Electrothermal Phenomena at the Contact of Two Conductors, with a Theory of a Class of Radiotelegraph Detectors

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1912 Proc. Phys. Soc. London 25 273

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XXXI.—*Electrothermal Phenomena at the Contact of Two Conductors, with a Theory of a Class of Radiotelegraph Detectors.* By W. H. ECCLES, D.Sc.

RECEIVED MAY 16, 1913. READ MAY 30, 1913.

WHEN an electric current is caused to pass across the interface between a pair of conducting masses, heat is in general liberated or absorbed in accordance with the law of Peltier. When the masses are in contact over a very small area, as, for example, when a cylinder of graphite is laid across a copper wire, there may be, in addition, appreciable generation of heat in accordance with the law of Joule. If the substances constituting the contact are bad conductors of electricity and of heat, and if they stand far apart in the thermoelectric series, the phenomena arising when a current is forced across the joint become very striking, for in such circumstances relatively large amounts of heat may be developed, the heat is conserved, and therefore the thermoelectric effects enhanced.

It is evident that the thermoelectric forces called up by the local heating may assist or may oppose the E.M.F. applied to produce the current and that the phenomena of asymmetric conduction at once arises. But besides the Joule and Peltier effects, the Thomson effect may contribute to the phenomena. In the case of bad conductors of heat the temperature gradients very near the contact will be very steep, and thus the Thomson effect will be localised in the immediate neighbourhood of the contact.

Further than this it is obvious that, on account of the temperature changes, the portions of the substances near the contact will suffer a change in the magnitude of their electrical resistivity. It has been shown* that this effect alone leads to remarkable and important results, and is sufficient to account for all the principal features of the single-point coherer used in wireless telegraphy.

The thermoelectric forces and the changes of electrical resistance that arise from differences of temperature are much greater in combinations of such substances as iron pyrites than in combinations of ordinary metals. A pyrites-lead couple yields an E.M.F. some 200 per cent. greater than a bismuth-

* "Proc." Physical Society of London, Vol. XXII., p. 869.

lead couple, between the same extremes of (ordinary) temperature; while the temperature coefficient of resistance of pyrites is about four times as great numerically as that of copper. But the thermal conductivity of pyrites is so very much smaller than that of lead that all these thermoelectric phenomena are greatly accentuated in the former case. Contacts between non-metallic conductors are of special interest for the reason that the bulk of the wireless telegraphy of the world is carried on by aid of detectors that consist of nothing else than a contact involving at least one non-metallic conductor.

The subject does not appear to have been discussed hitherto from the theoretical standpoint adopted in this Paper.

Thermoelectric Theory of a Contact.

The thermo-electric constants of such substances as pyrites, zincite, carborundum, &c., are not easy to measure accurately and their coefficients of increase of resistance with temperature are exceedingly difficult to determine. The author has made numerous determinations, and has found that all the materials examined follow with fair precision the ordinary thermoelectric law that their thermo-electric powers are linear functions of the temperature, and also that their temperature-resistance coefficients are all large and negative.

For a thermocouple formed of (say) pyrite and lead we may therefore assume the E.M.F. equation

$$e_{\tau} = \alpha\tau^2 + \beta\tau + \text{constant}, \quad \dots \dots \dots (1)$$

where τ represents the absolute temperature of the hot junction, α and β are constants, and e_{τ} is the E.M.F. when one junction is at temperature τ and the other at the absolute zero. The ordinary thermodynamic theory may legitimately be applied. Thus if σ represent the specific heat of electricity in the substance, P the Peltier effect and p the thermoelectric power at the temperature τ , the first and second laws of thermodynamics give, when applied to a cycle of infinitesimal range,

$$\sigma = \frac{dP}{d\tau} - p,$$

and

$$\frac{d}{d\tau} \left(\frac{P}{\tau} \right) = \frac{\sigma}{\tau}.$$

Whence we have

$$\sigma = \tau \frac{dp}{d\tau} = \tau \frac{d^2 e}{d\tau^2}.$$

If we represent the absolute temperature of the surroundings and of the cold junction by z , and the temperature of the hot junction by $\theta+z$, we find from these equations that at the temperature of the surroundings

$$\sigma=2az, \quad p=\beta+\sigma, \quad P=pz,$$

so that it is easy to calculate the thermoelectric quantities from the experimental equation (1).

