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THE CAPACITY OF MICA CONDENSERS.

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THE differences in the value of the capacity of the same condenser observed in determinations made at different times (with different galvanometers) have given the impression that mica condensers are not available for measurements of high precision. The manufacturers of the best condensers guarantee an accuracy of only one fourth of one per cent. and even this is not usually trusted by the purchaser. The reason for these differences is that with the method of measurement commonly employed¹ the capacity value is dependent upon the period of the galvanometer. The condenser is charged or discharged through the galvanometer and remains in circuit with it during the whole period of the first throw. The gradual liberation of the absorbed charge produces a current which flows through the coil of the galvanometer the whole period of the throw. The longer this period the greater is the amount of the absorbed charge that affects the throw, and the greater is the value obtained for the capacity of the condenser.

This difficulty can be avoided readily and the results made independent of the period of the galvanometer by opening the circuit

¹Jenkin, Report of British Association, 1863, p. 144; 1867, p. 483. Maxwell, Electricity and Magnetism, 1881, II., p. 383. Stewart and Gee, Practical Physics, 1891, II., p. 407. Kohlrausch, Physical Measurements, 1894, p. 382. Stratton and Milliken, Exps. in Gen. Physics, 1898, p. 83. Henderson, Practical Electricity and Magnetism, 1898, p. 232. Carhart and Patterson, Electrical Measurements, 1900, p. 227. Schluster and Lees, Exercises in Practical Physics, 1901, p. 357. Zeleny and Erikson, Manual of Physical Measurements, 1902, p. 107. Glazebrook, Electricity and Magnetism, 1903, p. 283. Miller, Laboratory Physics, 1903, p. 356.

after the whole measurable part of the free charge has passed and before a measurable amount of the absorbed electricity has been liberated. With this modification the capacities of good condensers can easily be determined to within 0.05 per cent. and an accuracy 0.01 per cent. is attainable. The intermittent current method¹ and the "instantaneous" charge or discharge method² have been described and used for measuring the capacity in terms of the free charge, and under proper conditions fulfil the above requirements. These methods, however, have not been employed generally because of the greater simplicity of the ordinary method and because no definite objection appears to have been raised to it, the dependence of the value of the capacity upon the period of the galvanometer not having been fully appreciated. The "capacity" although distinguished from "instantaneous capacity" appears to have been considered a definite quantity independent of the period of the galvanometer employed.

It is the object of this paper to point out the variations in the value of a capacity as determined by the ordinary method, to show the reasons for these variations and to describe a suitable and convenient apparatus for avoiding them and enabling the mica condenser to be used as a standard of precision.

THE DISCHARGE OF A MICA CONDENSER.

The curve of Fig. 1 shows how the quantity of electricity discharged by a good condenser varies with the time of discharge. It is seen that in the interval *AB*, which is usually less than .01 second, the whole measurable part of the free charge passes through the galvanometer. The absorbed charge begins to be liberated at the beginning of the discharge and continues to flow for a period greater than that of the galvanometer. In a good mica condenser,

¹ Maxwell, loc. cit., p. 384. Thomson, Phil. Trans., 1883. Glazebrook, Phil. Mag., 1884, Vol. 18, p. 98. Electrician, 25, 1890. Rosa, Phil. Mag., 1889, Vol. 28, p. 315. Thomson and Searle, Phil. Trans., 1890. Dittenberger and Gruneisen, Zs. f. Instrk., Apr., 1901, p. 112. Fleming and Clinton, Phil. Mag., 1903, Vol. 5, p. 493. Rosa and Grover, 1905, Bulletin No. 2, Bureau of Standards.

