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ON THE FREE VIBRATIONS OF A LECHER SYSTEM,
USING A LECHER OSCILLATOR. I.¹

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IT was Professor Rubens² who first quantitatively demonstrated the existence of the free vibrations of a Lecher³ system. Since then a great deal of work has been done on the subject of stationary waves in parallel wires, so much so in fact that it would seem that at best one could only repeat the work of others if he undertook a research along this line. Moreover, many theoretical papers⁴ have appeared on the subject and one has only to attempt to correlate the experimental and theoretical parts of the subject as exemplified in the literature to convince himself that there is something still to be desired in this field. But it was the work of Blake and Ruppertsberg⁵ that led us to repeat Rubens's experiments under the better working conditions now attainable. Their work had shown us that experimentally at least there ought to be found solutions of such vexing questions as these—(a) ought the bridge to be taken into account and to what extent?⁶ (b) are the currents in the Lecher system transverse, or longitudinal, or partly one and partly the

¹ Read before the American Physical Society at Chicago, November 26, 1910.

² Wied. Ann., 42, p. 154, 1891.

³ Wied. Ann., 41, p. 850, 1890.

⁴ Mention need be made of only three, viz., Cohn and Heerwagen, Wied. Ann., 43, p. 343, 1891; P. Drude, Wied. Ann., 61, p. 631, 1897; M. Abraham, "Theorie der Elektrizitat" (2d edition), Bd. 1, p. 347.

⁵ Blake and Ruppertsberg, PHYS. REV., Vol. XXXII., p. 445. 1911.

⁶ See for instance Rubens, *l. c.*, p. 161.

other?¹ Intimately related with each of these are two other more general questions, (*c*) are the various frequencies in a freely vibrating Lecher system related to each other? and (*d*) if so, what are the relative intensities of these harmonics? It was hoped that further experimentation on this old problem would help to answer these questions. To what extent this hope was justified will show further along in these papers. A discussion of the work of previous investigators is accordingly reserved until after our own experimental results are presented.

APPARATUS.

The oscillator was of the Lecher type connected through two air-gaps and two water-resistances to the induction coil after the manner of Blake and Ruppertsberg.² The coil used was a 10-inch coil attached directly to a 110-volt, 60-cycle A.C. lighting circuit, the condenser and interrupter being removed. As several different types of oscillators and primary circuits were used it seems best to describe them separately as the various phases of the work are touched upon. The bolometer used was the same as that employed by Blake and Ruppertsberg. However, these writers did not make use of a check receiver and while their results cannot be materially in error on that account, nevertheless it was evident to us as we witnessed their work that the introduction of a check receiver would be a great improvement if one were to try for the greatest accuracy. Such a receiver was accordingly employed and of course two galvanometers were used. The one was a duBois-Rubens iron-clad, the other a Broca instrument. Their sensitiveness was 4.07×10^{-9} and 6.45×10^{-8} respectively for a period of 4 seconds. After being once set up it was not necessary to adjust them throughout the work.

EFFECT OF VARYING THE DISTANCE BETWEEN THE LECHER WIRES.

Employing a check receiver, we tried at first to get the free oscillations of the parallel system. Blake and Ruppertsberg had found that the overtones of the free (?) system were not harmonic and after our results became known to them it appeared that the

¹ See Lecher, *l. c.*, p. 854; also J. J. Thomson, "Recent Researches," p. 466.

² Blake and Ruppertsberg, *l. c.*

reason for this was because of the closeness of the coupling employed by them. Accordingly, we started with what we deemed a fairly loose coupling and hoped thereby to get the ideal free vibra-

Fig. 1.

tions shown in Fig. 1. For this the oscillator consisted of two brass balls each $\frac{3}{8}$ in. in diameter, bored out and filled in with steel

Fig. 1a.

cylinders tapered off to a truncated cone¹ at the spark-gap in oil. Soldered to these cylinders were copper wires attached to primary tin plates 5 cm. square. This made the total length of the oscillator

¹ This was the oscillator used by Blake, *Phys. Rev.*, Vol. XXX., p. 678, 1910.

circuit up to the plates 28 cm. Opposite these plates, at a distance of 2 cm., stood the two secondary plates each 3 cm. square. The whole arrangement of the oscillator and Lecher circuits is shown in Fig. 2. The air-gaps were each 7 mm. long. The check receiver

Fig. 2.

was a thermal couple of iron and constantan (diameter of wire 0.0208 mm.). It proved to be quite a problem properly to locate the check receiver with respect to the rest of the apparatus so as to accomplish the sole purpose of correcting the oscillator deterioration. Trial revealed the fact that it was readily influenced by stray radiation when only a short distance from the oscillator, so it was symmetrically placed about 50 cm. from the oscillator and two long equal lead wires led from it to small capacity plates each 1 cm. square placed 3 cm. away from the primary plates of the oscillator though parallel to them, as shown in the figure. The total length of the check receiver circuit was thus 3 meters. With this arrangement a Lecher curve was taken by moving a single bridge along the parallel wires. Plotting the check receiver readings against the bridge position showed a curve of sinusoidal nature and it became at once apparent that the check receiver was influenced by what might be called the mutual interaction of the oscillator and Lecher circuits, or better the back-action of the Lecher circuit upon the oscillator. This *back-action* will be spoken of again. A test experiment showed that this mutual interaction was present even though the check receiver was metallically protected from any stray radiation effects. The back-action upon the check receiver plates, CC, of Fig. 2 must have come from the secondary plates SS through

the primary plates *PP*. As we could not use the check receiver readings the curve was plotted with the oscillator deterioration uncorrected for and is shown in Fig. 3.

Fig. 3.