Further, we obtain from (1)

$$\begin{aligned} e_{\theta} &= a\theta^2 + (\beta + 2az)\theta \\ &= p\theta + \theta^2/2z \\ &= \theta(P + \frac{1}{2}\sigma\theta)/z. \end{aligned} \quad (2)$$

When instead of lead a substance is used of which the thermoelectric properties are represented by σ' , p' , P' , the E.M.F. in the circuit, when the junctions are at temperatures z , $\theta+z$, is

$$e_{\theta} = \theta\{P - P' + \frac{1}{2}(\sigma - \sigma')\theta\}/z. \quad (3)$$

Now, suppose the circuit made up of these two conductors

FIG. 1.

to have one junction forming a loose contact, the other a very good contact, as indicated in Fig. 1, where one conductor may be regarded as a piece of pyrites and the other as a piece of metal. Let an E.M.F. be applied to the circuit in any manner so as to produce a current in the direction opposite to the E.M.F. that would be produced by heating the contact. The heat liberated near the contact, say h , is, per absolute unit of electricity flowing across the contact, equal to the thermoelectric force e_{θ} plus the heat absorbed at the cold junction. That is

$$h = e_{\theta} + P - P'.$$

This heat tends to be concentrated in a small volume of the substance near the contact, but is dispersed continually by thermal conductivity and radiation. We will assume that the

rate of loss of heat by these agencies is, as a whole, proportional to the excess of temperature of the junction over its surroundings. The rise of temperature causes an alteration of the electrical resistance of the joint; let the true resistance at any temperature θ above the temperature of the surroundings be expressed by $\rho(1+\gamma\theta)$, where γ is a temperature coefficient dependent on both the substances at the contact.

By equating the rate of accumulation of heat to the rate of liberation of heat by the electrical processes less the rate of loss of heat to the surroundings by purely thermal processes, we have, in mechanical units,

$$k \frac{d\theta}{dt} + m\theta = hy + \rho(1+\gamma\theta)y^2, \dots \dots \dots (4)$$

where y is the current in the direction defined as above, k is a constant involving the densities and specific heats of both substances and m is a constant involving the internal and external thermal conductivities of the substances near the contact.

Let the part of the circuit which suffers inappreciable temperature change possess the constant resistance r and inductance L , and let the applied E.M.F. be e , at any time t , then

$$L \frac{dy}{dt} + (r+\rho)y + e_\theta = e. \dots \dots \dots (5)$$

We have to deal in this Paper with a state of affairs in which the rate of variation of current and applied E.M.F. is relatively slow. In this case we may use the equations of the steady state, namely,

$$m\theta = hy + \rho(1+\gamma\theta)y^2 \dots \dots \dots (6)$$

$$(r+\rho)y = e - e_\theta \dots \dots \dots (7)$$

Substitute for e_θ from equation (3) above, eliminate θ , and we obtain an equation for the current y sent across the junction by the applied E.M.F. e . The equation is less cumbrous, however, if we obtain it in terms of the P.D. between the extreme parts of the conductors which vary in temperature. Regarding this as a portion of the applied E.M.F. and representing it by x , we have

$$x = e - ry + P - P' \dots \dots \dots (8)$$

Then the current equation becomes

$$\frac{\sigma - \sigma'}{2m^2z} x^2 y^2 + \frac{\rho\gamma}{m} xy^2 + \frac{P - P'}{mz} xy - x + \rho y + P - P' = 0, \dots \dots (9)$$

which may be written

$$ax^2y^2 + cxy^2 + bxy - x + py + b' = 0. \quad (10)$$

The constant coefficients a , c , b in this equation are, it will be seen, mainly dependent on the Thomson effect, the change of resistance with temperature, and the Peltier effect respectively. The term b' is always negligible except very near the origin of co-ordinates and will be put zero in all that follows. Subject to the proper interpretation of the variable x , the equation is that of the "steady current characteristic" of the combination of conductors. And since when b' is taken zero

$$x = e - ry, \quad (11)$$

the steady current characteristic, as usually drawn from observations of applied E.M.F. and consequent current, is identically that obtained by applying to the curve drawn from the above equation a homogeneous shear of amount r parallel to the x axis.

Particular Cases.

Case A.— $a = b = 0$.

The coefficient a is proportional to the difference between the Thomson effects in the two conductors at the absolute temperature z ; if they are equal it vanishes. The coefficient b is proportional to the Peltier E.M.F. at the junction at temperature z ; it vanishes if the neutral temperature of the substances happen to be z . For thermoelectric effects to be wholly absent a and b must both be zero. We are then left with the cubic

$$cxy^2 - x + py = 0, \quad (12)$$

which has been discussed in Papers* by the present writer. The quantity c depends mainly on γ , the coefficient of increase of resistance with rise of temperature. The most interesting cases arise when γ is negative, as it nearly always is in contacts including a native crystalline oxide or sulphide, or carborundum. In fact, this equation explains the action of the single-point coherer fairly perfectly; and thus, when in equation (10) this quantity c is finite and negative, we may say that this "coherer action" is accompanying the thermoelectric effects.

The kinds of curve that may be yielded by pure coherer action are shown in Figs. 2 and 3. In Fig. 2 the equation is

$$c'xy^2 + x - py = 0,$$

* "On Coherers," "Proc. Phys. Soc.," Vol. XXII. "On an Oscillation Detector," "Proc. Phys. Soc.," Vol. XXII.

when c' is put for $-c$. The curve of Fig. 2 has only one real asymptote, the axis of y . In order to exhibit the curve con-

FIG. 2.