² Gray, Measurements in Electricity and Magnetism, 1888, I., p. 443. Glazebrook and Shaw, Practical Physics, 1893, p. 595. Tory and Pitcher, Manual of Laboratory Physics, 1901, p. 272.

the part of the absorbed charge which is liberated in a few hundredths of a second, AC , is too small to be measurable, but in 2.5 or 5 seconds, the usual periods of the first throw of a galvanometer needle or coil, this quantity becomes considerable. If the galvanometer circuit remains closed during the whole of this time, the total quantity of electricity, DD' or EE' , affecting the throw varies in galvanometers of different periods. The value of the capacity determined in this manner is, therefore, a function of the period of the galvanometer as well as of the condenser itself. It is a variable quantity not adapted for use as an accurate standard. If, however, the galvanometer circuit is opened during the interval BC the quantity of electricity discharged is definite in amount and independent of the period of the galvanometer.

Fig. 1. Time in Seconds.
Quantity Discharged.

THE DISCHARGE KEY.

The special discharge key¹ adopted by the writer for determining the capacity in terms of the free charge is shown in diagram in

Fig. 2.

Fig. 3.

Fig. 2. In the position shown, the condenser C is being charged

¹ A well constructed (Pohl's) mercury-and-rocker commutator may be employed to accomplish the same result. The cross-bars are removed and connections made as shown in Fig. 3. The height of the mercury in the cups is adjusted so that when the rocker is turned from the charging position, shown in the figure, the cell circuit is opened first, at cup 1. The rocker then makes a contact in cup 2 before leaving cup 3, discharging the condenser during the time the two cups are in connection. The period of discharge is varied by changing the height of the mercury in cup 3.

by means of the cell *E*. On depressing the key,¹ the springs *K*, *L* and *M* are brought into metallic connection in succession. One tap on the key (1) opens the standard cell circuit, (2) discharges the condenser through the galvanometer during the time spring *K* is depressing the spring *L* to *M* and (3) opens the condenser circuit when the spring *L* depresses the spring *M*. The key must be held in the last position during the whole time of the first throw.

The period during which the condenser is allowed to discharge depends on the velocity of the tap and upon the length of the space *S* between the springs *L* and *M*. This period is varied by maintaining an approximately uniform velocity for repeated taps while regulating the length of the space *S* by means of the set screw *X*. In the key described above this space could be made of any length up to four millimeters.

Since, for a good condenser, as shown in the curve of Fig. 1, the period *BC* is several times greater than the period *AB*, the adjustment of the key and the maintenance of a sufficiently uniform velocity of tapping give no special difficulty. The operator makes the successive taps of a nearly uniform velocity and the space *S* is adjusted until increasing or decreasing its length does not change the amount of the throw of the galvanometer needle or coil. This space, at the writer's velocity² of tapping while using a galvanometer of 127 ohms resistance and observing to .05 per cent. could be of any length from 0.5 millimeter to the full range (4 mm.) of the key. If measurements were to be made to .01 per cent. the space *BC* would become much shorter, necessitating a greater care in adjustment. If, in any case, it were found necessary, the key could be depressed at a more uniform rate by means of a spring, or some other device could be used to accomplish the same results with greater precision.

When the space *S* of the key is of the proper length, successive throws agree to within one of the estimated parts of the smallest scale division, while, when too short, the usual variations in the velocity of tapping are sufficient to affect the amount of the successive throws.

¹ The springs *K*, *L* and *M* are not in the same vertical plane as shown in the figure. The points of contact only are in line.

² Roughly estimated to be about 60 centimeters per second.

The following tabulated throws of the galvanometer illustrate the action of the key. The discharges shown in Table I. were taken from a Leeds and Northrup Standard Condenser (one microfarad) using a very rapid velocity¹ in tapping the key. The space S regulating the period of discharge and the resistance of the galvanometer circuit were varied. The galvanometer resistance = 127 ohms. R = external resistance in the galvanometer circuit.

TABLE I.

