ART. XLI.—*The Bi-quartz Wedge Plate Applied to Polarimeters and Saccharimeters*; by FRED. EUGENE WRIGHT. *

IN the preceding paper on the measurement of the extinction angles of mineral plates in the thin section, the general theory of the relative intensity of illumination of the field for different positions of the nicols and also of the crystal section is developed. The different methods for crossing the nicols accurately and also for determining the position of total extinction for a crystal section are considered with respect to their sensitiveness, and the conclusion reached that the best results are obtained by dividing the field into parts in which the planes of polarization are inclined at equal but small angles ($90-\phi$ of the general formula) to the line of division between them (half shade system).† For given conditions of illumination and sensitiveness of the observer's eye, there is always a certain angle ($90-\phi$) for which the phenomena observed are most sensitive to slight movements from the position of true extinction. To reach the maximum efficiency of the half shade device, this angle ($90-\phi$) should therefore be adjustable within the limits prescribed by these conditions.

The conditions for maximum sensitiveness have been worked out for polarimetric purposes with sufficient care and accuracy by a number of investigators,‡ and several instruments have been designed in which high precision is possible, provided only that light of a certain quality and intensity is available and that the substance under investigation (sugar, for example) permits just the right quantity of this light to pass through it to fall within the rather narrow limits in which the observer's eye is most sensitive. In practice, these conditions have proved somewhat exacting, and considerably greater elasticity in the adjustment of the optical system of the instrument with a view to service under a greater variety of experimental conditions, without loss of accuracy, is desired. Mr. Frederick Bates§ of the Bureau of Standards has recently successfully designed an instrument in which this feature has received attention. His polariscope follows the Lippich system with the addition of sets of gear wheels for revolving the large polarizer and analyzer simultaneously in such a way that the angle of revolution of the polarizer at every instant is twice that of the analyzer. He has also demonstrated that the small angle (δ) between the

* The author is indebted to Dr. Arthur L. Day of this laboratory for several important suggestions in the preparation of this paper.

† S. Nakamura, *Centr. f. Min.*, 1905, p. 267-279; P. G. Nutting, *Bulletin. Bureau of Standards*, iii, 249, 1906.

‡ See Winkelmann, *Handbuch der Physik* (2), vol. vi, pp. 1362-3.

§ *Bulletin of the Bureau of Standards*, iv, 461, 1907.

normal to the principal plane of the analyzer and the bisector of the angle between the nicols of the Lippich polarizer can be readily compensated by a slight shift of the zero in the quartz compensator scale. This apparatus is easily superior to the ordinary quartz compensating polarimeter and is of more general application, but it is mechanically difficult and expensive to build and its adjustments are rather sensitive to wear and tear. It therefore occurred to the writer that the bi-quartz wedge plate described in the foregoing paper might serve the same purpose much more simply and therefore be of some practical utility in polarimetric and saccharimetric measurements. By means of this wedge the plane of vibration of waves from the analyzer is made to rotate from 0° to any specified angle by varying the thickness of the wedge employed, the rotation of the one half being right-handed and the other left-handed.

Since the problem of saccharimetry from the practical standpoint reduces to the determination of the exact angle of rotation of the plane of polarized light after its passage through the sugar solution, or, in brief, to the accurate setting of the analyzer or the compensating system with the analyzer, it is apparent that the bi-quartz wedge plate is directly applicable.

The mode of application is the same whether a monochromatic or a white light source is used. If monochromatic light is employed a feasible arrangement of the apparatus is shown in diagram in fig. 1. The monochromatic light, properly restricted in wave length and as intense as possible, is polarized by the nicol prism P. On passing through the solution T its plane of polarization is rotated and the angle of its rotation is determined by revolving the analyzer (fitted with an accurately divided degree circle C) until darkness ensues. The exact position of extinction is then found by inserting the bi-quartz wedge, which offers opportunity for a finer adjustment, and ascertaining that the intensity of illumination of both halves of the wedge is precisely equal for all positions of the wedge plate.*

* The wedge plate should be mounted in a metal frame arranged to slide like the quartz compensator in accurate grooves. The quartz wedge plate used in the examination of extinction angles in crystals is 35^{mm} long, 12^{mm} wide, and is made up of wedges 0.3^{mm} thick at one end and 0.5^{mm} at the other, which are underlain by two quartz plates each .35^{mm} thick. The rotary angle range in each half is from -1.1° to $+3.2^\circ$.