FIG 3.

necting applied E.M.F. and resulting current the curve must be submitted to shear of amount r , where r is the non-varying

part of the resistance of the whole of the circuit containing the contact. In this way we obtain curves of the kind shown in Fig. 3, which resemble those obtained* from a contact consisting of two pieces of galena. There are two vertical tangents if $8r < \rho$ (case I.) and none if $8r > \rho$ (case III.). The two vertical tangents coincide if $8r = \rho$ (case II.). Here ρ is the starting value of the variable part of the true resistance of the contact.

The curve I. shows the existence of instability; near the right-hand vertical tangent a very small increase of E.M.F. will produce discontinuously a great increase in the current; case III. is quite stable and illustrates a "self-restoring" coherer.

For contrast, the curve corresponding to the case when resistance increases with rise of temperature is shown in Fig. 4. The equation is

$$cxy^2 - x + \rho y = 0.$$

The part of immediate physical interest is that passing through the origin. It shows that, as the current increases, greater and greater increments of E.M.F. are needed to produce an assigned increase of current, as might be concluded without calculation. This is the kind of curve obtained in any circuit in which a conductor with positive resistance coefficient becomes heated; for example, a fuse wire raised towards fusion in air. The curve of case III., Fig. 3, arises when the heated conductor possesses a negative coefficient—*e.g.*, the carbon filament of a lamp.

The contact or aggregate of contacts that occurs between the carbon brushes and the commutator of a dynamo affords a striking instance of the phenomena. Many of the known properties of brush contacts have been explained by W. H. F. Murdoch † on these lines.

Case B.— $a=0$, $c=0$.

When the specific heats of electricity in the two conductors are equal $a=0$, and when there is no coherer action $c=0$. In this case the equation reduces to

$$bxy - x + \rho y = 0, \quad . \quad . \quad . \quad . \quad . \quad (13)$$

which gives the hyperbola shown in Fig. 5. Only the branch

* "On an Oscillation Detector," *loc. cit.*

† Murdoch, "Commutator Resistance and Energy Losses," "Electrical Review," June 16, 1911.

through the origin appears to be physically relevant. When sheared the equation is

$$bry^2 - bey + e - (r + \rho)y = 0.$$

This represents a hyperbola with its centre at the point $(r - \rho)/b, 1/b)$, and with asymptotes $by = 1$, $b(e - ry) + \rho = 0$; it is shown in Fig. 6. It is clear that the curve is not symmetrical about the origin, and, therefore, that the existence of the Peltier effect makes the contact a rectifier of alternating current of any amplitude.

Case C.—No coherer action. $\gamma = 0$, and, therefore, $c = 0$.

In this case the full effects of thermoelectric action, free from coherer action, are brought out. The equation is

$$ax^2y^2 + bxy - x + \rho = 0. \quad . \quad . \quad . \quad . \quad (14)$$

Two sub-cases arise—namely, a positive and a negative.

1. *a Positive.*—This sub-case is shown in Fig. 7. The branch through the origin shows that the contact will act as an excellent rectifier, unless, indeed, the invariable part r of the resistance is relatively very large; for even after considerable shearing the curve will have its relevant negative portion much steeper than the positive portion. The contrast with the last case (Thomson effect zero) is especially noteworthy. In that case the difference of gradient on the two sides of the axis of y , after shearing, is not nearly so pronounced as in the present case.

2. *a Negative.*—When a is negative we obtain the curve shown in Fig. 8. It has its steepest part in the positive quadrant, and the points of inflexion near the origin indicate that the rectification is more pronounced than in the last case. Thus we may expect a combination in which a is of opposite sign to b to behave as a good detector for the purposes of wireless telegraphy, other things being not unsuitable. The meaning of this condition for good rectifying properties may easily be expressed by aid of the diagrams of thermoelectric power in Figs. 9 and 10.

In both figures the Peltier E.M.F.s P, P' divided by the absolute temperature z are represented by zp and zp' ; and the specific heats of electricity σ, σ' divided by the temperature z are represented by the gradients of the lines at the temperature z . Now

$$a = \frac{\sigma - \sigma'}{2m^2z} \quad \text{and} \quad b = \frac{P - P'}{mz}.$$

Hence in Fig. 9 both a and b are positive, while in Fig. 10 a is negative and b remains positive. In the one case the neutral temperature is below z , in the other above z . This is to be

FIG. 9.

FIG. 10.

seen also from the familiar fact that the neutral temperature T is given by

$$T = -\frac{P - P'}{\sigma - \sigma'}$$

which in our notation is written

$$T = -\frac{b}{2ma}$$

In words the theorem now arrived at is that the best rectifying

contact obtained by purely thermoelectric effects is that in which the neutral temperature (or "temperature of inversion") for the pair of substances forming the contact is above the temperature of the cold junction.