R in Ohms.	Length of Space S in Millimeters.					
	0.2	0.5	1	2	3	4
0	19.25	19.82	19.82	19.82	19.82	19.82
	9.15	19.82	19.82	19.82	19.82	19.82
	11.05	19.82	19.82	19.82	19.82	19.82
500	4.59	12.45	17.95	18.50	19.82	19.82
	3.45	10.70	18.82	19.30	19.82	19.82
	3.88	11.63	18.23	19.70	19.80	19.82

Table II. gives the throws obtained with the above apparatus using different external resistances and a less rapid velocity² of tapping the key. A judgment of the approximate uniformity of successive velocities was obtained by a few up and down movements of the hand before striking the key. $S = 1$ millimeter.

TABLE II.

$R = 0$	$R = 500$	$R = 1,000$	$R = 2,000$
19.82	19.82	19.82	19.40
19.82	19.82	19.82	19.45
19.82	19.82	19.82	19.45

With this velocity of tapping the condenser can be used with any resistance in the galvanometer circuit up to 1,000 ohms.

FREE CHARGE AND ORDINARY CAPACITIES OF CONDENSERS OF DIFFERENT ABSORBING POWERS.

Table III. gives the observations taken with three condensers of different absorbing powers for the purpose of testing the possible

¹ Roughly estimated to be about 60 centimeters per second.

² Roughly estimated to be about 12 centimeters per second.

limits of accuracy that can be attained in measuring the free charge capacities of such condensers.

(I.) = a Leeds and Northrup mica condenser (one microfarad).

(II.) = a mica condenser of large absorbing power made in the laboratory.

(III.) = a paper condenser ("2.8 microfarads").

The space S of the key was measured in terms of the number of turns made by the set screw X , Fig. 1. Each turn represents a distance of one thirty-sixth of an inch or 0.706 mm. Resistance of galvanometer = 127 ohms. Period of first throw = 2.44 seconds. Velocity of tapping key = about 60 cm. per second. Two crystal cadmium cells in series were used in charging. The readings were taken on a circular scale.

TABLE III.

Conden- sers.	Space S in Terms of the Number of Turns of Set Screw X .										Throws, Using the Ordinary Method.
	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	$3\frac{1}{2}$	
(I.)	9.60	13.11	15.70	15.71	15.71	15.71	15.71	15.71	15.71	15.71	15.83
	.10	13.54	15.71	.71	.71	.71	.71	.71	.71	.71	.83
	.25	14.10	14.60	.71	.71	.71	.71	.71	.71	.71	.83
	9.32	13.58	15.30	15.71	15.71	15.71	15.71	15.71	15.71	15.71	15.83
(II.)	11.09	12.50	15.45	15.55	15.51	15.60	15.66	15.72	15.81	15.91	18.05
	9.33	11.20	.48	.51	.56	.62	.65	.65	.70	.91	.05
	8.20	12.19	.43	.49	.56	.55	.65	.73	.75	.85	.06
	9.54	11.96	15.45	15.52	15.54	15.59	15.65	15.70	15.75	15.89	18.05
(III.)	4.54	13.26	14.70	15.00	15.45	15.82	16.33	16.92	16.92	17.81	47.24
	11.81	.63	15.82	.60	16.02	.80	.01	.90	17.50	.72	.24
	8.68	.61	14.63	.39	15.52	.84	.10	.81	16.95	.70	.26
	8.34	13.50	15.05	15.33	15.66	15.82	16.15	16.88	17.12	17.74	47.25

The observations essential for determining the point at which the whole of the free charge has passed through the galvanometer in each of the three cases are plotted in Figs. 4 and 5.

Curve I. shows that for a good mica condenser the point at which the free electricity is all discharged is definite and the residual charge produces no measurable effect for a considerable period thereafter.

Curve II. shows that for the mica condenser of high absorbing power a measurable part of the absorbed charge has affected the

throw before the whole measurable part of the free electricity has been discharged. A close estimate of the free charge can be made from the curve. It is, however, better to consider the part of the

Fig. 4. Length of space S in terms of number of turns of set screw X .
Throws in centimeters.

absorbed charge liberated during the interval the free electricity is discharging as a part of the free charge. With this interpretation the curve shows that the free charge capacity of this condenser can be determined to within 0.2 per cent.

