the number in the 1st Column, *i.e.* when you subtract k -times this number from the 2nd period with its prefix, it will sometimes happen that the subtrahend exceeds the minuend. In this case the subtraction will end with a *minus* digit, which may be indicated by an asterisk. Now find what number of Divisors must be added to the 2nd Column to cancel this *minus* digit, and enter that number, marked with an asterisk, in the 1st Column, and that multiple of the Divisor in the 2nd; and then draw a line below the 2nd Column, and add in this new item.

As an example, let us take a new Dividend, but retain the previous Divisor, changing the sign of k , so that it will become 7003 (*i.e.* $7 \cdot 10^3 + 3$). The sum, as set for working, would stand thus:—

$$\begin{array}{r|rrrr|l} 7003 & 6504 & 318 & 972 & 526 & \text{Rem.} \\ \text{Quot.} & & & & & \end{array}$$

After working, it would stand thus:—

$$\begin{array}{r|rrrr|l} 7003 & 6504 & 318 & 972 & 526 & \text{Rem.} \\ \text{Quot.} & 928 & 790 & 371 & 4473 & \\ \hline & 929 & 2^*531 & 2602 & & \\ & 1^* & 7003 & & & \\ & & & 371 & & \\ & & & 5534 & & \\ & & & \hline & & & 790 & & \end{array}$$

the Mental Process being as follows:—

Divide the 6504 by 7, and enter the Quotient, 929, in the 1st Column, and the Remainder, 1, above the 2nd period. Then subtract, from the 1318, 3-times the 929, entering the result in the 2nd Column, thus. "27 from 8 I can't, but 27 from 28, 1." Enter the 1, and carry the borrowed 2. "48 from 1 I can't, but 8 from 11, 3." Enter the 3, and carry the borrowed 1. "28 from 3 I can't, but 28 from 33, 5." Enter the 5, and carry the borrowed 3. "3 from 1, *minus* 2." Enter it, with an asterisk. Observing that, to cancel this *minus* 2, it will suffice to add *once* the Divisor, enter a (-1) in the 1st Column, and 7003 in the 2nd; and then draw a line below the 2nd Column, and add in this new item: the result is 5534. Then add up the 1st Column, and enter the result, 928, in the Quotient. Now take the 5534 as a new 1st period, and the 3rd period, 972, as a new 2nd period, and proceed as before.

The Rules for case (2) may be derived, from the above, by making $k=1$; and those for case (3) by making $k=1$. I will give worked examples of these; but it will not be necessary to give the Mental Processes.

By making $k=1$, we get Divisors of the form $(h \cdot 10^n \pm 1)$: let us take $(11 \cdot 10^4 - 1)$ and $(6 \cdot 10^5 + 1)$.

$$\begin{array}{r|rrrr|l} 109999 & 107523 & 9 & 10 & 4 & \\ \text{Quot.} & 9774 & 9813 & 0861 & 41846 & \text{Rem.} \\ \hline & 9774 & 107942 & 119474 & & \\ & & 9812 & 1 & & \\ & & & 9475 & & \\ & & & 861 & & \\ \hline & & & & & \\ 600001 & 7239 & 3 & 2 & 3 & \\ \text{Quot.} & 1206 & 58431 & 94595 & 219230 & \text{Rem.} \\ \hline & & 350592 & 4^*7572 & & \\ & & 58432 & 600001 & & \\ & & 1^* & 567573 & & \\ & & & 94595 & & \end{array}$$

In this last example, there is no need to enter the Quotient, produced by dividing the 7239 by 7, in the 1st Column: we easily foresee that the number at the top of the 2nd Column *will be* less than the Divisor, so that there will be no new item in the 1st: hence we at once enter the 1206 in the Quotient.

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By making $k=1$, we get Divisors of the form $(10^n \pm k)$: let us take $(10^4 - 7)$ and $(10^5 + 12)$.

$$\begin{array}{r|rrrr|l} 9993 & 867 & 8159 & 2485 & 7031 & 5276 & 4092 & \\ \text{Quot.} & 868 & 4238 & 2153 & 2104 & 0004 & 4120 & \text{Rem.} \\ \hline & 867 & 14228 & 32130 & 22088 & 19990 & & \\ & 1 & 7 & 21 & 14 & 14 & & \\ & & 4235 & 2151 & 2102 & 4 & & \\ & & 3 & 2 & 2 & & & \end{array}$$

$$\begin{array}{r|rrrr|l} 100012 & 7185 & 6 & 2039 & 10327 & 53118 & \\ \hline & 7184 & 7 & 5822 & 00463 & 47562 & \\ & 7185 & 3^*5819 & 9^*00355 & & & \\ & 1^* & 100012 & 900108 & & & \\ & & 75831 & 9^* & 463 & & \end{array}$$

The first of these two sums is the one I gave to illustrate Mr. Collingwood's Method of working with Divisors of the form $(10^n - k)$.