Case D.— $c=0$ and $b=0$.

The above conclusion suggests an inquiry as to the rectifying properties to be expected of a contact composed of two substances which have the same Peltier E.M.F. with respect to lead and still have no coherer action. The equation is now, c being still zero,

$$ax^2y^2 - x + \rho y = 0. \quad (15)$$

Two sub-cases arise, a positive and a negative.

1. *a Positive.*—The curve is given in Fig. 11. It indicates good rectification; a *negative* E.M.F. causes a greater flow of current than a positive E.M.F.

2. *a Negative.*—The curve is given in Fig. 12. In this case again there is good rectification, but now a *positive* E.M.F. causes the greater flow of current.

The principal difference between this case and the case C2 is that there is no point of inflexion near the origin in the present case. This again indicates that it is advantageous to have the neutral temperature high. In fact, referring back to case C, when b is not zero, and solving the corresponding equation as a quadratic in y , we have

$$y = \frac{-(bx + \rho) \pm \sqrt{(bx + \rho)^2 + 4ax^3}}{2ax^2}.$$

The vertical tangent to the curve is, therefore, given by the cubic equation for x

$$-4ax^3 = (bx + \rho)^2.$$

First, taking a positive and of fixed value we see that the one real root of this equation is numerically greater the greater b , being $-(\rho/4a)^{1/3}$ in the limiting case $b=0$. Thus the greater the value of b the further does the vertical part of the physical branch of the curve depart from the axis of y . Since the neutral temperature is proportional to b , we may say that the rectification* produced by such a contact is greater the higher the neutral temperature. This is, moreover, the more marked the smaller the value of ρ . Now, taking a negative we see that the real positive root of the equation is greater the greater b ,

* See Appendix p. 286.

and thus again the rectification is greater the higher the neutral temperature. Further, supposing that in turn b is fixed in value, the form of the equation shows at once that the smaller the numerical value of a the greater numerically is the relevant root in x . Thus again it appears that a high neutral temperature corresponds to good rectification. But in all these conclusions there is always to be kept in mind the practical condition that the root of the cubic shall be a value of x that is attainable experimentally.

Taken as a whole the argument establishes the theorem that the rectifying properties of a contact at which there is no coherer action are the more pronounced the higher (within practical limits) the temperature of inversion.

Case E.—The Resistance Coefficient not Zero.

The most interesting sub-cases occur when the Thomson effect in the circuit is zero. The steady current equation is

$$cxy^2 + bxy - x + \rho y = 0. \quad (16)$$

There are three sub-cases :—

1. c Positive—that is, the resistance of the substances near the contact on the whole increases with rise of temperature. There are two real asymptotes parallel to the axis of x and on opposite sides of that axis. The curve (Fig. 13), is unsymmetrical about the origin and the contact possesses rectifying properties to some extent, but the asymmetry is smaller the greater c .

2. c Negative and $16c^2 < b^4$. Here coherer action enters, but only to a relatively small extent. The two real asymptotes parallel to the axis of x are now on the negative side of that axis, and consequently the curve (Fig. 14) is very unsymmetrical about the origin. The contact will be a good rectifier, and will be a good detector of oscillations when submitted to a suitable positive E.M.F.

3. c Negative and $16c^2 > b^4$. Here coherer action is pre-dominant. The curve is shown in Fig. 15. The asymptotes parallel to the axis of x are imaginary, and the contact, when employed as a detector, is sensitive under a particular negative as well as a particular positive E.M.F. Its difference from a pure coherer would be seen experimentally by a difference in the numerical values of the particular positive and negative E.M.F.s that give maximum sensitiveness. The vertical tangents occur at values of x given by

$$\rho/(2\sqrt{-c}+b) \text{ and } -\rho/(2\sqrt{-c}-b).$$

GENERAL CASE.

We have now to analyse the general case when none of the constants in the steady current curve vanish. From what has gone before it may be expected that, broadly, the effect of the thermoelectric terms is to produce asymmetry and the effect of the resistance terms is to encourage symmetry in the physical portions of the curves.

The general equation is

$$ax^2y^2 + cxy^2 + bxy - x + \rho y = 0. \quad (17)$$

By definition b is positive, and the positive direction of the current is that in which it must flow in order to absorb heat by the Peltier effect at the junction that remains at the temperature of the surroundings. The resistance ρ is essentially positive; the Thomson term a may be negative or positive, and c has the same sign as the aggregate resistance temperature coefficient of the contact.

There is no need to discuss here the method of plotting the curves, nor is there any necessity for plotting the curves accurately. Therefore in the figures only the general shape of each curve is intended to be conveyed, and though the non-physical branches have been inserted, isolated ovals have all been omitted. Asymptotes are shown by dotted straight lines. It is convenient to divide the discussion into cases delimited by the signs of the constants a and c . The reader may be reminded that the curves ought to be sheared in the manner already described before being compared with experimental curves.