Fig. 5. Length of space S in terms of number of turns of set screw X .
Throws in centimeters.

Curve III., Figs. 4 and 5, shows that in a paper condenser the free charge capacity as interpreted for condenser (II.) can be measured to within 2 per cent.

The free charge and ordinary capacities of the three condensers as determined from Table III., with the aid of the curves of Fig. 4, are given in Table IV. The free charge capacity of condenser (I.), for the purpose of comparison, is assumed to be unity.

TABLE IV.

Condenser.	Capacities.		Difference or the Capacity Value of the Absorbed Charge Effect.	Increment to Free Charge Capacity.
	Free Charge.	Ordinary.		
(I.)	(1.0000)	1.0076	0.0076	0.76 %
(II.)	0.988	1.149	0.161	16.3
(III.)	0.96	3.008	2.05	213

The effect of the absorbed charge in condenser (II.) is seen to be more than twenty times that in condenser (I.). It has been shown that the free charge capacity of condenser (II.) can be determined to within 0.2 per cent. The free charge capacity of condenser (I.), therefore can be measured to within 0.01 per cent. An inspection of curve I, Fig. 4, confirms this conclusion.

A further test was made on condenser (I.) by taking observations to .01 per cent. The period of the first throw of the galvanometer was increased to 9''.14 by suspending lead balls on the lower end of the coil. The observations are given in Table V.

TABLE V.

Length of Space S in Turns of Set Screw X.	
3	1
94.53 cm.	
94.52
	94.50 cm.
	94.50
	94.51
94.53	
94.52	
94.525	94.503

This shows a difference of more than .02 per cent. for a variation of the space S by two turns of the set screw. From this in connection with an inspection of the curves of Figs. 4 and 5 it is seen that the free charge capacity of this condenser can have a well defined value only to within about .005 per cent.

CAPACITIES OF CONDENSERS COMPARED BY MEANS OF GALVANOMETERS OF DIFFERENT PERIODS.

Table VI. shows the ordinary and the free charge capacities of three condensers as obtained with galvanometers¹ of different periods of vibration. The results were obtained by comparison with the free charge capacity of one of the condensers.

TABLE VI.

	Period of 1st Throw = 2''.25.		Period of 1st Throw = 4''.45.		Variation of Value of Ordinary Capacity.	Increment to Free Charge Capacity.	
	Free Charge.	Ordinary.	Free Charge.	Ordinary.		$t = 2''.25.$	$t = 4''.45.$
Leeds & Northrup (1 mcf.) (A).	(1.0000) ¹	1.0073	(1.0000) ¹	1.0114	0.41 %	0.73 %	1.14 %
Mica Condenser (poor) (B).	1.015	1.123	1.015	1.151	2.5	10.6	13.4
Paper Condenser ("1.2" mcf.)	0.46 ²	1.473 ³	0.46	1.559	5.8	220	239

Fig. 6. Time of first throw in seconds.

Fig. 7. Time of first throw in seconds.

Increment due to liberation of absorbed charge.

The capacities in terms of the free charge are shown to be independent of the period of the galvanometer while the ordinary "capacities" vary considerably even in the best mica condenser.

¹ The galvanometers used were both of the d'Arsonval type. The one having the shorter period has an open coil and the other a cylindrical one.

² Value assumed for purposes of comparison.

³ In this case the period of the first throw of the galvanometer coil was 2''.44.

The curves of Figs. 6 and 7 show, approximately, the increments to the free charge capacities of condensers *A* and *B* as the period of the first throw increases.

DAMPING OF THE GALVANOMETER DUE TO THE CONDENSER.

The damping correction,

$$\sqrt{\rho} = \sqrt{\frac{\theta_1}{\theta_2}},$$

was determined on open circuit and in parallel with a condenser, to test experimentally whether the condenser as used in the ordinary method had a measurable effect on the damping. An open coil d'Arsonval galvanometer was used whose coil had a resistance of 55 ohms. The capacity of the condenser was 5 microfarads.

