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James Ivory M.A. F.R.S.

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XLIII. *Observations relating to the Depression of Mercury in Glass Tubes ; occasioned by an Article in the last Quarterly Journal of Science.* By JAMES IVORY, M.A. F.R.S.

To Mr. Tilloch.

SIR, — I HAVE to beg the favour of your inserting in your next publication, the following observations occasioned by an article in the last Quarterly Journal of Science. They relate to the depression of mercury in glass tubes, for computing which two tables have been published in the Supplement of the *Encyclopædia Britannica* ; one under the head of COHESION OF FLUIDS, and the other under that of FLUIDS.

It is necessary to begin with saying a few words of the *series* by which the first of the two tables is constructed. If we neglect all the terms of each of the coefficient-serieses, except the first, as may be done in capillary tubes with very minute bores, the series for computing the depression will become

$$s = (bx) + \frac{gx^3}{48} \cdot (bx)^3 + \frac{gx^5}{128} (bx)^5 + \frac{gx^7}{256} (bx)^7 + \&c.$$

Now here, all the coefficients after the first being small, the product bx , which I consider as the quantity sought, will not be much different from s ; and it is manifest that, when s is considerable, a great number of the terms must be taken in, if the approximation is to be carried to many figures, the coefficients decreasing slowly. In the case of glass and mercury, s is nearly $\frac{3}{4}$; and, taking into account the rate of decrease of the coefficients, the total convergency of the series may be reckoned at $\frac{1}{4}$; and the first four or five terms will give an approximation extending to five figures. Accordingly it will be found that, in the table, the depressions for the smallest bores are nearly exact.

But the case is different when the diameters of the tubes have a greater magnitude. Then the values of the coefficient-serieses increase greatly, and cannot be computed with any degree of certainty from two or three of their first terms. In reality all these serieses, excepting the first which is very regular and convergent, diverge in their first terms instead of converging, when $\frac{gx^2}{4}$ is nearly equal to, or greater than, unit. The divergency indeed goes on to a certain point only, after which the serieses will converge, and they will ultimately converge very rapidly as in the first one. In this manner of computing therefore, when the tubes have large bores, the degree of exactness in the result will depend entirely on the three first terms of the value of the sine of depression ; since these terms are the only ones in which the co-

efficients of the unknown quantity can be computed with any tolerable precision.

If the exactness of the result would be affected in the fourth and fifth decimal places by omitting entirely all the terms in the value of the sine except the three first, it is to be feared that a like inaccuracy will occur although some of the omitted terms be taken in, but with inexact coefficients, greatly short of the real values, not amounting perhaps to the 10th, or the 100dth, or even the 1000dth, part of the truth.

Such appears to be the objection to this mode of computation; and the coefficient-serieses, after the first and second, are so complicated that the disadvantage does not appear to admit of a remedy. Still however the method is possessed of considerable accuracy, which it owes to the circumstance, noticed by the author, that the first term alone brings out the truth within a $\frac{1}{36}$ th of the whole.

The rules by which the other table is constructed are investigated on this principle; that the quantity denoted by f is very nearly equal to unit in the case of small capillary tubes, and even in the largest bores it decreases only to a certain limit which is greater than $\frac{2}{3}$. A formula is therefore sought for determining f when it differs sensibly from the limit; and in all other cases, the same quantity is supposed to coincide with the limit. The rules in the article *FLUIDS* in the Supplement to the *Encyclopædia* will determine the depression to five places of figures, and are therefore more than sufficient for any practical purpose. But it is by no means supposed that it is impossible to find other and better rules for the same purpose; as I shall presently show by giving another formula, which has the advantage of determining the depression directly from the given quantities without the solution of an equation. I put q for the depression sought; l , for the diameter of the tube; z , for the sine of depression $= .735$; $t = \left(\frac{7l}{2}\right)^2$; and

$$\lambda = 1 + \frac{1}{1^2} \cdot \frac{t}{2} + \frac{1}{1^2 \cdot 2^2} \cdot \frac{t^2}{3} + \frac{1}{1^2 \cdot 2^2 \cdot 3^2} \cdot \frac{t^3}{4} + \&c.; \text{ then}$$

$$\begin{aligned} q = \frac{z}{49 \cdot l \cdot \lambda} - \frac{l}{4} \cdot \frac{z^3}{\lambda^3} \cdot \left\{ \frac{1}{12} + \frac{t}{36} + \frac{11 \cdot t^2}{1440} + \frac{t^3}{1200}, \right. \\ - \frac{l}{4} \cdot \frac{z^5}{\lambda^5} \cdot \left\{ \frac{1}{32} + \frac{7 \cdot t}{384} + \frac{19 \cdot t^2}{1920} + \frac{523 \cdot t^3}{161280}, \right. \\ - \frac{l}{4} \cdot \frac{z^7}{\lambda^7} \cdot \left\{ \frac{1}{64} + \frac{29 \cdot t}{1920} + \frac{419 \cdot t^2}{32256}, \right. \\ \left. - \frac{l}{4} \cdot \frac{z^9}{\lambda^9} \cdot \left\{ \frac{7}{768} + \frac{41 \cdot t}{3072} + \frac{41029 \cdot t^2}{2580480}, \&c. \right. \right. \end{aligned}$$

This formula will extend to all tubes not much exceeding six-tenths of an inch; and by means of it the results in the table may

may be confirmed. If two independent methods of computation agree in bringing out the same numerical values, a greater proof of exactness can seldom be obtained. Now having computed the depressions in the table by the formula just set down, I have found the same results as before, with slight differences in the last place of figures, except in the cases of the bores 0.5 and 0.6, in each of which a numerical error has been detected. In the first instance, $l = .5$; $t = 3.0625$; $\lambda = 3.546$; then

$$q = \frac{.01445}{l \lambda} = .00815.$$

In the other instance, $l = 0.6$; $t = 4.41$; $\lambda = 5.579$; and

$$q = \frac{.01445}{l \lambda} = .00431.$$

These results are confirmed by the new formula. The numbers in the table, viz. .00835 and .00443 are therefore both erroneous; although the real state of the case is not on this account much altered.

I may observe here that, in the long run, or for tubes large enough, the expression,

$$q = \frac{.01445}{l \lambda},$$

will coincide with the formula given long ago by Laplace at p. 65 of the Supplement to his Theory of Capillary Action. The truth is, that λ continually tends to a final expression as the bore of the tube becomes greater; and it is only when that quantity has attained its ultimate value that the two rules will coincide. Laplace's formula fails in giving an approximation near enough the truth for tubes contained in the table; but it would be found to have all the exactness to be wished for in larger tubes, lying beyond the scope of the table.

In conclusion, sir, I am sorry that in this investigation I was obliged to introduce such obnoxious quantities as exponentials, &c., and I must express my regret that I could not accomplish what I had in view by means of the *Taylorian Theorem*, that universal solvent of all analytical difficulties, as it has lately been discovered to be. Believe me to be, &c.

April 7, 1821.

J. IVORY.

XLIV. *True apparent Right Ascension of Dr. MASKELYNE'S 36 Stars for every Day in the Year 1821. By the Rev. J. GROOBV.*

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1821.