For general polarimetric work, in view of the limited sensitiveness of the eye and the difficulty in obtaining homogeneous illumination of sufficient intensity, it is desirable that the angle $\frac{\alpha}{2}$ should be capable of being increased

to 15° . To accomplish this in a single wedge it would be best to make it 50^{mm} long, 10 or 12^{mm} wide with the thin end of each wedge 35^{mm} and the thick end 1.10^{mm} in thickness, combined with quartz plates 0.4^{mm} thick. This gives a pitch to the wedge of 1.5 : 100 or 8° . The point of zero deflection or the position of the black band will fall 3.3^{mm} from the thin end and the deflection on the thick end will be $\pm 15^\circ$ for sodium light. The total thickness of the wedge plate is 1.5^{mm}. Canada balsam should be used for the cementing material and care must be taken to have the wedge surfaces adjacent to the Canada balsam and also the two surfaces of the complete wedge plate parallel.

Moreover, if the wedge is inserted horizontally and at such a point that its effect on the plane of polarization of transmitted light is precisely zero, a straight, black, vertical band appears in each half of the field similar to the bands in the Babinet compensator (fig. 2). By means of this band, the true position

FIG. 1.

FIG. 1. Proposed arrangement of parts in polarimeter, using bi-quartz wedge plate as sensitive device. P, polarizer; T, sugar solution tube; W, bi-quartz wedge; C, degree circle of analyzer; O, Ramsden ocular; A, analyzer. Although the bi-quartz plate wedge may or may not be attached to the revolving circle mechanism, it seems preferable that its carriage should remain in one plane. The ocular O may be used either in front or back of the analyzer A, and may be a single acromatic lens in place of the positive ocular. Observations can also be made without the aid of the ocular and thus an increase of light intensity gained.

of the analyzer can be found with great accuracy, for the setting is thus made to depend upon the exact alignment of two black bands and the photometric principle of comparing two dimly lighted fields is for the most part eliminated.

Another advantage of this system lies in the fact that the boundary lines between the halves of the wedge can be made of knife-edge sharpness without the disturbing division line in the center of the field produced by the total reflection and con-

sequent depolarization of light waves on the edge of the small prism of the Lippich system. The edges of the two halves of the bi-quartz plate are first polished and then cemented with Canada balsam, which has practically the same refractive index (1.540 compared with 1.544), so that no appreciable total reflection with the accompanying depolarization occurs. This is the same result which was successfully attained by Brace,* who

FIG. 2.

expended an extraordinary amount of care and ingenuity in an effort to free the field from this disturbing dividing line; but his instrument is not of such mechanical construction as to lend itself readily to ordinary laboratory use.

Still another favorable feature of the wedge plate scheme for adjusting the sensibility of a polarimeter is the position of the accumulator or magnifying lens of short focal length, which is necessarily of greater light intensity than the telescopic ocular usually used to view the polarizing system.

FIG. 3.

FIG. 3. Proposed arrangement of parts in quartz compensating polariscope using bi-quartz plate wedge as sensitive device. P, polarizer; T, sugar solution tube; Q, quartz compensating system; W, bi-quartz wedge plate; O, ocular; A, analyzer. As in fig. 1, the ocular O may be used either in front of or back of the analyzer; a single achromatic lens may be substituted for the Ramsden ocular, or the magnifying lenses may be done away with altogether.