When it is remembered from Fig. 2 that the detecting system is at the free ends of the Lecher wires it is plain that the free vibrations shown in Fig. 1 will appear as Fig. 1*a* when properly interpreted. After comparing Fig. 3 with Fig. 1*a* no one will question the statement that we have in Fig. 3 a first approximation to the ideal case depicted in Figs. 1 and 1*a*. The following calculation shows, however, that Fig. 3 will not in all parts bear close theoretical scrutiny. The zero end of the 2-meter stick was placed vertically below the last bridge position obtainable on the Lecher wires at the bolometer end. Beyond this the wooden supports for the bridge and for the means of keeping the wires under tension occupied a linear space of 1.8 cm. The Leyden jars were themselves 1 cm. long and the small disc with its protruding knob of solder added 2 mm. The actual end of the wire was thus at - 3 cm. Just as Blake and Ruppertsberg found a decidedly better agreement between theory and experiment by considering that the wave is reflected only after it reaches *the end of the wire*, so do we find this true also. That this interpretation is justified will appear from the work using cylindrical capacities reported later in this work. Table I. shows that the mean fundamental wave-length λ_0 is 434

cm., of which one fourth is 108.5 cm. From the end of the wire to the bridge reading for the fundamental tone is 105.0 cm. giving as the equivalent value of the end-capacity 3.5 cm. of wire. The rest

TABLE I.

Distribution of Energy. [Plate Capacities.] *Potential Difference* = $\Phi_1 - \Phi_2 = 2 \frac{|A|}{K} \cos \left(\frac{vx}{w} - \gamma \right) \cdot \cos(vt) \propto \sin \left[\frac{\pi}{2} \left(\frac{1}{n+1} \right) - 2\gamma \right]$. *Phase Change* = 2γ (where $\tan \gamma = \frac{v}{w} \cdot \frac{K_0}{K} = \frac{2\pi K_0}{\lambda K}$). (n = no. of overtone.)

$\frac{K_0}{K}$ (cms.).	l cms.	λ cms.	2γ	Tone.	Intensity.		
					Theory (Per Cent.).	Experi- mental (Per Cent.).	Difference.
3.5	105	434	5° 47' 54"	Fund.	100	100	—
				1st har.	62.7	62.7	0.00
				2d har.	41.0	49.0	+8.00
				3d har.	29.0	31.7	+2.70

of the calculation is collected into the table and it is seen that the agreement between theory and experiment is very satisfactory except for the second harmonic where the difference is almost 8 per cent. It will be shown later that undoubtedly an oscillator vibration swings in here *along with* the free second harmonic, thereby increasing its intensity.

Other curves were taken for distances between the wires as follows: 1 cm., 3 cm., 5 cm., 11.5 cm. Two of these are shown in Fig. 4. Curve *I* is for 3 cm., Curve *II* for 11.5 cm. For each of these curves the oscillator deterioration was corrected for by employing the check receiver readings in the following manner. Plotting the check receiver readings against the bridge position the curves of Fig. 5 were obtained. For 11.5 cm. distance between the parallel wires the sinusoidal nature of the curve was clearly shown, the oscillator evidently being in first class condition. Although the oscillator for the 3 cm. curve was not in such good condition, the curve shown cannot be much in error. The beginning of the 1 cm. curve is also shown, the oscillator being in unusually good condition, though the last half of the curve (not plotted) shows that the oscillator "went badly to pieces." The figures on

the right of the 3 cm. curve for the bridge position 10 cm. represent check readings at that position taken immediately after the bridge position indicated by the number written opposite the points. For

Fig. 4.

Fig. 5.

all the curves shown in Fig. 5 the bridge was moved toward the oscillator and the downward slope of the axes of the curves, as well as the 10-cm. check readings of the 3-cm. curve, shows the need of correcting for the oscillator deterioration. Since the check receiver

readings are themselves influenced by the bridge position a little thought will convince the reader that the oscillator deterioration is properly corrected for by multiplying the bolometer reading for a given bridge position by the ordinate for the sine curve of Fig. 5 and dividing by the ordinate for the axis of such curve. To show the order of magnitude of this correction it should be stated that it served to even up the intensities of any particular harmonic at the two ends of the Lecher system but in no case did it displace a maximum more than 3 mm.

It is seen from the curves of Fig. 4 that for increasing distance between the Lecher wires the fundamental maximum broadens out and the minima are less prominent. If one plots the ratio of the

Fig. 6.

fundamental maximum to its own minimum (at 85 cm. say) for the different distances between the wires the curve shown in Fig. 6 is obtained. This curve shows a maximum at 3.4 cm. It would appear that, starting with this distance, as one decreases the distance between the wires the magnetic fields surrounding the two parallel wires nullify each other more and more and hence

decrease the ratio of maximum to minimum. For zero distance between the wires one should expect this ratio to become unity and we have so extrapolated it in the curve. On the other hand, if we increase the distance between the Lecher wires it is plain that there must be an upper limit beyond which any further increase will begin to lessen the above ratio. For, theoretically, the free vibrations are essentially *longitudinal* vibrations such that the bridge serves only the purpose of enabling us to detect the vibrations without distorting them as to intensity or position. It is to be noticed that in Curve II of Fig. 4 (for 11.5 cm.) the system of vibrations is not the regular symmetrical system that it is for the lesser distances. It will be shown later (as indeed the curve for 5 cm. distance with the arrange-

ment of Fig. 2 shows when plotted) that up to a distance of 5 cm. at least with a Lecher wire length of 175 cm. or more the presence of the bridge introduces no disturbing factor. That is, so long as the bridge length is not more than 3 per cent. of the Lecher length we may safely conclude that, although there is *necessarily always* current in the bridge wire whatever its position the two ends of the bridge wire may be said to have the same phase and hence the phase relations at the bolometer end of the Lecher system are the simple ones the theory demands. For a distance as great as 11.5 cm. this is not the case, as Curve *II*, Fig. 4, plainly shows. An inspection of this curve reveals an oscillator vibration at 9, 76.5 and 144 cm., giving 67.5 cm. for $\lambda/2$. Now it seems to us that the distance between two adjacent maxima of Fig. 5, viz., 45 cm., is the half wave-length of *some* vibration of the oscillator. These two oscillator vibrations of 45 cm. and 67.5 cm. half wave-length bear the ratio 1.50. In the light of the work of Blake and Ruppertsberg and of evidence still to be spoken of we think these represent the two fundamental vibrations of the oscillator, due to the comparatively close coupling. This makes the true oscillator wave-length 115.8 cm. Now the distance *NON* (Fig. 2) was 28 cm. When one allows for the plates *PP* and for the balls at the spark-gap it is plain that the oscillator half wave-length could easily be 57.4 cm.