On open circuit $\sqrt{\rho} = 1.0300 \pm 2$.

In parallel with capacity $\sqrt{\rho} = 1.0300 \pm 2$.

In the ordinary method for determining capacity, therefore, no appreciable error¹ is introduced on account of the damping due to the condenser.

RESIDUAL CHARGE VARIES AS THE E.M.F.

The amount of the first residual discharge was tested for different voltages up to 100 volts. It was found to be proportional to the electromotive force employed in charging. The capacity as determined by the ordinary method, therefore, is not influenced in value by varying the charging electromotive force.

The part of the absorbed electricity discharged during the first throw of the galvanometer coil is about one half of the first residual obtained in one minute after a discharge of the free electricity. This varies with the condenser and the period of the galvanometer.

TIME REQUIRED TO CHARGE A CONDENSER.

The difference of potential between the plates of a condenser begins to diminish immediately after the removal of the charging electromotive force on account of the gradual absorption of a part of the electric charge. The longer the time of charging the slower and smaller is the absorption after charging. A condenser must

¹ W. Einthoven, Ann. der Physik, 1905, 16, p. 20.

therefore be charged for a sufficient length of time so that during the interval between the opening of the battery circuit and the discharge no appreciable absorption takes place.

The following observations, Table VII., were taken with the Leeds and Northrup condenser. A Crystal cadmium cell having an internal resistance of about 720 ohms was used as the charging battery. The condenser was short-circuited after each observation. It was then charged for a known period and after charging allowed to stand ten seconds before the discharge.

TABLE VII.

Time of Charging in Seconds.	Throw After Standing 10".	Per Cent. of Charge Absorbed.
1	19.68 cm.	.75
5	19.73	.50
10	19.76	.35
15	19.79	.20
30	19.81	.10
60	19.815	.07
200	19.82	.05
300	19.83	.00
600	19.83	.00

The last two observations are the same as those obtained when the discharge was made immediately after opening the battery circuit. The condenser during the five minutes of charging had absorbed almost its maximum amount so that during the interval of ten seconds the additional part of the charge absorbed was not measurable. Fig. 8 is drawn from the data of Table VII.

When the observations were repeated without short-circuiting the condenser after each discharge, practically the same

Fig. 8. Time of charging in seconds.
Part of free charge absorbed.

throws were obtained as before. This is due to the residual charge being nearly all liberated during the interval the condenser remains discharged. The condition of the condenser in the two cases is therefore practically the same.

When the condenser is used with the special key of Fig. 1, it is discharged almost immediately after the opening of the battery circuit. After charging the condenser one second a discharge with this key gave the maximum throw of 19.83 cm. The amount of absorption during the short interval between the opening of the cell circuit and the discharge is not measurable. When using this key, therefore, it is not necessary under ordinary conditions to give any attention to the time of charging.

ABSOLUTE MEASUREMENTS.

The method here described necessitates for absolute measurements the determination of the constant of the ballistic galvanometer and the observation of a scale reading. These limit the attainable accuracy to one part in 10,000 or to .01 per cent. The intermittent current method when properly used is capable of a somewhat higher accuracy in terms of a resistance standard. For ordinary laboratory determinations, however, the method described in this paper will be found simpler and more convenient.

COMPARISON MEASUREMENTS.

The free charge capacities can be compared to .01 per cent.

Fig. 9.

by the direct deflection method. It is however preferable to use a differential method when measurements are to be made with an accuracy greater than 0.1 per cent. The writer

has found the two following methods satisfactory.

I. Each condenser is connected to a battery and a special discharge key in the usual manner except that the galvanometer used is common to both circuits and the poles of the batteries are placed in reverse order so that only the difference in the two quantities

held by the condensers affect the galvanometer throw. The essential arrangement of the apparatus is shown in Fig. 9. The two keys are depressed by a lever (not shown in the figure) so that both condensers are connected to the galvanometer at the same instant.