Case I.— a Positive, c Positive.—The curves take the forms shown in Figs. 16A, 16B, 16C, according as b/ρ is less than, equal to, or greater than a/c , i.e., $2\gamma(P - P')$ less than, equal to, or greater than $\sigma - \sigma'$. The greatest asymmetry is indicated in Fig. 16A, which corresponds to the condition that the Thomson coefficient shall be great compared with that of the Peltier effect and of the temperature coefficient of increase of resistance.

Case II.— a Positive, c Negative.—The existence of coherer action brings about two distinct sub-cases which both indicate that the contact would operate well as a detector for radio-telegraphy. In one of the cases the contact would be a sensitive detector only when the E.M.F. is negative; in the other it would be sensitive at two unequal and opposite values of the applied E.M.F. The analytical condition separating these cases is cumbrous (Figs. 17A and 17B).

Many samples of galena form with lead contacts of these types.

Case III.—a Negative, c Positive.—There is only one type of curve. A contact belonging to this class will behave as a sensitive detector only for positive applied E.M.F. (Fig. 18).

Case IV.—a Negative, c Negative.—There are four sub-cases shown in Figs. 19A, 19B, 19C, 19D. In A, b/ρ is much smaller than a/c —that is to say, $2\gamma(P-P') < \sigma - \sigma'$. In B, $2\gamma(P-P')$ is slightly smaller than $\sigma - \sigma'$. A case scarcely different from the last in its physical branch is given in C, where $2\gamma(P-P') = \sigma - \sigma'$. Finally, in D is seen the case when $2\gamma(P-P') > \sigma - \sigma'$. If we regard the passage through these sub-cases as being due to a gradual increase of the share of coherer action in the whole phenomenon, it is evident that coherer action always tends to make the realisable part of the curve symmetrical about the origin of co-ordinates. If, on the other hand, we regard the transition as due to gradual elevation of the neutral temperature, we see cases A to D correspond to higher and higher values of this temperature.

Contacts including zincite usually fall in one of these sub-cases.

Most of the curves drawn in this Paper possess a vertical tangent on one side of the origin of co-ordinates, and many of them a vertical tangent at each side. After allowing for the shearing process required to convert the curves into "characteristic curves," it may be concluded broadly that when a is positive the current produced by a definite voltage is greater when the voltage is negative than when it is positive; while, when a is negative or zero the positive voltage produces the greater current. Now it is well known that when a contact is warmed by direct communication of heat, the thermoelectric force is for some pairs of substances in *same* direction as the larger, and for other pairs in the *opposite* direction to the larger of the currents produced across the contact by reversing a constant applied E.M.F. The latter case has frequently been quoted as showing that these contacts did not owe their rectifying properties to thermoelectric effects. This is a fallacy. For the observation of the direction of the thermoelectric force produced by direct heating often shows only the sign of the Peltier effect and not the sign of the Thomson effect in the circuit; and it is the latter alone which decides, through the constant a of this Paper, the accordance or disaccordance indicated above.

Considerations relative to the Determination of the Constants involved in the Theory.

The constants occurring in the general steady current equation are a , b , c and ρ . By aid of the equations of the section dealing with the thermoelectric theory, two of these constants may be expressed partially in terms of the more fundamental constants appearing in the thermo E.M.F. equation

$$e_{\theta} = \theta(\beta + 2\alpha z + \alpha\theta) \\ = \theta(\beta' + \alpha\theta), \text{ say.}$$

Here θ is the temperature of the contact above that of the cold junction and the surroundings z . We thus obtain

$$a = \frac{\alpha}{m^2}, \quad b = \frac{\beta'}{m}.$$

The quantity m is the measure of the rate of loss of heat by radiation and conduction.

Now, for any particular two substances forming a contact the constants a and β' may be determined by measurement of the thermoelectric force for two values of θ , and thus a and b become known if m can be determined. Unfortunately this quantity cannot be determined. Of the other constants in the current equation—namely c and ρ —the first is given by the equation

$$c = \rho\gamma/m.$$

The coefficient γ is, for the crystals that form the most interesting contacts, very difficult to measure. In all the methods the author has employed, difficulty has arisen through the high thermoelectric effect at the electrodes; but so far as the measurements go it is negative in many conducting native oxides and sulphides. The quantity ρ can be roughly estimated by shearing an experimental curve till its asymptote is parallel to the axis of y and then noticing that the tangent at the origin is $x - \rho y = 0$.

The data accumulated up to the present by the author will be published in a later communication, when also the experimental work of other observers will be discussed.

APPENDIX.

On the Operation of a Contact as a Radio-telegraphic Detector.

It appears that the earliest statement of the general connection between the steady current curve of a contact and its pro-

FIG. 4.

FIG. 5.

FIG. 6.

FIG. 16A.

FIG. 16B.

FIG. 16C.

FIG. 7.

FIG. 8.

FIG. 11.

FIG. 12.

FIG. 17A.

FIG. 17B.

FIG. 18.

FIG. 13.

FIG. 14.

FIG. 15.

FIG. 19A.

FIG. 19B.

FIG. 19C.

FIG. 19D.

perty of acting as a detector of oscillations was made by H. Brandes* in 1906. He pointed out that any kind of conductor—not necessarily a contact—which departs from Ohm's law will rectify alternating current in some degree. If with such a substance (or contact) the volt-ampere curve is symmetrical in the first and third quadrants, a direct current of suitable magnitude must be kept flowing and the oscillatory current to be rectified must be superposed on this current. On the other hand, he pointed out, if the characteristic curve is not symmetrical about the origin, no direct E.M.F. is actually needed. And he showed that the same general rules hold good for all sorts of detectors, such as the electrolytic, as well as for the so-called crystal rectifiers. Again, Pickard† has indicated that the greatest sensitiveness of crystal contacts is at the E.M.F. where the rate of change of resistance is greatest, and J. A. Fleming‡ has published diagrams showing that in the case of his vacuum valve the E.M.F. of maximum sensitiveness is near that of the maximum ordinate of the second derived curve of the characteristic. These principles may now be regarded as well known and have been tacitly assumed in the preceding pages. It would be well, however, to examine the matter from the purely theoretical standpoint of this Paper.

Let the equation of the characteristic of any detector (Fig. 20) be $y=f(e)$, where y is the current produced by applying the steady E.M.F. e to the detector. Suppose the E.M.F. to increase by the small amount h and the current to increase in consequence by the amount k . Then, by Taylor's theorem,

$$k=hf'(e)+\frac{h^2}{2}f''(e)+\frac{h^3}{3}f'''(e)+,$$

where f' , f'' , f''' ... indicate the first, second, third... derived functions of $f(e)$. We will suppose that h is a very small alternating E.M.F., say, $h=a \sin \omega t$, where ω is small.

The whole matter may now be regarded from two points of view. In the first place we may concentrate attention on the rectifying action of the contact, and calculate the excess of electricity passing in one direction round the circuit over that passing in the other direction, when the alternating E.M.F. is acting. In the second place we may endeavour to estimate,

* H. Brandes, "E.T.Z.," 27, p. 1015, Nov. 1, 1906.

† G. W. Pickard, "El World," November, 1906.

‡ J. A. Fleming, Royal Institution Discourse, June 4, 1909, and British Patent Specification 13518, 1908.

not what may be called the rectified current, but, instead, the energy possessed by the portion of the current in the circuit due to the harmonic E.M.F.

In the former case the average value of the increased current flowing in the circuit while the alternating E.M.F. is acting is

$$\delta y = \frac{1}{T} \int_0^T k dt,$$

where T is the period of the alternation and $T = 2\pi/\omega$. Hence

$$\delta y = \frac{a^2}{2} f''(e) + \frac{3a^4}{8} f^{iv}(e) + \dots$$

Thus the magnitude of the average rate of flow of the rectified

FIG. 20.

electricity depends on the derived functions of even orders as might be expected. The equation shows that if the steadily applied E.M.F. e is adjusted to a value corresponding to a sharp bend on the characteristic curve, then the rectified current may be very large, according to the value of $f''(e)$ principally. It is on this account that a coherer with a perfectly symmetrical characteristic can be made to act as a rectifier of slow alternating current.

On the other hand, from the energy point of view, the increase of the mean square value of the current round the

circuit, when alternating E.M.F. is imposed on the steady E.M.F., must be calculated. This increase is given by

$$\begin{aligned}(y+k)^2 - y^2 &= \left\{ f(e) + hf'(e) + \frac{h^2}{2} \cdot f''(e) + \right\}^2 - \{f(e)\}^2 \\ &= 2hf(e) \cdot f'(e) + h^2[\{f'(e)\}^2 + f(e) \cdot f''(e)] \\ &\quad + h^3[\frac{1}{3}f(e)f'''(e) + f'(e) \cdot f''(e) +] + \dots\end{aligned}$$

Thus, the additional work done in the circuit per second, in consequence of the imposition of the harmonic E.M.F. h , is a function of the gradient of the characteristic as well as of the higher derived functions. In particular, work will be done in the circuit even when using a straight portion of the characteristic curve where all the derived functions of even order might be zero, notwithstanding that in this case the rectified current would be zero. The apparent paradox that work may be done in the telephone circuit although there is no rectified current is removed by reflecting that a sine wave alternating current will not deflect a direct-current galvanometer though it may be doing work in the circuit.

We therefore draw the conclusion that when the steady E.M.F. is adjusted to a sharp bend on the characteristic curve the detector gives the best rectification; but when the greatest deflection of an instrument of hot-wire type is required, we must have the quantity $\{f'(e)\}^2 + f(e) \cdot f''(e)$ near its maximum, which will usually occur very near the E.M.F. corresponding to the point of inflexion on the curve.* In this latter case the mode of operation may be styled the "mean square mode." In general the amplitude of the oscillatory E.M.F. will perhaps not be so minute as is here assumed, and consequently the local steady E.M.F. applied to the detector cannot be so adjusted as to separate the two modes of action just described.

