500. Required, a Solution of the Following

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∴ the chance that the approximation $\pi = 3\frac{1}{2}$ will give an answer incorrect to 4 figures is

$$\begin{aligned} & \left\{ \int_{1000}^{\frac{\pi}{3\frac{1}{2}-\pi}} \left(\frac{3\frac{1}{2}c}{\pi} - c \right) B \frac{dc}{c} + \int_{\frac{\pi}{3\frac{1}{2}-\pi}}^{10000} B \frac{dc}{c} \right\} / \int_{1000}^{10000} B \frac{dc}{c} \\ &= \left\{ \left(\frac{3\frac{1}{2}}{\pi} - 1 \right) \left(\frac{\pi}{3\frac{1}{2}-\pi} - 1000 \right) + \log_e 10000 - \log_e \left(\frac{\pi}{3\frac{1}{2}-\pi} \right) \right\} / \log_e 10 \\ &= \left\{ 1 - 1000 \left(\frac{3\frac{1}{2}}{\pi} - 1 \right) \right\} \log_{10} e + 4 + \log_{10} \left(\frac{3\frac{1}{2}-\pi}{\pi} \right) = \cdot 864. \dots\dots\dots(3) \end{aligned}$$

This gives us the complete answer to the question, "Why should I give the answers to circle questions correct to 3 figures and not to 4?" "Because if you try for 3 figures, you will get them right more than 5 times out of 6: but if you try for a 4th figure, it will be wrong more than 6 times out of 7."

The result (3) may be generalised thus:

Defining p and P as before, $\frac{P \sim p}{p}$ being between 10^{-n} and 10^{-n-1} , the probability that the approximate answer is incorrect, if given to $n+1$ figures, is

$$\left\{ 1 - 10^n \left(\frac{P}{p} \sim 1 \right) \right\} \log_{10} e + (n+1) + \log_{10} \left(\frac{P}{p} \sim 1 \right). \dots\dots\dots(4)$$

The minimum value of this being $\cdot 3909$, the $(n+1)$ st figure is not, in general, trustworthy.

The following table is of value in deciding how many figures we should give up, or teach our pupils to give up, in answers obtained by using certain common approximations:

Approximation.	Probability of obtaining answer correct to		
	2	3	4 significant figures.
$3\frac{1}{2}$ for π	$\cdot 984$	$\cdot 843$	$\cdot 136$
$\sqrt{3\frac{1}{2}}$ for $\sqrt{\pi}$	$\cdot 992$	$\cdot 921$	$\cdot 349$
$6\frac{1}{2}$ gallons for 1 ft ³ .	$\cdot 888$	$\cdot 232$	0
$\frac{5}{8}$ mile for 1 Km.	$\cdot 772$	$\cdot 053$	0
32 for g (London)	$\cdot 768$	$\cdot 050$	0
$\sqrt{32}$ for $\sqrt{g}$ (London)	$\cdot 884$	$\cdot 223$	0

(To be continued.)

MATHEMATICAL NOTES.

500. [X. iv. a.] *Required, a Solution of the following:*

EW is a straight line, Pp is parallel to EW at a distance A ; Qq is parallel to Pp at a distance B ; Rr is parallel to Qq at a distance C . P, Q, R are any points on Pp, Qq, Rr .

EW represents a road running E and W . The three strips, which we may call A, B, C , are strips of cultivated land, the speeds across which are respectively $1/a, 1/b$, and $1/c$ of the speed on the road. Speeds equal to that on the road are possible along the three parallel tracks Pp, Qq, Rr .

At a given point on EW an order is received to reach the position P in the shortest time. What is the course? What would be the course were

the order to be given at a given point on Pp to reach Q , and at a given point on Qq to reach R ?

(2) What would be the course were the order received at a given point on JW to reach R ?

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501. [v. 1. a. 8.] "*Russian Peasant*" *Multiplication in Roman Numerals.*

A method of multiplication stated to be in common use among the peasantry of the villages in Russia has been brought into notice in a note recently published in America. The method involves only the operations of doubling, halving and adding, and the object of this note is to show that it can be applied equally well to numbers expressed in Roman numerals.

The method is illustrated by the following example; say 37×39 .

37	39
18	[78] omit
9	156
4	[312] omit
2	[624] omit
1	1248
Product 1443	

The numbers in the first column are got by repeatedly halving one factor, omitting remainders; those in the second column by repeatedly doubling the other factor. The numbers in the second column which stand opposite even numbers in the first are struck out, and the sum of those standing opposite odd numbers gives the required product.

It will be seen that the process of halving the numbers in the first column is exactly the same as would be required to express this factor in the binary scale of notation, only, instead of the remainders being written down, their positions are indicated by the odd numbers in the column. Thus $37 = 100101$ or $2^5 + 2^2 + 1$; and the numbers added in the second column represent 39 multiplied by 1, 2^2 , 2^5 respectively.

When applied to Roman numerals, the process stands thus, for example:

Multiply XLVI by LXIII.

XLVI	[LXIII] omit
XXIII	CXXVI
XI	CCLII
V	DIV
II	[MVIII] omit
I	MMXVI

MMDCCCXVIII

G. H. BRYAN.

502. [C. 1. e. g.] *Note on the Teaching of the Mean Value Theorem and its Extensions.*

Two of the greatest stumbling blocks to the ordinary student of the Differential Calculus are the Mean Value Theorem and Taylor's Theorem. The argument given in Lamb's *Calculus*, § 56, is easy to follow, but there is always difficulty in writing down the function in (2). I suggest that the idea of comparison of $y=f(x)$ with another curve, as in § 66, be used for the Mean Value Theorem. Not only does that course simplify the proof of the Theorem itself, but it forms an easy introduction to the work of §§ 66 and 67, often found difficult. Starting by comparing $y=f(x)$ with $y=Ax+B$, we have the proof of the Mean Value Theorem. Proceeding then to a comparison with $y=Ax^2+Bx+C$, we obtain the results of §§ 66 and 67. The student then finds it fairly easy to extend the method to the proof of Taylor's Theorem as follows: