shell is a cylindrical box, suspended by gimbals upon two pivots fixed to the shell, with their geometrical axis coinciding with that of the two outer pivots, by which the whole instrument is suspended. This box contains a simple train of clockwork, and in its upper part a bar magnet, with a light graduated ring, balanced upon a fine pivot, the point of which coincides with the centre of the shell. The clockwork is so connected with the magnet that, at a given instant, it lifts it suddenly off the pivot and fixes it firmly against a glass plate immediately above. The clockwork can be set, by a time wheel, so as to fix the magnet after the lapse of any time up to one hour.

In Plate 9, Fig. 7 is a side elevation, and Figs. 8 and 9 are respectively vertical and horizontal longitudinal sections. Fig. 10 shows a general plan of the mechanism, and Fig. 11 a plan of the underside of the suspended box.

When the instrument is lowered into a current the principal axis of the instrument takes the direction of the current, while the magnet places itself in the magnetic meridian, and it is only necessary to secure the magnet in this position before drawing up the instrument. This is done by setting the clock to act upon the magnet at a known time, previous to lowering the instrument into the water, and it is sufficient to give the magnet four or five minutes to settle itself in the meridian before the lapse of this time. When the instrument is drawn up the magnet will be found fixed in position, showing at once the angle between the magnetic meridian and the direction of the current, or the magnetic

bearing of the current below. The direction of the compass is not affected by the rise and fall of the ship; this merely causes a small oscillation of the case in a vertical plane about the axis of its pivots, which axis is always horizontal, and this motion of the case does not affect the direction of the instrument. When the current indicator is lowered from a ship at anchor, or otherwise at rest, or from a ship moving with the same velocity as the surface current but in the opposite direction, so that it remains vertically above the same point of the earth's surface, the reading obtained from the magnet shows, directly, the magnetic bearing of the submarine current. Under the same circumstances the deep-sea current meter will determine the velocity of the same current. When, however, the ship is in motion with the surface current, or in any other direction, the indications both of the current meter and current indicator are affected by this motion; but when these instruments are used in combination they still suffice to determine the velocity and direction of the submarine current, provided the course of the ship and its speed are known.

It should be remarked that in no case will the instrument be in the vertical line through the point of support except the ship be at rest in motionless water. The horizontal motion of the ship will, however, be impressed upon the instrument suspended from it. Consider first the case of a ship at rest upon the surface of the water; let S , Fig. 1, be the position of the ship, SM the

FIG. 1.

magnetic meridian, and SC the direction of the submarine current; then the instrument will point in the opposite direction SC_1 , and the angle shown by it will be the angle MSC_1 , marked ϕ in the

diagram. If the current is in the opposite direction $S C_1$, Fig. 2, the instrument will point in the direction $S C$, and will register the angle $M S C$, marked ϕ_1 . Figs. 1 and 2 also show vertical ideal

FIG. 2.

sections along the line $C C_1$, $S O$ being a vertical from the ship S , and I the position of the current indicator, when the current runs in the direction $S C$, Fig. 1, and I_1 , its position when the current runs in the opposite direction $S C_1$, Fig. 2.

The current meter will, under the same circumstances, measure the velocity of the current in whichever direction it may run, and in the case of a ship being stationary no calculations are required.

When the ship is in motion the problem is more complicated. Before considering the general case it will be well to examine the simplest case, viz., that in which the motion of the ship is in the same direction as the submarine current, or in the opposite direction. Fig. 3 represents the case in which the ship and current

FIG. 3.

move in the same direction, the speed of the current being greater than that of the ship; and Fig. 4 that in which the speed of the current is less than that of the ship. Fig. 4 also represents the case

in which the ship and current move in opposite directions. In each figure SM represents the magnetic meridian, and SS_1 the motion of the ship in a given time.

The suspended instrument will travel through the same distance SS_1 in the same time, and it will point in the direction SC , if the velocity of the submarine current is less than that of the ship, and in the opposite direction SC_1 when the velocity of the submarine current is the greater. In the former case it will indicate the angle ϕ_1 , and in the latter the angle ϕ_2 , so that the current indicator alone will not determine in which of these two directions the current flows, unless it is known whether the speed of the ship is greater or less than that of the current.