An attempt was made from the curves of Figs. 3 and 4 to test out the law that the potential difference between the Lecher wires is proportional to the logarithm of the ratio of the distance between the wires to their radius, but not enough attention had been paid to the equality of the other conditions to make this possible. To insure success in this effort one must have absolute control over the oil-gap and this we did not have at the time these curves were taken.

MODIFIED LECHER SYSTEM.

In the course of the work referred to above it became plain to us that two features of the experimental arrangement needed improvement. First, it was plain that the check receiver must be located so as to be free from the back-action of the Lecher system. And secondly, it became very desirable to have the degree of coupling readily alterable. Both of these conditions called for a modi-

fication of the apparatus. As to the first, it was clear that if one were able to free the check receiver from the back-action it would certainly be accomplished if at all by placing the check receiver as a high resistance shunt directly across the spark-gap in oil. Accordingly this was done and it proved at once the solution of the difficulty. Small platinum plates 1 cm. in diameter were arranged as electrodes of a high resistance, one on each side of the thermal couple. After considerable experimentation the length of each olive oil resistance was chosen as best at 0.5 cm. The notion was to steal as little energy from the oscillator circuit as possible for checking purposes and at the same time not to alter its mode of vibration or wave-length.

For changing the coupling readily the experimental arrangement shown in Fig. 7 was finally chosen. Cylindrical capacities 15 cm.

Fig. 7.

long by 3.5 cm. in diameter were attached both to the oscillator circuit and to the bolometer circuit. In this way the Lecher wires themselves were as free as possible at both ends. They were held in place at a distance of 5 cm. apart by supports of soft wood placed at HH' of the figure. To H' a tension of 4 kilograms was attached after the manner already described.

The degree of coupling was changed by changing the distance d to which the Lecher wires extended into the oscillator cylinders VV . Curves were taken with the Lecher wires protruding various distances into VV and BB but they served no useful purpose except to point out the best way of experimentation; for the type of curve materially altered as the distances d and d' increased much beyond 5 cm. Before describing further curves it seems well to illustrate the checking power of the check receiver and to show that in its new position it is uninfluenced by the location of the bridge. Fig. 8 demonstrates the latter point clearly enough, the check receiver readings being plotted against the bridge position for three different curves

chosen at random. Although the need for a check receiver is clearly seen one cannot possibly detect any sine nature of the curves such as Fig. 5 shows. The method of checking a bolometer reading was as follows. A preliminary experiment showed the location of the fundamental maximum for a given set of conditions. With the bridge at the position of this maximum and with galvanometer shunts adjusted for large deflections a set of from ten to twenty readings both of the bolometer and of the check receiver was taken. These readings were plotted against each other with the readings of the check receiver as abscissæ; a straight line was drawn

Fig. 8.

through the points thus obtained. Then a regular curve was taken, the bridge being moved along the wires, consisting usually of about as many points as are shown in any of the curves of Fig. 8, both galvanometers being read of course. This usually took from one and a half to three hours. Then another set of ten or more readings was taken with the bridge again at the fundamental maximum. On account of temperature changes producing a slow unbalancing of the bolometer (shown by the wandering of the zero of the galvanometer) the straight line obtained from this last set of readings never coincided with the straight line obtained at the beginning of the curve. It usually lay several per cent. above it, though parallel to it. Trial had shown us that taking the ratio between the two galvanometer readings for any given bridge position would not be accurate enough, although better than the bolometer readings alone (see column 4, Table II.). If, for two successive readings with the bridge position constant the spark passed over a longer distance one time than the other, the ratio of the galvanometer readings should not necessarily be wholly constant. By increasing and decreasing the spark gap length we observed that the check re-

ceiver was always more strongly affected than the bolometer. If, however, we divided the bolometer reading for any given bridge position by its reading for the bridge at the position of the fundamental maximum for the same check receiver reading (making use of the straight line referred to above) we eliminated this error. The slow unbalancing of the bolometer was corrected for as follows. We added the per cent. of difference " x " between the two straight lines referred to above to 100 per cent. and plotted the two points 100, $100 + x$, with the mean zero of the bolometer galvanometer in each set as corresponding abscissa; these two points were joined by a straight line. For any given bridge position then we can get the

TABLE II.

Column 1.	Column 2.	Column 3.	Column 4.	Column 5.
Bridge Position.	Bolometer Reading.	Check Receiver Reading.	Ratio.	Relative Energy.
76	475	370	1.283	0.976
	449	341	1.318	0.972
79	201	287	0.700	0.485
	221	346	0.639	0.474
118	71	273	0.260	0.176
	77.5	320	0.242	0.175
149	32.8	380	0.086	0.066
	29.1	328	0.089	0.065
10	69	220	0.314	0.226
	75	260	0.289	0.224
88	51	240	0.212	0.158
	55	280	0.196	0.158
125	151	297	0.508	0.423
	135	248	0.554	0.423
149	111	232	0.478	0.351
	108	222	0.487	0.351

final correction factor by reading off the proper ordinates on this line. In this way we eliminate both oscillator deterioration and bolometer unbalancing entirely.¹ How nearly we have succeeded

¹ This bolometer unbalancing seems to be due to several things; among others may be mentioned direct radiation from the bodies of the observers and from the electric lamps used for illuminating the galvanometer-scales and the heat that comes from the electrical oscillations which on account of the cotton-wool packing immediately around the instrument cannot be readily equalized. Moreover, it is, of course, a function of the resistance in the auxiliary battery circuit and of the value of the galvanometer shunt.