It may interest the Reader to see the 3 Methods of working the above example—ordinary Division, Mr. Collingwood's Method, and my version of it—compared as to the amount of labour which each entails in the working:—

	Ordinary Division.	Mr. C.'s Method.	My version of it.
Digits written:	202	82	44
Additions, or Subtractions:	204	97	25
Multiplications:	0	70	22

I am assuming that any one, working this example by ordinary Division, would begin by making a Table of Multiples of 9993, for reference: so that he would have *no* Multiplications to do. Still, the great number of digits he would have to write, and of Additions and Subtractions he would have to do, involving a far greater risk of error than either of the other Methods, would quite outweigh this advantage.

By whatever process a Question in Long Division has been worked, it is very desirable to be able to test, easily and quickly, the correctness of the Answer. The ordinary test is to multiply together the Divisor and Quotient, add the Remainder, and observe whether these together make up the given Number, as they ought to do.

Thus, if N be the given number, D the given Divisor, Q the Quotient, and R the Remainder, we ought to have

$$N = D \cdot Q + R.$$

This test is specially easy to apply, when $D = (h \cdot 10^n \pm k)$; for then we ought to have

$$N = (h \cdot 10^n \pm k) \cdot Q + R; \\ = (hQ \cdot 10^n + R) \pm kQ.$$

Now $hQ \cdot 10^n$ may be found by multiplying Q by h , and tacking on n ciphers. Hence $(hQ \cdot 10^n + R)$ may be found by making R occupy the place of the n ciphers. If R contains less than n digits it must have ciphers prefixed; if more, the overplus must be carried on into the next period, and added to hQ .

Having found our "Test," viz. $(hQ \cdot 10^n + R)$, we can write it on a separate slip of paper, and place it below the working of the example, so as to come vertically below N , which is at the top. When the sign in D is '-', we must add kQ to N , and see if the result = T ; when it is '+', we must add kQ to T , and see if the result = N .

Now it has been already pointed out that when, in the new Method, the 1st and 2nd Columns have been worked, the 1st period of the Quotient and the number at the foot of the 2nd Column are the Quotient and Remainder that would result if the Dividend ended with its second period. Hence the Test can be at once applied, before dealing with the 3rd Column. This constitutes a very important new feature in my version of Mr. Collingwood's Method. Every two adjacent columns contain a separate Division-sum, which can be tested *by itself*. Hence, in working my Method, as soon as I have entered the 1st period of the Quotient, I can test it, and, if I have made any mistake, I can correct it. But the hapless computator, who has spent, say, an hour, in working some gigantic sum in Long Division—

whether by the ordinary process or by Mr. Collingwood's Method—and who has chanced to get a figure wrong at the very outset, which makes every subsequent figure wrong, has no warning of the fatal error till he has worked out the whole thing “to the bitter end,” and has begun to test his Answer. Whereas, if working by *my* Method, he would have been warned of his mistake almost as soon as he made it, and would have been able to set it right before going any further.

As an aid to the Reader, I will give the Mental Process in full, for the 2nd and 3rd Columns of the first of the examples worked above.

The Divisor is 6997 (where $h = 7$, $k = 3$). Here you are supposed to have just entered the 281 in the Quotient. The Dividend, for these 2 Columns, is 1972 | 103; the Quotient is 281, and the Remainder 5946. The Test is $hQ.m + R$, (i.e. $7 \times 281000 + 5946$), the Mental Process being as follows.

You write, on a separate slip of paper, the last 3 digits of R , viz. 946, and carry the 5 into the next period, adding it to the 7×281 : thus. “5 and 7, 12.” Enter the 2, and carry the 1. “1 and 56, 57.” Enter the 7, and carry the 5. “5 and 14, 19.” Enter it. Having got your Test, try whether $(N + hQ)$ is equal to it. This you compute, comparing it with your Test, digit by digit, as you go on, thus. “3 and 3, 6.” Observe it in the Test. “0 and 24, 24.” Observe the 4, and carry the 2. “3 and 6, 9.” Observe it. “1972 and 0, 1972.” Observe it. The Test is satisfied.