But the current meter will supply this information directly; for, when the ship is moving in the same direction as the submarine current, the current meter registers the difference of their velocities, and when the ship and submarine current are moving in opposite directions the current meter registers the sum of their velocities. Thus all ambiguity will be removed by using the two instruments in combination; for, let V be the velocity of the ship and S that of the current, and conceive the whole mass of water to have a motion impressed upon it equal and opposite to that of the ship. The ship will then be at rest, and the instrument will be acted on by a current having a velocity $S - V$ when the ship and current move in the same direction, and a velocity $S + V$ when they move in opposite directions.

If then p be the reading of the current meter,

$$p = S - V \text{ in the former case,}$$

and

$$p = S + V \text{ in the latter.}$$

When S is greater than V , the angle shown by the current indicator will be the angle ϕ_2 , and when S is less than V the instrument will give the angle ϕ_1 , and thus the velocity and direction are completely determined, and

$$S = p \pm V,$$

+ or -, according as the ship and current move in the same or in opposite directions.

Consider lastly the general case :—

Let SM (Fig. 5) represent, as before, the magnetic meridian,

FIG. 5.

SS_1 the motion of the ship in a known time T , with a velocity of v miles per hour, in a direction making an angle α with the magnetic meridian.

Let SC represent the direction of the submarine current making an angle θ with this meridian, and suppose s the velocity of this

current in miles per hour, and SC the run of the current in the time T. As before, the ship and instrument may be supposed at rest, if the velocity v be impressed upon both in the direction $S_1 S$, or it be conceived that this velocity is applied to the whole mass of water in the same direction. Join CS_1 and draw SP parallel to it: then SP is the direction in which the current indicator will point while the ship moves from S to S_1 . For, the ship being reduced to rest, the instrument is subject to the action of the current which moves parallel to SC, through the distance SC in the time T, while the whole mass of water moves in the direction $S_1 S$ through the distance SS_2 (equal to $S_1 S$) in the same time T, consequently SO, the diagonal of the parallelogram constructed on the lines SC and SS_2 , is the effective direction of the moving water with respect to the instrument, and as this diagonal is in the line SP, the instrument will turn in that direction.

The same reasoning applies to the current meter, and therefore this instrument will measure the distance CS_1 , or SP, while the ship moves from S to S_1 .

The calculations for determining the direction and velocity of the current present no difficulty. Let p represent the distance SP, and ϕ the angle MSP.

$$\text{Then} \quad \frac{SC}{SS_1} = \frac{\sin S S_1 C}{\sin S C S_1},$$

$$\text{and} \quad \frac{CS_1}{SS_1} = \frac{\sin C S S_1}{\sin S C S_1},$$

$$\text{that is} \quad \frac{s}{v} = \frac{\sin(a - \phi)}{\sin(\theta - \phi)} \quad . \quad . \quad . \quad (1)$$

$$\text{and} \quad \frac{p}{v} = \frac{\sin(\theta - a)}{\sin(\theta - \phi)} \quad . \quad . \quad . \quad (2)$$

$$\begin{aligned} \text{From (2)} \quad \frac{\sin \theta \cdot \cos a - \cos \theta \cdot \sin a}{\sin \theta \cdot \cos \phi - \cos \theta \cdot \sin \phi} &= \frac{p}{v}, \\ \therefore \frac{\tan \theta \cdot \cos a - \sin a}{\tan \theta \cdot \cos \phi - \sin \phi} &= \frac{p}{v}, \\ \therefore \tan \theta &= \frac{v \sin a - p \sin \phi}{v \cos a - p \cos \phi} \quad . \quad . \quad (3) \end{aligned}$$

This last equation will determine θ when v , p , a and ϕ are known, and then s can be found from equation (1).