(A) If the two charging electromotive forces are equal

$$q = Q - Q' = (C - C')E$$

$$C' = C - \frac{q}{E}$$

The quantity q can be made as large as desired by using the proper charging electromotive force. It is measured by comparison with a discharge from the standard condenser.

(B) If the two charging electromotive forces are only approximately equal

$$q = Q - Q' = CE - C'E'.$$

Interchanging electromotive forces,

$$q' = Q_2 - Q_3 = CE' - C'E.$$

Then

$$q + q' = (C - C')(E + E')$$

and

$$C' = C - \frac{q + q'}{E + E'}.$$

(C) When the two capacities differ greatly in value the charging electromotive forces used are such as to give approximately equal quantities of electricity in the two condensers. Then

$$q = CE - C'E'$$

and

$$C' = \frac{CE - q}{E'}.$$

In case (A) comparisons can be made to the limit of agreement of the two electromotive forces. In case (B) the quantity $(q + q')/(E + E')$ can be determined to within .01 per cent. if the condensers can withstand the electromotive forces necessary to produce a throw of sufficient magnitude. If two capacities differing

in value by 1 per cent. have this difference measured to .01 per cent. a comparison is made to one part in a million or to .0001 per

Fig. 10.

cent. This lower limit, however, has little meaning and ordinarily is difficult or impossible to be attained on account of the inequalities of the absorbed charges.

II. Another method of comparison is the well-known Thomson's method of mixtures except that the special discharge keys are substituted for the Pohl commutator ordinarily used.

The diagram of the apparatus is shown in Fig. 10. When R and R' are adjusted until no throw is obtained while depressing the keys

$$\frac{C}{C'} = \frac{R'}{R}.$$

CONCLUSION.

The capacity of a mica condenser as determined by the ordinary direct deflection method is a variable quantity and a function of the period of the galvanometer. It represents neither the unabsorbed free charge nor the total quantity of electricity held by the condenser. The capacity determined from the free charge is independent of the period of the galvanometer and dependent on the condenser only. Standards of precision must be rated, therefore, in terms of the free charge.

An interval of several hundredths of a second exists between the time of discharge of the free electricity and before the liberation of a measurable quantity of the absorbed charge. The condenser circuit must be opened during this interval. This is most conveniently accomplished by means of a special key. To avoid a loss of the free charge due to the absorption of the condenser the discharge should preferably take place immediately after charging, or the time of charging must be continued for several minutes.

The ordinary methods employed for comparing the capacities

of condensers must also be modified so as to compare the free charges only.

The free charge capacity and the temperature coefficient¹ of a condenser being known, it becomes an instrument of precision especially useful for the accurate determination of the constants of ballistic galvanometers. For this purpose each physical laboratory should contain a condenser whose free charge capacity, including the temperature coefficient, is known to .01 per cent. The accuracy of the constants of ballistic galvanometers as determined by the condenser method employing the usual discharge key cannot be assured even to within one per cent., while with the method described in this paper it should be limited only by the accuracy with which the ballistic throws can be read.

From the considerations given in this paper and since the absorbed charge is not in the true sense a measure of a part of the capacity of a condenser the following definition appears to be more appropriate than that in present use: The capacity of a condenser is the quantity of electricity it retains *unabsorbed* while its plates are at a difference of potential of one volt.

In some cases the condenser is employed in a manner where the time is too short to allow it to become fully charged or discharged or the time is of sufficient length to cause either a part or the whole of the absorbed charge to become effective. In such cases the observed capacity value should be designated by some such term as *effective capacity*, the value being definite for that particular condition of employment only.

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¹ Appleyard, Phil. Mag., 1896, Vol. 46, p. 148. Cassie, Phil. Trans., 1890, Vol. 181, A, p. 1. Terry, PHYS. REV., Sept., 1905, p. 193. Rosa and Grover, *loc. cit.*, p. 185.