in doing this is shown in column 5 of Table II., where the relative energy was obtained in the manner just indicated. The table contains random duplications for two different curves. The great superiority of column 5 over column 4 or column 2 (unchecked) will readily be admitted. So far as the oscillator is concerned we say without hesitation that we are able, by the employment of a check receiver situated as a shunt of high resistance across the spark gap, to eliminate all errors except an occasional error due to a mistake in reading a large deflection on a quick swing. A glance at Table II. will convince the reader that in general it is possible to duplicate any reading to a degree well within one per cent. Even for a badly worn oscillator one can duplicate to within two per cent., though it pays to repolish the oscillator occasionally. The results of Table II. and the curves yet to be spoken of in this paper were taken with the oscillator shown in Fig. 7 where the spark passed between two cylindrical hard steel rods each 5 cm. long and 3 mm. in diameter. At first we tried aluminium rods, after the manner of Schaefer and Laugwitz.¹ They worked better than the steel rods for the first few readings (perhaps twenty) but beyond that their action became more and more irregular and capricious and so they had to be abandoned. For the steel oscillator, on the other hand, it was no uncommon thing to take as many as six curves (say 600 readings) without repolishing. The oscillator was so constructed that the spark gap could readily be polished and cleaned by simply removing the rods from their ebonite support.

EFFECT OF CHANGING THE COUPLING.

Using the steel oscillator and applying both corrections referred to above, the effect of changing the coupling between the oscillator and Lecher circuits was studied. To preserve the fundamental maximum fixed at a certain bridge position any change in the coupling was made at *both* ends of the Lecher system, though for some reasons it would have been better not to have changed the bolometer ends of the wires. Curves were taken for the following couplings, expressed as the distance in centimeters the Lecher wires

¹Schaefer and Laugwitz, *Ann. d. Phys.*, 23, p. 953, 1907. See also Laugwitz, *Phys. Zeit.*, 8, p. 378, 1907.

were inserted into the cylindrical capacities at both ends: + 5 cm., + 3 cm., + 2 cm., + 1 cm., 0 cm., - 1 cm., - 2 cm., - 3 cm. For the latter distance the coupling was so loose that the energy detected by the bolometer was very small, too small to make the results quite as reliable as for the other distances. Three of these curves are shown in Fig. 9, viz., + 2 cm., 0 cm., - 2 cm. One sees at a glance

Fig. 9.

in contradistinction to Fig. 3, the practical absence of all third harmonics. The maxima are all drawn as sharp peaks in the figure, whereas in fact they are all rounding as shown in Fig. 10, where for a given case the fundamental maximum and the first harmonics are drawn to a larger scale. One sees at once from Fig. 10 that it makes no difference whether one chooses the peaks or the true maxima when comparing the various tones as to their relative intensities.¹ It should be noted also from the figure that the center

¹For this reason we drew the curves of Fig. 9 as peaks for it was easier in this way to locate the maxima accurately. In fact for the data collected together in Tables III. and IV. each maximum was carefully drawn to a larger scale after all the corrections had been applied and, following Rubens, the center of gravity lines were drawn for locating the exact position of a maximum. When the center of gravity line was not vertical the position of the maximum was taken as the point of intersection of the actual curve with this center of gravity line drawn from the peak downwards. In this way any maximum is located accurately to within 1 mm. in general, though in certain cases we do not claim closer than a 2 mm. accuracy.

of gravity line of each maximum is vertical. It will be shown later that when this is not the case it reveals two maxima in one, the extra one being due to the oscillator.

If we study Fig. 9 we note the following points. Figured from the fundamental at 76.4 cm. the semi-internodal spaces for the first harmonics are greater on the left than on the right of the figure, the disparity growing less as the coupling grows looser. The same

Fig. 10.

is true for the internodal distances for the second harmonics, only to a greater extent. This is more clearly seen in the two lower curves of Fig. 11, in which the data for several curves not shown in Fig. 9 are included. We see from these curves that the lack of symmetry in the internodal spaces disappears for the looser couplings, for practical purposes vanishing at the coupling — 1 cm. We see, moreover, from Fig. 9 that the two first harmonics decrease, but remain equal to each other as the coupling is changed, a thing they should do. For the second harmonics we notice that as the coupling gets looser, the two second harmonics (omitting the central second which is concealed in the fundamental and hence cannot be argued about) increase in intensity, but the one nearer the bolometer end is always the larger; for the looser couplings it overtops the first harmonics. If we examine closely the second harmonic near

the oscillator end we notice that it is much broader at the base in proportion to the other maxima than it ought to be. Moreover, its center of gravity line is usually distinctly out of the vertical.

Fig. 11.

This shows two maxima run together into one and we found by experiment in certain cases that the true position of either one could not be determined accurately. This gives us a clue also to the very high value of the second harmonic near the bolometer end as the coupling gets looser. The fundamental vibration of the oscillator falls in more fully with this second harmonic and hence greatly increases its intensity. However, the oscillator vibration is still somewhat longer than this free second, for the slope of the fundamental on the oscillator side is less steep than on the bolometer side, while both the oscillator swing and the free second appear, as has been said, in the broad maximum near the oscillator end, the former lying closer to the oscillator by reason of its greater length. For this reason the internodal spaces for the seconds are even more unsymmetrical than for the firsts as Fig. 11 plainly shows. Moreover, the presence of two maxima in one makes it harder to determine the actual top of the double maximum, hence in Fig. 11 the points for the second harmonics lie further from the smooth

curve than for the firsts. For the latter it is seen that the greatest deviation of any point from the curve is 1 mm.