As an example, suppose a ship to sail due N.N.E. for 3 miles in

a known time, at a part of the earth's surface where the magnetic variation is 25° west, and that the current meter registers $2\frac{1}{2}$ miles in the same time, and the current indicator bears 18° to the east of the magnetic meridian.

Then $\alpha = 25^\circ + 22^\circ 30' = 47^\circ 30'$; $\phi = 18$;
 $v = 3$, and $p = 2.5$.

$$\begin{aligned}\text{Therefore } \tan \theta^\circ &= \frac{3 \sin 47^\circ 30' - 2.5 \sin 18^\circ}{3 \cos 47^\circ 30' - 2.5 \cos 18^\circ} \\ &= \frac{12 \times .7372773 - 10 \times .3090170}{12 \times .6755902 - 10 \times .9510565} \\ &= \frac{8.8473276 - 3.0901700}{8.1070824 - 9.5105650} \\ &= -\frac{5.7571576}{1.4034826} \\ \therefore \tan (180^\circ - \theta^\circ) &= -\tan \theta^\circ = \frac{5.7571576}{1.4034826}\end{aligned}$$

$$\therefore \log \tan (180^\circ - \theta^\circ) = 10 + \log 5.7571576 - \log 1.4034826$$

$$\begin{aligned}10 + \log 5.7571576 &= 10.7602083 \\ \log 1.4034826 &= 0.1472056\end{aligned}$$

$$\begin{aligned}\therefore \log \tan (180^\circ - \theta^\circ) &= 10.6130027 \\ \text{but } \log \tan 76^\circ 18' &= 10.6130131\end{aligned}$$

$$\begin{aligned}\therefore 180^\circ - \theta^\circ &= 76^\circ 18' \text{ omitting seconds,} \\ \therefore \theta^\circ &= 103^\circ 42' .\end{aligned}$$

To find the velocity of the current,

$$s = v \frac{\sin (\alpha - \phi)}{\sin (\theta - \phi)}$$

$$\therefore \log s = \log v + \log \sin (\alpha - \phi) - \log \sin (\theta - \phi)$$

$$\begin{aligned}v = 3, \alpha - \phi &= 47^\circ 30' - 18^\circ = 29^\circ 30' \\ \theta - \phi &= 103^\circ 42' - 18^\circ = 85^\circ 42'\end{aligned}$$

$$\begin{aligned}\log v &= \log 3 &= 0.4771213 \\ \log \sin (\alpha - \phi) &= \log \sin 29^\circ 30' &= 9.6923388\end{aligned}$$

$$\begin{aligned}&10.1694601 \\ \log \sin (\theta - \phi) &= \log \sin 85^\circ 42' = 9.9987758\end{aligned}$$

$$\begin{aligned}\therefore \log s &= 0.1706843 \\ \text{but } \log 1.4814 &= 0.1706723 \\ \therefore s &= 1.4814 .\end{aligned}$$

Thus the current runs with a velocity of 1.4814 mile per hour, and its bearing is $103^{\circ} 42'$ from the magnetic meridian, or $78^{\circ} 42'$ east of north.

The velocity and direction of the current may also be found by a simple geometrical construction as follows, giving the same results as the preceding calculation:—

In Fig. 6 lay down the meridian S N, the magnetic meridian

FIG. 6.

S M, and the ship's course S S₁, and draw S P in the direction pointed out by the current indicator; mark off S P for the distance measured by the current meter on the same scale on which S S₁ represents the run of the ship, and join P S₁. This line will represent the submarine current in magnitude and direction. Producing S₁ P to meet S M in Q, the angle P Q M shows the bearing of the current from the magnetic meridian.

The Paper is illustrated by drawings, from which Plates 8 and 9 and the woodcuts have been compiled.

APPENDICES.

1. FLOW OF THE RIVER THAMES, JAN. 1, 1876.

THE flow of the river Thames on the 1st of January, 1876, estimated from measurements taken below the railway bridge at Staines, was as follows:—

The transverse section of the river was taken at right angles to the main direction of the stream, and measurements were made of the velocity of the water, at the surface, bottom, and mean depth, at a number of equidistant stations in this transverse section. From these velocities and the corresponding depths, the areas of a series of parallel sections of the volume of water passing through the transverse section in a given time were obtained, and from these areas and the known dimensions of the transverse section the volume of water was computed. The parallel sections alluded to are parabolic in form, each being approximately a portion of a parabola, the axis of which is horizontal, near to and below the surface. The areas of these sections may therefore be readily calculated from the formula,

$$\text{Area} = \frac{d}{6} \{v_s + v_b + 4 v_m\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{A})$$

in which

d is the depth of the water,

v_s the velocity at the surface,

v_b " " bottom,

and

v " " mean depth.

When these areas have been computed, the volume of the mass of water passing through the transverse section, between the two most distant velocity-stations, can be found from the prismoidal formula.

$$\text{Volume} = \frac{h}{3} \left\{ \sum A \text{ (extreme)} + 2 \sum A \text{ (odd)} + 4 \sum A \text{ (even)} \right\}. \quad (\text{B})$$

in which h is the distance between any two consecutive stations,

ΣA (extreme) the sum of the areas of the extreme parallel sections,

ΣA (odd) the sum of the areas of the parallel sections which are of an odd order, omitting the extremes, and

ΣA (even) the sum of the areas of the parallel sections which are of an even order.

To the quantity so obtained must be added the small quantities of water passing between each of the two most distant velocity-stations and the nearer bank.

From the formula (A) the following table of areas has been calculated:—

Velocity Station.	Area in Square Feet in Twelve Seconds.
No. 1.	114·48
A	267·36
No. 2.	288·40
B	264·90
No. 3.	244·42
C	225·03
No. 4.	200·42
D	169·03
No. 5.	166·44
E	159·50
No. 6.	150·36
F	136·60
No. 7.	120·92
No. 8.	106·50

The volume of the water passing between the stations No. 1 and No. 7 may now be found by the formula (B):—

$$\begin{aligned}\text{For } \Sigma A \text{ (extreme)} &= \text{sum of areas at stations Nos. 1 and 7:} \\ &= 114\cdot48 + 120\cdot92 \\ &= 235\cdot40.\end{aligned}$$

$$\begin{aligned}\Sigma A \text{ (odd)} &= \text{sum of areas at stations Nos. 2, 3, 4, 5, and 6:} \\ &= 288\cdot40 + 244\cdot42 + 200\cdot42 + 166\cdot44 + 150\cdot36 \\ &= 1,050\cdot04.\end{aligned}$$

$$\begin{aligned}\Sigma A \text{ (even)} &= \text{sum of areas at stations A, B, C, D, E, and F:} \\ &= 267\cdot36 + 264\cdot90 + 225\cdot03 + 169\cdot03 + 159\cdot50 + 136\cdot60 \\ &= 1,222\cdot42\end{aligned}$$

$$\text{and } h = 12\cdot5 \text{ feet}$$

$$\begin{aligned}\therefore V &= \frac{12\cdot5}{3} \left\{ 235\cdot40 + 2 \times 1,050\cdot04 + 4 \times 1,222\cdot42 \right\} \\ &= 30,104\cdot75 \text{ cubic feet in twelve seconds.}\end{aligned}$$

Therefore the volume per minute is 150,524 cubic feet.

The mass of water between station No. 1 and the bank is approximately pyramidal in form, and its volume is $114\cdot48 \times \frac{7\cdot5}{3} = 286$ cubic feet.

The volume of the water between the stations No. 7 and No. 8 is $\frac{121 + 106}{2} \times 5 = 567$ cubic feet; and the volume of the end portion, between the station No. 8 and the bank, is approximately equal to the parabolic area inclosed by the velocity-curve at the mean depth multiplied by the depth, i.e. it is equal to $\frac{2}{3} \times 5\cdot5 \times 18\cdot3 \times 6 = 402$ cubic feet. Therefore the total quantity outside the stations Nos. 1 and 7 is $286 + 567 + 402 = 1,255$ cubic feet in twelve seconds, and, therefore, the quantity per minute is 6,275 cubic feet. Adding this quantity to the main volume already obtained, the total flow is 156,799 cubic feet per minute.