If a quartz compensating system be used and a white light source, the arrangement outlined in fig. 3 might prove advantageous. The quartz compensating system has been adopted on

* D. B. Brace, *Phil. Mag.* (6), v, 161, 1903.

most commercial saccharimeters in order that white light may be used, taking advantage of the fact that the rotatory dispersion of sugar solutions is approximately that of quartz. According to the scheme of fig. 3, plane polarized white light emerges from the polarizer *P*, passes through the sugar solution *T*, the different wave lengths being rotated through different angles and thence through the quartz compensating system *Q*, where they are again united and reduced to a common plane of vibration,—that of the original polarizer. The quartz compensating system is inserted until the field when observed through the analyzer appears totally dark. As in the preceding case, this condition is verified with the greatest sensitiveness by inserting a bi-quartz plate wedge *W*.

Up to this point, the advantages offered by the bi-quartz plate wedge are mainly those of simplicity, both in construction and in manipulation, and of greatly decreased cost, without any corresponding sacrifice of accuracy or sensitiveness. Its sensitiveness is adjustable within any limits likely to arise in usual polarimetric work; it is equally adaptable to the monochromatic and to the quartz compensating systems; it is possible with it to do away in considerable part with the photometric principle by the use of Landolt's bands; and finally, the line of division between the hemispheres is so narrow as to be practically invisible as an independent line.

Furthermore, in so far as it avoids the small prism of the Lippich polarizing system, it also avoids an error which is inherent in this system, and incidentally also in the Bates system, due to the loss of light in one half of the field in passing into and out of this superposed prism.* In his analysis of the problem, Bates has shown that the small angle δ between the normal to the principal plane of the analyzer and the bisector of the angle between nicols of the Lippich polarizer, can be allowed for by a slight shift of the zero of the quartz compensating scale for which provision is made in his apparatus. A slight additional correction of the same character made at the same point will also serve to correct for the loss of light by reflection at the end surfaces, and by absorption within the small prism. Supposing this to amount to 10 per cent of the total intensity, the situation can be summed up as follows: If a be the angle between the two prisms of the Lippich polarizer, I the intensity of the light from the large polarizer, then

* It will be recalled that the Lippich system consists, either for monochromatic light or with the quartz compensator and white light, of a large nicol in front of which is placed a small nicol covering one-half the field of the former. The plane of polarization of this nicol makes a small angle with the plane of the large nicol. In the matter of the loss of light at the surfaces and by absorption within prisms, the system is therefore not symmetrical with respect to the line dividing the fields.

the intensity I' of the light emerging from the small nicol will not be precisely $I' = I \cos^2 a$, but $I' = .9 I \cos^2 a$.

If the two fields are matched to show equal intensity, the plane of the analyzer will not in general coincide with the bisector of the angle a , but will include with it a small angle δ (fig. 4).

FIG. 4.

An equation from which the angle δ can be figured when a is given has been derived by Bates, but his expression as noted above does not take cognizance of the loss of light in the extra prism and accordingly requires slight modification.

Let OP_1 = direction of plane of large polarizer

OP_2 = direction of plane of small polarizer

a = angle between two polarizers; angle between the normal (OC) to the plane of analyzer AA_1 , and I_1 intensity of light in the field covered by the large polarizer alone when viewed through analyzer; and I_2 intensity of light of the field covered by the small polarizer. The angle P_1OA angle between large polarizer and analyzer is

$$\angle P_1OA = \frac{\pi}{2} - \frac{a}{2} + \delta$$

while

$$\angle P_2OA = \frac{\pi}{2} - \frac{a}{2} - \delta$$

From the intensity formula of the preceding article :

$$I_1 = \cos^2 \angle P_1OA = \sin^2 \left(\frac{a}{2} - \delta \right)$$

$$I_2 = .9 \cos^2 a \cos^2 \angle P_2OA = .9 \cos^2 a \sin^2 \left(\frac{a}{2} + \delta \right)$$