Another thing should be said about Fig. 9. As the coupling became looser we saw that the first harmonics were pulled down in intensity while the seconds were pulled up. This is undoubtedly partly due to this oscillator vibration. That this is so is shown clearly in Curve I, Fig. 12, where for a longer Lecher system than

Fig. 12.

those illustrated in Fig. 9 the coupling was as follows: vibrator end -1 cm., bolometer end $+3$ cm. Here the oscillator vibration was in exact unison with the free second harmonic, hence both of these harmonics are very high. The first harmonics at 33 and 125 cm. are correspondingly low. Without changing the actual length of the Lecher wires we then changed the coupling to -1 cm. at the bolometer end leaving the coupling at the oscillator end as it was. In this way, since the effective length of the parallel wires was lessened, it ought to be possible to throw the free second out of tune with the oscillator vibration and hence they ought to stand out separately. That this was accomplished is shown in Curve II, Fig. 12. The oscillator swing comes in at 11, 73 and 135.5 cm.

with an internodal space of 62 cm. The free second appears at 18.5, 77 and 135.5 cm. giving an internodal space of 58.5 cm. Both vibrations occur together at 135.5 cm. and make the maximum very high (one and a half times the fundamental). For Curve *I* both oscillations came in at 18.2, 79.5 and 140.8 cm. with an internodal space of 61.3 cm., in substantial agreement with the oscillator internodal space of Curve *II*.¹

That the oscillator vibration can be made to swing in with some other harmonic is shown in Curve *III* of Fig. 12. This curve was taken earlier with a different oscillator of somewhat greater length, though the same cylindrical capacities were used as for the other curves. The coupling was + 1 cm. at both ends. In this curve there are two fundamental oscillations present, one at 27.8 and 122.3 cm., the other at 59 and 132.5 cm.,² the first one falling in with the free first harmonic to make it of abnormal intensity. Here we notice that when the firsts are very high the seconds at 15.4 and 137 are abnormally low.³

Referring again to Fig. 11 we saw that as the coupling at both ends got looser and looser the internodal spaces on the two sides of the fundamental became more nearly equal until for a very loose coupling (— 1 cm. or less say) the equality is exact. If from the two lower curves of Fig. 11 we were to calculate the fundamental wave-length, we would obtain curves entirely similar in nature to the curves of Fig. 6 of Blake and Ruppertsberg. It is accordingly

¹ In all the curves of Fig. 12 no corrections were applied. The bolometer readings, reduced to 100 for the fundamental, alone are plotted. Hence the maxima are not located with great accuracy.

² These internodal spaces of 73.5 cm. and 94.5 cm. for the oscillator vibrations make the half wave-length of the oscillator when isolated 84.6 cm. These figures are checked thus; adding 47.2 to 122.3 gives as the end of the wire including the end capacity 169.5 cm.; subtracting 36.8 from 169.5 gives 132.7 cm., in agreement with 132.5 cm. Now the length of this oscillator up to the cylindrical capacities was 35.6 cm. This makes the equivalent length of wire for each cylindrical capacity 24.5 cm. or roughly 1.5 times its natural length.

³ We are not entirely clear why apparently the intensifying of one harmonic by the oscillator vibration should lower the intensity of a neighboring harmonic. The fact that merely changing the coupling at the bolometer end from + 3 cm. to — 1 cm. changed Curve *I*, Fig. 12, into Curve *II* of the same figure, seems to point to the fact that the nature of this coupling plays an important part in determining the relative intensities of neighboring harmonics.

plain that the closeness of the coupling in their work was the cause of their overtones not being harmonic.

A glance at Fig. 9 shows that as the coupling gets looser the ratio of maximum to minimum decreases. However, for some of the curves not shown (*viz.*, + 3 cm., + 5 cm.) as the coupling gets closer this ratio also decreases. All the data are collected together into Curve *A* of Fig. 11. We see that this ratio attains a maximum for the coupling + 1.4 cm. Its value is 40 to 1. This is roughly four times as good as Rubens got and one third as good as the value obtained by Blake and Ruppertsberg.

Curve *B* of Fig. 11 was obtained by carefully comparing the bolometer readings with no bridge on the Lecher system against those with a single bridge placed at the fundamental maximum, the coupling being changed symmetrically at both ends. For close coupling there is more energy detected at the bolometer for no bridge on the parallel wires than for a bridge placed at the fundamental maximum and the closer the coupling the greater this energy. For couplings less than 0 cm. the reverse is true. It is interesting to note that for this particular coupling of 0 cm. the readings with and without a bridge are identical. It seems, perhaps, unprofitable at this time to speculate as to the meaning of this curve *B*. One might expect that all the tones ought to be present on the parallel wires when there is no bridge on them. If this were true the reading for no bridge ought to be roughly 2.3 times the reading for a bridge at the fundamental maximum. It should be borne in mind, however, that the free vibrations are only maintained in the Lecher system (so far as our detecting devices register) by a steady in-pour of energy at the oscillator end of the system. Very likely this plays a part that must yet be taken into account. In this respect the case is different from that of pulling a pendulum back and then letting go.

COMPARISON OF THEORY WITH EXPERIMENT.

An attempt was made, from the curves of Fig. 9 and from the five other curves not there shown but taken with equal care, to compare the results with the theory outlined by Kirchhoff and fully developed by Abraham.¹ How well the attempt succeeded is shown

¹ Abraham, *Theorie der Elektrizitat*, Vol. 1, 2d ed., page 347.

TABLE III.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Coupling cms. Insert of Wires.	Tone.	Position on Wire.	Actual End of Lecher Wire.	End of Wire In- cluding K_1K_2 .	K_1K_2 (cms.).	l	Method for λ .	λ	$a = K_1l/K_0$	$a = \xi \tan \xi$	Per Cent. Variation.	$\xi = \pi \frac{\lambda}{l}$	γ ($\tan \gamma = 2\pi K_0 \lambda K_1$)	$\xi + \gamma = \pi/2$.	Total Actual Length of Wire, l_0 .	Capacity at Oscillator End.	$\lambda_0 =$ wave-length
+5	Fund.	76.3	168.4	174.2	5.8	92.1	A	391.8	15.88	15.63	1.6	84° 36'	5° 19'	89° 54'	181.3	8.8	2.16
	1st har.	125.4				43.0	A	196.4	74.20	69.32	6.0	78° 46' 54"	10° 29'	88° 47'			
	1st har.	25.0				143.4	B	205.2	24.72	13.15	47.0	71° 32' 24"	10° 4'	81° 36'			
	2d har.	141.6				26.8	B	130.6	4.64	4.46	4.0	73° 51' 15"	15° 36' 30"	90° 17'			
	2d har.	4.2				164.2	B	144.2	28.31	12.64	50.0	60° 30'	11° 58'	72° 28'			
+3	Fund.	76.6	166.4	171.2	4.8	89.8	A	378.3	18.71	18.66	1.5	85° 25' 48"	4° 33' 30"	90° 0'	177.4	6.8	2.14
	1st har.	124.1				42.3	A	190.0	8.82	7.92	11.0	79° 56'	9° 0' 30"	88° 57'			
	1st har.	27.8				138.6	B	195.2	28.87	17.66	40.0	75° 48' 36"	8° 47' 30"	82° 37'			
	2d har.	139.6				26.8	A	126.0	5.58	5.58	0.0	76° 32' 50"	13° 29' 20"	90° 1'			
	2d har.	9.7				156.7	B	133.8	32.65	12.98	62.0	60° 31' 12"	12° 22' 30"	72° 45'			
+2	Fund.	76.4	165.4	170.2	4.8	89.0	A	374.4	18.54	19.20	-3.5	85° 34' 40"	4° 13' 30"	89° 48'	175.5	6.4	2.14
	1st har.	28.7				136.7	C	189.6	28.48	24.33	14.0	79° 28' 20"	9° 2' 20"	88° 31'			
	1st har.	28.7				136.7	B	190.8	28.48	20.64	24+	77° 48'	11° 12' 20"	89° 0'			
	1st har.	123.5				41.9	D	186.4	8.72	8.81	-1.1	80° 54'	9° 12' 40"	90° 7'			
	2d har.	11.0				154.4	B	130.4	37.16	16.82	55.0	66° 9'	13° 1' 20"	79° 22'			
+1	2d har.	76.4				89.0	A	124.8	18.54	18.94	2.5	76° 42'	13° 35'	90° 17'			
	2d har.	138.8				26.6	A	124.8	5.54	5.65	2.0	76° 42' 30"	13° 35'	90° 17'			
	Fund.	76.4	164.4	168.5	4.1	88.0	A	368.8	21.47	21.26	1.0	85° 57' 40"	4° 0'	89° 58'	173.6	6.8	2.12
	1st har.	29.9				134.5	C	185.2	32.80	30.04	9.0	81° 21' 24"	7° 51'	89° 15'			
	2d har.	76.4				88.0	A	122.8	21.47	20.94	2.5	77° 52'	11° 52'	89° 42'			

0	Fund.	76.45	163.4	166.5	3.10	87+	A	350.4	28.06	27.79	1.4	86° 53'	3° 5' 40"	89° 59'	171.6	5.5	2.10
	1st har.	31.45				132.0	A	180.1	42.58	42.44	0.3	83° 48' 10"	6° 10' 20"	89° 58'			
	1st har.	31.45				132.0	C	180.0	42.58	43.11	-1.3	83° 54'	6° 10' 30"	90° 4'			
	1st har.	121.5				41.9	D	180.2	13.51	13.42	0.6	83° 46' 48"	6° 10' 30"	89° 58'			
	2d har.	15.4				148.0	B	122.0	47.74	31.99	32+	76° 36'	9° 4' 10"	85° 40'			
	2d har.	76.45				87+	A	120.1	28.06	28.00	0.25	80° 46'	9° 12' 20"	89° 58'			
	2d har.	136.50				26.9	D	120.1	8.67	8.49	2.1	80° 36'	9° 12' 40"	89° 48'			
-1	Fund.	76.4	162.4	165.6	3.20	86.3	A	356.8	26.25	26.15	0.4	86° 41'	3° 13' 30"	89° 56'	169.5	5.7	2.10
	1st har.	31.8				130.6	B	178.4	40.75	40.40	0.85	83° 28'	6° 25' 40"	89° 54'			
	2d har.	76.4				86.3	A	118.8	26.25	26.64	-1.4	80° 20'	9° 36' 20"	89° 57'			
	2d har.	16.8				145.6	B	119.0	45.50	45.10	0.9	80° 21'	9° 35' 20"	89° 57'			
-2	Fund.	67.4	162.4	164.9	2.5	86.0	A	354.0	34.40	34.42	-0.01	87° 27' 45"	2° 32' 30"	90° 10'	169.5	5.0	2.09
	1st har.	32.2				130.2	B	176.8	52.08	51.93	0.3	84° 54' 44"	5° 4' 20"	89° 59'			
	1st har.	120.5				41.9	C	176.6	18.38	18.58	-1.1	85° 24' 54"	5° 3'	90° 27'			
	2d har.	17.4				145.0	B	118.0	58.02	58.08	-0.1	80° 26'	7° 34'	90° 0'			
	2d har.	135.4				27.0	D	118.0	10.80	10.74	0.6	82° 22' 40"	7° 35'	89° 58'			
-3	Fund.	76.1	161.4	163.8	2.4	85.3	A	350.8	35.54	35.48	0.02	87° 32' 6"	2° 27' 40"	90° 0'	168.5	4.5	2.08
	1st har.	32.3				129.1	B	175.4	53.79	52.89	1.8	84° 59' 40"	4° 54' 50"	89° 55'			
	1st har.	120.0				41.4	C	175.6	17.20	16.60	3.6	84° 56'	4° 54' 30"	89° 50'			
	2d har.	17.7				143.7	B	116.8	61.60	59.90	3.0	82° 51'	7° 21' 20"	90° 11'			
	2d har.	134.6				26.8	D	117.0	11.16	10.90	3.5	82° 26'	7° 41'	90° 7'			

A = wave-length calculated from the interval between the fundamental and the harmonic at *bolometer* end.B = wave-length calculated from the interval between the fundamental and the harmonic at *oscillator* end.C = wave-length calculated from the *internodal* space for a given harmonic.D = wave-length calculated from the *quarter wave* distance of the harmonic from the corrected end of Lecher wire.

in Tables III. and IV. Both tables are self-explanatory. In order to see whether the maxima of the curves fall at their theoretical positions columns 10 and 11 of Table III. are to be compared, the percentage variation being given in column 12. It is seen from the

TABLE IV.

Distribution of Energy. [Cylindrical Capacities.] *Potential Difference* = $\Phi_1 - \Phi_2 = 2 \frac{|A|}{K} \cos\left(\frac{vx}{w} - \gamma\right) \cdot \cos(vt) \propto \sin\left[\frac{\pi}{2}\left(\frac{1}{n+1}\right) - 2\gamma\right]$. *Phase Change* = 2γ (where $\tan \gamma = \frac{v}{w} \cdot \frac{K_0}{K} = \frac{2\pi K_0}{\lambda K}$). (n = no. of overtone.)

Coupling (cms.).	$\frac{K_0}{K}$ (cms.).	l (cms.).	λ (cms.).	2γ	Tone.	Intensity.		
						Theory (Per Cent.)	Experi- mental (Per Cent.)	Diff.
-2	2.5	86	354	5° 5'	Fund.	100	100	—
					1st har.	64.1	58.1	- 6.0
					2d har.	42.1	79.0	+36.9
+1	4.1	88	368.8	8° 0'	Fund.	100	100	—
					1st har.	60.2	62.8	+ 2.6
					2d har.	37.5	62.8	+25.3
+2	4.8	89	374.4	8° 27'	Fund.	100	100	—
					1st har.	60.2	61	- 0.8
					2d har.	37.1	43.5	+ 6.4
+3	4.8	89.8	378.3	9° 7'	Fund.	100	100	—
					1st har.	60.35	60	-0.35
					2d har.	36.70	38	+1.30
+5	5.8	92.1	391.8	10° 28'	Fund.	100	100	—
					1st har.	57	69.2	+12.2
					2d har.	33.4	34.6	+ 1.2

table that here the agreement is entirely satisfactory for *loose couplings*.¹ For the couplings - 1 cm. and - 2 cm. one can say that there is agreement to within 1 per cent. As the coupling gets closer the agreement is less good until for the closest couplings (+ 3 cm. and + 5 cm.) the disagreement for the maxima near the oscillator reaches values around 50 per cent. We regard this as a complete experimental verification of the truth of the Kirchhoff-Abraham generalization of the Thomson formula

¹ It should be borne in mind that for the -3 cm. coupling the energy available was too small to make accurate work possible.

$$\frac{2\pi l}{\lambda} \tan \frac{2\pi l}{\lambda} = \frac{Kl}{K_0}.$$

So far as we are aware this verification up till now has been lacking. It is to be said, of course, that the formula has been proven to hold only for the case where the end-capacity is small in comparison to the total capacity of the discharge wire. The reverse case, the Thomson formula, has been verified by many investigators. The general case, viz., when the capacities of the condenser and discharge wire are comparable in value, remains to be verified. We have assigned ourselves the task of attempting its verification and hope to be able to make a report in the near future.

It is of course not to be expected that there should be good agreement between theory and experiment for the *close* couplings. For the theory takes no account of the mutual interaction of the oscillator and Lecher circuits. In view of the importance of the subject both from the theoretical and practical standpoints some of the mathematical physicists might find it desirable to consider the effect of coupling upon the free vibrations of such a system. Furthermore, agreement between theory and experiment is not to be expected as to the relative intensities of the various maxima. For the theory assumes the entire absence of oscillator vibrations. In no case, so far, have we been able to avoid them entirely. Experiments are now in progress where by employing a very long oscillator we hope to free the Lecher system from the oscillator vibrations. For the case that seemed the most promising, viz., the right half of the + 3 cm. curve we have recorded the comparison in Table IV. The agreement is satisfactory. However, the figures for the - 2, + 1, + 2, and + 5 cm. curves attest that there are two causes present in our experiments tending to vary the relative intensities of the harmonics, neither of which the theory takes into account; namely, the presence of the oscillator vibrations and the effect of coupling at both ends of the Lecher wires. These two causes are to be differentiated from each other. For the figures given in the table, which are only for the harmonics near the bolometer end, the coupling at this end as it gets closer (*e. g.*, + 5 cm.) raises the first harmonic above its theoretical value. For the loose

couplings (*e. g.*, -2 cm.) the reverse is true, but superimposed upon this effect is the swinging in of the oscillator vibration in tune with the second harmonic to raise it far above its normal value. The figures for all the curves not shown in the table bear out this idea.

Referring again to Table III. it should be remarked that since both α and $\xi \tan \xi$ involve l it is plain that the per cent. of variation between theory and experiment is but little affected by a mistake of a few millimeters in determining¹ the actual end of the wire. It is, of course, difficult to determine accurately the true reading of K_0/K . If we plot the values as given in column 6 against the

Fig. 13.

couplings we obtain Curve I of Fig. 13. Although the points do not fit the curve very closely the change in capacity seems to be a linear function of the extent to which the Lecher wires extend into the cylinders. Curve II of the same figure shows the change in wave-length as the wires were shortened. In both these curves it should be remembered that a change of one or two millimeters in locating a given maximum or the actual end of the wire could readily account for the variations from the curves shown.² We have seen

¹ The actual end of the wires was determined within 0.5 mm. for the coupling -3 cm., the other values being obtained by adding the amount clipped off of the bolometer end as we lessened the coupling. No great care was used in doing this cutting. The amount cut at the two ends was generally though not always the same.

² As a matter of fact the maxima were located more accurately than the measurements of the linear dimensions of the system would warrant. We were not aware of the need for the closest measurement until after these experiments were completed

that a close coupling displaces the harmonics near the oscillator toward the oscillator. For a coupling as close as $+5$ cm.¹ it is very probable that the fundamental itself is also somewhat displaced in the same direction. This would account for the fact that on the wave-length curve of Fig. 13 the $+5$ cm. point is too high.