Cor.—The mean velocity of the water for the whole section may be found by dividing the total flow by the area of the transverse section.

This gives a mean velocity of 127·12 feet per minute.

2. MECHANICAL PRINCIPLES ON WHICH THE CURRENT METER IS CONSTRUCTED.

FIG. 1.

A B represents the axis of the instrument, A being the point of the ogival head, and B the extremity of the tail, and C its centre of gravity. B C is about three times the length of A C.

The forces which act upon the instrument are: W, its weight in water; R, the resistance which it offers to the stream; and P, the pull upon the stirrup, which inclines forward through some angle θ from the vertical.

These three forces, P, W, and R, are in equilibrium at the point C.

$$\therefore P \cos \theta = W, \text{ and } P \sin \theta = R.$$

The position of the axis A B is manifestly one of stable equilibrium when in the direction of the current; for if it be displaced about the point C, the action of the current will restore the axis to its former position. An increased velocity in the current increases R, and therefore both P and θ . This increase of θ can be prevented, when desired, by suspending a weight from the two ends of the stirrup.

Let W' be the effect of this weight in the vertical direction, then the former of the above equations becomes $W + W' = P \cos \theta$.

Thus θ may be made as small as desired by sufficiently increasing W'.

The Ship's Log is constructed on the same principles as the current meter.

FIG. 2.

Let A B represent its axis, G the centre of gravity, C P the direction of the pull of the log-line at C, W the weight of the log in water, and R the resistance of the water. Then $P \cos \theta - W$ is the lifting force upon the log, which will consequently rise through the water until this force vanishes, i.e. until $\theta = \cos^{-1} \frac{W}{P}$.

θ can never be so great as a right angle, and therefore this log can never rise to the surface in smooth water, and its tendency to rise in rough water is

reduced to a minimum. Again, $P \sin \theta - R$ is the effective tractive force in a horizontal direction, and $W \times CG$ is the moment tending to raise the point A and to give the axis of the log an inclination upwards. This moment can be made as small as desired by diminishing the distance CG , and therefore this log may be made to travel with its axis horizontal, and below the surface of the water.

Let AB represent the axis of a log constructed in the usual manner, with the log-line attached to its front A by a ring.

FIG. 3.

Let G be its centre of gravity, W its weight in water, and P the pull on the log-line, the direction of which at A makes an angle θ with the vertical.

In this case the moment $W \times AG$, tending to raise the point A , and give the log an inclination upwards, can never be made small; and a log so constructed can never travel with its axis horizontal. Its inclination upwards causes the water to strike against the under side of the log, which speedily brings it to the surface.

Scale $\frac{1}{3}$ Full Size.

CURRENT METER.

VERTICAL CURVES OF FLOW FROM VELOCITY STATIONS, IN 12 SECONDS, in planes perpendicular to the Transverse Section.

Scale. 30 40 50 Feet.

SHIPS' LOG.

Fig: 1.

LONGITUDINAL SECTION.

Fig: 2.

Fig: 3.

DEEP SEA CURRENT INDICATOR.

Fig: 7.

SIDE ELEVATION, SHEWING POSITION OF THE INSTRUMENT IN A STRONG CURRENT.

PROTECTED LOG AND DEEP SEA CURRENT METER.

Fig: 4.

LONGITUDINAL SECTION.

Fig: 6.

SELF ACTING LOCKING GEAR
for Deep Sea Current Meter.

Fig: 5.

FRONT ELEVATION.

Fig: 8.

VERTICAL SECTION OF CASE THROUGH LONGITUDINAL AXIS
Shewing mechanism in Suspended Box.

Fig: 9.

HORIZONTAL SECTION OF CASE WITH PLAN
of Suspended Box and Compass.

Fig: 10.

PLAN OF TRAIN OF WHEELS
in Lower Part of Suspended Box.

Fig: 11.

PLAN OF UNDERSIDE,
of Suspended Box.

Scale 1/3 Full Size.