For Divisors of the form $(m \pm k)$ there is no need to write out the Test: the numbers, which compose it, already occur in the working, and may be used as they stand.

CHARLES L. DODGSON.

Ch. Ch., Oxford, December 21, 1897.

Optical Illusions produced by Observation of Rotating Spirals.

It is well known that if a spiral, such as represented in the figure, rotating in the opposite direction to the movement of the hands of a clock, be observed for some minutes, an impression of circles arising at the centre and disappearing at the periphery is produced; after protracted observation, on looking at printed matter or a person's face, the letters appear to move towards the centre while the face appears to become smaller and recede. If the spiral be rotated in the opposite direction, the circles

appear to be passing towards the centre, in which case the after-effect is more marked and lasts longer, the letters naturally pass centrifugally while the face apparently increases in size.

The latter effect in my own observations has sometimes continued for fifty seconds, while the former never for more than twenty seconds. This effect is practically the same as the “waterfall phenomenon,” in which case the banks appear to move upwards after gazing for some time at the falling water.

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What I wish to draw attention to in this note is the effect which may be observed on closing the eyes after watching a well-illuminated rotating spiral.

If the direction of rotation has produced the impression of centripetally travelling rings, a star composed of radial lines consisting of small granular patches of light passing centrifugally will be noted for a second or two after closing the eyes. The granules pass centripetally if the direction of rotation be reversed. The star will appear symmetrical if the spiral be viewed normally, but if sideways the lines composing it will be curved and the star distorted: to some observers the centre of the star always appears to be further distant than the periphery. I have not been able to find any difference in time of duration of this after-image on rotating the spiral in the two directions.

The colour of the granules is always yellowish, whatever be the colour of the spiral and background. It is interesting to note that, as a rule, after-images of central production can be prophesied on psychological reasoning; while in the above case no one to my knowledge has been able to foretell the appearance of the star, and therefore I am inclined to think that the effect is retinal.

O. F. F. GRÜNBAUM.

Poisonous Koda Millet.

THERE have been several well-ascertained examples of poisoning from diseased or improperly-prepared *Koda* millet (*Paspalum scrobiculatum*) during the past year in India. Owing to the prevailing scarcity of the usual food-grains, it is probable that *Koda* millet has been extensively sold and eaten in localities where its use is ordinarily unknown.

I hoped to undertake an investigation myself into this matter, which is one of great importance both from the hygienic and economic points of view. After consultation, however, with Surgeon-Major van Geyzel, chemical examiner to the Government of Madras, I have decided that any investigation worthy of the name would occupy far more time than I have to give to it. I therefore write to you with a view to eliciting the help of some worker in Britain or elsewhere who has the necessary leisure and facilities for work of this sort. I have samples of the grain, husked and unhusked, from different localities, and can supply the necessary references to the literature of the subject. The investigation, I take it, would have to be of a chemico-biological nature, and would be most appropriate, say, for a thesis, or in connection with some fellowship. I hope that some one who can carry the work right through, and will ascertain the exact source and constitution of the poison (? a volatile alkaloid), will write to me at the address given below. In another ten years, perhaps, there will be some attempt to provide the men and the facilities for such work, even in India, when the Victoria Institute is an accomplished fact.

A. E. GRANT.

Hygiene Laboratory, Medical College, Madras.

Hermaphroditism in the Herring.

I WISH, with your permission, to record a singular “freak of nature” and, at the same time, to ask whether any similar observation has been made by others.

Some fresh herrings were served us here last evening for dinner, and among them was one which, on being opened, disclosed a roe, half of which was “hard” and the other half “soft.” The ova and milt were respectively quite normal in appearance (and flavour), and melted imperceptibly one into the other about midway in the length of the roe, which was, itself, in an obviously natural and undisturbed condition.

Sea Lawn, Torquay, January 12.

DAN. PIDGEON.

A Bright Meteor.

ON Sunday night (December 12), at about 8.15, I witnessed the appearance of a magnificent meteor which seemed to travel from the south-east in the direction of Ursa Major. It broke up into a number of fiery balls of most brilliant colours. The atmosphere was clear and frosty and the moon very bright, but it was for the moment quite eclipsed by the brilliancy of this striking meteoric display. This may be the same phenomenon as that referred to in NATURE for January 6 (see page 228). I did not hear any sound like thunder at the time.

SUSANNA LEHMANN.

Wales Lodge, Wales, Sheffield, January 16.