The total length of the Lecher wires is given in column 16 of Table III. If we add to this length twice the value of K_0/K and multiply the result by 2, in every case we obtain a figure which is several centimeters short of the value for λ_0 given in column 9. Now physically the capacities at the two ends of the Lecher wires were always identical. The only possible interpretation of this discrepancy is this, it seems to us—electrically there is a real difference in value between the capacities at the two ends in spite of their identical geometrical configuration. The values of the end-capacity at the oscillator end are given in column 17 of the table. In general they are from 2 to 3 cm. higher than the corresponding capacities at the bolometer end of the system. Nor can this difference be materially altered by the “juggling” of the points of Curve I, Fig. 13, to fit the curve. In attempting an interpretation of this difference in the end capacities one should remember that there is a constant in-pour of energy at the oscillator end and an out-pour of energy at the bolometer end. This would necessarily introduce phase differences at the two ends. Indeed, the essential difference between the facts of our experiments and the theory lies here, it seems to us. Theory calls for a permanent train of waves continually reflected from end to end of the parallel wires.² In our experiments, to account for all the facts, one would have to calculate the phase relations between the incoming and reflected waves at the oscillator end of the Lecher system and between the outgoing and reflected waves at the bolometer end of the system. The original

and the comparison with theory was undertaken. The figures in column 4 are accurate only to within 2 mm. due to a lengthening of the Lecher wires (unfortunately they were soft copper wires) under tension. We have no means of correcting this, however, and it certainly can affect the arguments or the conclusions arrived at not at all.

¹The terms “close” and “loose” couplings in this paper are relative terms only. It is probable that even the closest coupling we used would be called “loose” in wireless telegraphy.

²We are safe, we think, in assuming with MacDonald (Electric Waves, p. 82) that the train of waves is permanent except for radiation at the ends.

notion of Professor Rubens was to have the detecting instrument steal but an inappreciable fraction of the energy from the swinging Lecher system. This is undoubtedly correct. However, we ought to expect that as the coupling gets looser the difference between the two end capacities would tend to vanish. It seems from comparing columns 6 and 17 of Table III. that this is the case but more accurate measurement would be necessary to determine this.

If we plot the ratio between the fundamental wave-lengths, λ_0 , to the total actual wire lengths, l_0 , for the different couplings we obtain Curve *III*, Fig. 13. For the loosest coupling the ratio is 2.08, increasing in a linear ratio with the coupling. Extrapolated backward this curve cuts the horizontal axis 2.00 for the coupling — 11.5 cm. Now for the coupling — 3 cm. the energy available at the bolometer end was as small as our apparatus would detect. Great increases in the sensibility of the detecting instruments would have to be attained before this limiting ratio 2.00 could ever be reached. The question of the true value of this ratio in a given case and the factors that control it is very important. We regard it as unsettled. It demands a special investigation. It will be conceded we think that a greater degree of care has never been used in work on electric waves than we have used in these experiments. Curve *III* shows that the ratio can vary with the conditions. We mention this in the hope that other experimenters in this field will stop wasting time working on this question using the earlier crude methods.

When in the work of Blake and Ruppertsberg it was stated that our experiments would show that there was always an “end-correction” of four per cent., it was thought that Curve *I* of Fig. 13 sloped off asymptotically toward the X-axis for looser couplings. Careful revision of the data shows that a straight line would fit the points as well as a curve however. We still have the feeling that such an end-correction is present even in an absolutely isolated Lecher system. But we are not able at present to insist on this. We must investigate further. Nor would the presence of this end-correction mean necessarily radiation taking place from the free ends. We can conceive that some of the energy might swing past the free end before reflection occurs and yet get back again instead of being lost to the system by radiation.

SUMMARY.

1. We have shown by these experiments how it is possible to get the free vibrations of a Lecher system by using a Lecher oscillator with suitable water resistances and air-gaps between the induction coil and the vibrator spark-gap. It was proven that in this case the frequencies were in the ratio of the natural numbers 1, 2, 3, 4, etc., while in the case of the Lecher circuit closed at the end (Blondlot type) the frequencies present bear the ratios 1 : 3 : 5 : 7, etc.¹

2. The effect of changing the coupling between the various circuits was studied and it was found that for a loose coupling the tones present were strictly harmonic. When this was the case it was found possible to verify the Kirchhoff-Abraham generalization of the Thomson formula for a discharge circuit to within one per cent. This was accomplished only by employing a check receiver and introducing all possible refinements of experimental methods.

3. The effect of changing the distance between the Lecher wires was studied and it was found to introduce complications as this distance and hence the bridge lengths became appreciable in amount in comparison to the length of the Lecher circuit, a result to be expected. The best set of free vibrations was obtained when the distance between the Lecher wires was 1.6 per cent. of their own length.

4. Our experiments were compared with the theory of free vibrations completely developed by Abraham and they were found to differ from the theory because of two conditions which the theory does not allow for, viz., the presence of oscillator vibrations on the Lecher circuit and the necessary proximity of the source of energy supply and of the means of detection of the vibrations.

5. When the coupling between the oscillator and Lecher circuits was close two fundamental oscillator vibrations were usually present. It was found that the oscillator vibration could be made to superpose itself upon any given harmonic to make it of abnormal intensity. No overtone of the oscillator seemed to be present.

¹ The consistency (?) of the usual notation is here clearly shown. It is customary in alternating current work to say that only the odd harmonics are present, or in other words to *include* the fundamental as one of the harmonics. The same is true for closed organ pipes. For open organ pipes on the other hand we *exclude* the fundamental from the list of harmonics, for we say the octave of the fundamental is the first harmonic.

6. For the best set of free vibrations the ratio of maximum to minimum was 40 to 1. This was obtained when the coupling at both ends of the Lecher circuit was + 1.4 cm. in arbitrary units for an unusual type of coupling.

7. The ratio of wave-length to wire length was found to vary with the coupling, ranging from 2.16 to 2.08, the last figure in agreement with the result of Blake and Ruppertsberg.

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