The above analysis leads immediately to an expression for the efficiency of transformation of energy. The instantaneous rate of working of the applied oscillatory E.M.F. is hy , and thus, if W be the average rate of working,

$$\begin{aligned}W &= \frac{1}{T} \int_0^T hy dt \\ &= \frac{a^2}{2} f'(e) + \frac{a^4}{16} f'''(e) + \dots\end{aligned}$$

* In the case of the magnetic detector it has been shown experimentally by the author that the point of maximum sensitiveness is near, but not quite coincident with, the point of inflexion of the hysteresis loop.—"Proc. Phys. Soc.," Vol. XX., June, 1906.

The average rate of working in the circuit of the detector is

$$w = m \cdot \frac{a^2}{2} [\{f'(e)\}^2 + f(e) \cdot f''(e)] + m \cdot \frac{a^4}{32} [3\{f''(e)\}^2 + 4f'(e) \cdot f'''(e) + f(e) \cdot f^{(iv)}(e)] + \dots,$$

where m is a proportional constant. The efficiency is the ratio $w : W$.

The degree of the equation connecting w and W when a is eliminated between the above equations depends on the number of terms in powers of a^2 included in the process. The simplest case occurs when w is near its maximum and also

FIG. 21.

$f'''(e)=0$; for near the maximum of w the coefficient of a^4 in the equation for w vanishes, and then we have

$$wf'(e) = m[\{f'(e)\}^2 + f(e)f''(e)]W.$$

When W and w are taken as abscissæ and ordinates this equation connecting input and output of energy is represented by a straight line through the origin. More generally, writing the equation for w in the form

$$w = \alpha_1 a^2 + \alpha_2 a^4$$

and

$$W = \frac{1}{2}f'(e)a^2 + \frac{1}{16}f'''(e) \cdot a^4,$$

we have

$$\{\alpha_2 W - \frac{1}{16}f'''(e)w\}^2 = \{\frac{1}{2}f'(e) \cdot \alpha_2 - \frac{1}{16}f'''(e) \cdot \alpha_1\} \times \{\frac{1}{2}f'(e)w - \alpha_1 W\},$$

which represents a parabolic locus.

In the cases in which $f'''(e)$ and higher derived functions may

be neglected—and these are often practical cases—this equation becomes of the form

$$4a_2(W+A)^2=f'(e)(w+B),$$

where A and B are positive constants involving $f(e)$, $f'(e)$ and $f''(e)$. The locus is shown in Fig. 21. The thick portion is that with which radio-telegraphy is concerned, for elsewhere W is negative or it is so small as to give no audible sound. Experimental observations made by the author on the magnetic detector,* the coherer,† the electrolytic detector and various contact rectifiers‡ have yielded points lying approximately on a straight line passing to the right of the origin in the manner suggested by a straight line drawn through the thickened part of the last figure.

From the experimental results mentioned, it would appear that this theory of the “detecting” property of any apparatus with a characteristic curve like Fig. 20, under low-frequency E.M.F. of small amplitude, holds good in some measure for the high frequencies and large amplitudes employed by the various observers.

ABSTRACT.

The Paper is a purely theoretical one, and deduces mathematically the laws connecting the current and the applied E.M.F. in a circuit containing a light contact of two conductors. When an electric current passes across a light contact of two different substances, heat is liberated or absorbed in accordance with the law of Peltier, heat is generated in accordance with the law of Joule, and, in the regions of the conductors where there is a temperature gradient, heat is liberated or absorbed in accordance with the laws of the Thomson effect. These thermal actions are very noticeable in contacts made of badly conducting natural oxides or sulphides on account of the high resistivity and the large thermoelectric effects in these substances. The low thermal conductivities of these substances exalt the electrical consequences by conserving the heat. The bulk of the wireless telegraphy of the world is carried on by such contacts as these, and the present Paper, therefore, constitutes a theory of the action of these detectors.

The general equation connecting the current across a contact and the P.D. between two points of the circuit is deduced, and is found to be of fourth degree when the difference of the specific heats of electricity in the conductors is not zero. The curves are discussed and plotted for all the principal variations of value of the Thomson effects, the Peltier effects, and the resistivity-temperature coefficients of the substances. It is shown that, in general, when a contact is used as a radiotelegraph detector, there may be some coherer

* “Proc. Phys. Soc.,” Vol. XX., 1906.