The condition of matched fields is

$$I_1 = I_2$$

or

$$\sin^2 \left(\frac{a}{2} - \delta \right) = .9 \cos^2 a \sin^2 \left(\frac{a}{2} + \delta \right)$$

or

$$\sin \frac{a}{2} \cos \delta - \cos \frac{a}{2} \sin \delta = \sqrt{.9} \cos a \sin \frac{a}{2} \cos \delta + \sqrt{.9} \cos a \cos \frac{a}{2} \sin \delta$$

$$\sin \frac{a}{2} \cos \delta \left(1 - \sqrt{.9} \cos a \right) - \cos \frac{a}{2} \sin \delta \left(1 + \sqrt{.9} \cos a \right) = 0$$

$$\therefore \quad \operatorname{tg} \delta = \frac{1 - \sqrt{.9} \cos a}{1 + \sqrt{.9} \cos a} \cdot \operatorname{tg} \frac{a}{2}$$

or

$$\operatorname{tg} \delta = \frac{1.05409 - \cos a}{1.05409 + \cos a} \cdot \operatorname{tg} \frac{a}{2}$$

The formula of Bates expressed in this notation reads

$$\operatorname{tg} \delta = \frac{1 - \cos a}{1 + \cos a} \cdot \operatorname{tg} \frac{a}{2} = \operatorname{tg}^3 \frac{a}{2}$$

and differs from the above only in the constant in the fraction. For the sake of comparison the angles δ have been figured for $a = 1^\circ$ to 15° by both formulas and included in the following table:

$\frac{a}{2}$	δ (new formula)	δ (Bates formula)
0°	0'	0'
1°	2'	0'
2°	3'	0'
3°	4'	1'
4°	7'	1'
5°	10'	2'
6°	14'	4'
7°	18'	6'
8°	22'	10'
9°	28'	14'
10°	35'	19'
11°	43'	25'
12°	52'	33'
13°	1°03'	42'
14°	1°16'	53'
15°	1°30'	1°06'

This table as well as a discussion of the two formulae shows that the angle δ is increased in every instance by reason of the loss of light by reflection in the small nicol prism.

These differences can be allowed for upon the scale of the quartz compensator. The operation for any angle a consists in first adjusting the zero of the quartz compensator scale with respect to the analyzer without intervening sugar solution

until the illumination of the two halves of the Lippich polarizer system is the same. After the introduction of the sugar solution, the quartz compensator is inserted until the original equal intensity of the halves is restored and the angle of rotation derived directly from the compensator scale.

This being the case, it would considerably simplify the construction to allow the analyzer to remain rigidly fixed and to revolve the two nicol prisms of the Lippich polarizer system, thus eliminating the complicated gearing of the Bates polariscope, which mechanically is an exceedingly difficult piece of apparatus to construct and to operate without lost motion. The revolution of the polarizing prisms in equal and opposite angles can be accomplished either by means of the worm thread device adopted in the binocular of fig. 11 of the preceding article, or by a grooved arm into which pins connected with the supporting collars of the nicols fit and slide as the arm is inserted. Mechanically these devices are not difficult of construction and the angle of revolution can be read off accurately.

In the foregoing pages the bi-quartz plate wedge is suggested as a simple and effective basis for the construction of a polarimeter of adjustable sensibility in which the error from the asymmetry of the Lippich system, together with all the serious complications of mechanism, are completely avoided without loss of accuracy. Such a plate has been constructed and successfully applied to the exact location of crystal extinctions, but unfortunately pressure of other duties has prevented the writer from actually constructing a saccharimeter. Through the courtesy of Dr. Bates of the Bureau of Standards, however, an opportunity was given to test the wedge on a large and accurate standard polarimeter illuminated by homogeneous green light from a mercury quartz-glass arc. Its performance was entirely satisfactory, minute displacements of the analyzer from its position of true extinction being readily detected.

Geophysical Laboratory,
Carnegie Institution of Washington,
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