† Vol. XXII., 1910.

action mixed up with the thermoelectric "rectifying" action; and that, whether there is or is not any coherer action, the principal features of the characteristic curve connecting current and E.M.F. are determined by the Thomson rather than by the Peltier effect. Thus, if two contacts could be made having all their electrical and thermal properties of equal measure excepting the Thomson property, of which the measures were of opposite signs, the curve connecting current and E.M.F. for one contact would rise more rapidly on, say, the side of positive E.M.F. than on the side of negative E.M.F., and vice versa for the other contact; and this although a direct experiment of applying heat to the contacts in turn and observing the consequent thermoelectric forces, showed these to have the same direction in both.

DISCUSSION.

Dr. A. RUSSELL considered Dr. Eccles' fundamental equation very valuable. He thought Dr. Eccles' definition of the efficiency of a detector useful and the method of arriving at it very neat, but he did not think the curve of efficiency given by Dr. Eccles would be much use. What was wanted to tell a good detector was the shape of the function showing the relation between the current and the E.M.F.

Prof. C. H. LÆS thought it was important to have shown that the well-known phenomena of thermoelectricity and variation of resistance could explain the behaviour of contacts. The theory was quite elastic enough to explain more complicated characteristic curves than those now considered, as the resistance was not strictly a linear function of the temperature and the heat conductivity also was not a constant.

Mr. P. R. COURSEY referred to the tests he had recently carried out for Dr. Fleming on the subject of crystal detectors, some of the results of which were before the meeting, and said they were undertaken more with the view to finding the relation (if any) that exists between the characteristic and sensitiveness curves for the crystals (taking impressed voltages as abscissæ in both cases) than to discovering a reason for the shape of the characteristic curves and the more or less abrupt changes of curvature, or "kinks," that occur in them, as had been done by Dr. Eccles in his Paper. It had been known for some time past that when the second differential of the characteristic curve of a Fleming oscillation valve is plotted it bears a considerable resemblance to the sensitiveness curve of the valve. This relation was found in the above tests to hold good to a greater or less degree in the majority of crystal detectors, the most notable exception being the galena-plumbago combination, for which practically no agreement between the two curves could be found. Similar remarks applied to the electrolytic detector which was also tested; and hence one was led to the conclusion that in most detectors we have at least two actions taking place tending to produce sounds in the telephones, and that these may or may not assist one another, it being probable that in the galena detector the effect due to the changes of curvature are almost entirely masked by some other, and opposing, effect due to a different cause—perhaps some type of electrolytic or thermal action. Hence to analyse or explain the shape of the characteristic curve of a detector in the manner outlined by Dr. Eccles was not necessarily going to be of assistance in finding a good detector, since, as seen from the above tests, the sensitiveness is not always dependent merely upon the shape of the characteristic curve, although nevertheless in many cases it will doubtless lead to valuable results.

Dr. J. A. FLEMING communicated the following: With reference to Dr. Eccles' remarks on the characteristic curves of certain rectifying detectors, I believe I was the first to point out, in a British patent specification, No. 13,518 of 1908, the conditions under which certain rectifiers

of oscillations, such as my glow-lamp detector, can be operated with an added E.M.F. as a more sensitive detector than by use as a mere rectifier without such E.M.F. Long before that date a large number of measurements of characteristic curves of my valves and other detectors had shown that there were sudden changes of curvature in them. The method of detecting these was to plot a first and second differential curve from the volt-ampere characteristic. In the above specification I described simple means of applying the boosting voltage in the case of a glow-lamp detector by using a potentiometer wire in connection with the same battery which incandesces the glow lamp. With certain detectors this is a more sensitive arrangement for detecting oscillations, but it is unfortunately as yet impossible to construct glow-lamp detectors or oscillation valves with predetermined characteristics. I have, however, one glow-lamp detector which when worked in this manner is more sensitive than any electric wave detector I have ever met. We have not yet found the way to reproduce it. I referred to these experiments in a Friday evening discourse at the Royal Institution, and exhibited a diagram showing the characteristic curve of a certain valve and its first and second differential curves. The scheme of the circuits and the method of working is fully explained with diagrams on p. 481 of Chapter VI. of the second edition of my book on "The Principles of Electric Wave Telegraphy and Telephony." Since that date a very large amount of information has been collected in my laboratory on the form of the characteristic curves of a number of contact and crystal detectors. Mr. Coursey made for me a very extensive collection of such measurements about eight or nine months ago, but this information and a numerous set of curves depicting the observations has never yet been published. The results do not, however, fall in with any very simple theory of such rectifying detectors. The point to be explained is the reason for this sudden change in curvature in the characteristic. It is obvious that if there is such a change of curvature then the mean value of the current through the detector and, therefore, in a telephone, in series with it, will be increased if we add to the alternating E.M.F. acting on the detector a steady unidirectional E.M.F. of value exactly corresponding to the abscissa of the characteristic at that point of change of curvature.

The AUTHOR, in reply to Prof. Lees, stated that all the coefficients of his fundamental equation were variable with temperature. In reply to Prof. Fleming, he stated that the Paper fully explains the main changes of curvature in the characteristic of a crystal detector, but does not touch on the theory of the vacuum valve.