Joint Resource Allocation in Multicarrier Based Cognitive Networks with Two-Way Relaying

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***Abstract*—The problem of resource allocation in two-way relay network is considered in this paper. The cognitive radio (CR) context is tackled where a amplify and forward (AF) OFDM-based dual-hop multiple relay system is investigated. The joint allocation of subcarrier pairing, relay selection and power allocation is performed under both interference and individual power constraints where the target is to maximize the achievable data rate of the network. The dual decomposition is adopted to obtain the optimal solution. Two sub-optimal schemes are proposed also to reduce the computational complexity of the optimal scheme. Simulations are presented to compare the performance of the optimal scheme with that of the proposed sub-optimal schemes.**

Keywords*—Cognitive radio, OFDM, Two-way relaying, Resource allocation.

I. Introduction

Enabling unlicensed users to utilize the licensed spectrum opportunistically will improve the overall spectral efficiency. Cognitive radio (CR) is considered recently as the enabling technology for such a flexible spectrum usage. In CR networks, the secondary users (SUs) are allowed to use the licensed spectrum as long as the interference introduced to the primary user (PU) is not harmful. The CR principle can be applied in a relay network specially when there is no direct link between nodes that need to exchange information. Instead, intermediate nodes act as relays such that they guarantee better channel conditions and less transmission power, implies low level of interference caused to PUs than the direct transmission case.

Spatial diversity gains inherited in multiuser systems can be utilized by using the relay networks without the need of multiple-antenna per node The relying schemes are divided, generally, into two main categories: one-way and two-way relay networks. In one-way relaying, the relay nodes receive a signal from a source node in the first time slot and retransmit it to a destination node in the second time slot, using the common cooperative schemes like decode and forward (DF) or amplify and forward (AF), which takes two time slots for one direction transmission. Consequently, in one way relaying, four time slots are needed if two nodes want to establish full transmission since they cannot transmit at the same time. On the other hand, in the two-way relaying scheme the relay nodes receive signals from transceivers in the first time slot, also called multiple access phase (MA), and then in the second time slot -broadcast phase (BC)- they broadcast the received signals to the transceivers. This overcomes the one way relay scheme and doubles the spectral efficiency since two time slots, only, are needed to full exchange of information between the two nodes. Employing the OFDM technique increases the spectral efficiency by transmitting information over multiple orthogonal narrowband subcarriers besides being very effective in mitigating inter-symbol interference (ISI) and combating frequency selective fading.

Resource allocation in relay communication networks has been extensively discussed in literature. In [1], Shaat and Bader discussed the joint power and subcarrier allocation in OFDM based cognitive one-way relay network. Vu and Kong studied in [2] the optimal power allocation in non-cognitive two-way decode-and-forward OFDM relay network where three time slot transmission is considered. In [3] , a joint resource allocation was designed in AF OFDM based two-way non cognitive system where power allocation, subcarriers assignment and relay selection are jointly optimized. Jang et al. showed in [4] a two-step approach to power allocation for OFDM two-way AF network where a total power constraint scenario is proposed. In [5], Ubaidulla and Aissa proposed a joint relay selection and optimal power allocation among the SU nodes in a twoway relay CR network achieving maximum throughput under transmit power and PU interference constraints. Multiuser twoway AF relay methods for beamforming systems were discussed in [6] where multiple-input multiple outputs (MIMO) relay transceiver processing was proposed. Ho et al. in [7] considered an AF scheme for two-way relaying over OFDM, in which two nodes exchange information via a relay where they performed power allocation for the relay and transceiver nodes. The work in [2], [6], and [7] discuss two-way relaying systems in non-cognitive environment which is not efficient in cognitive one due to the additional interference constraint. A two-way relay network in CR system was adopted in [8], where linear signal processing is done at the relay station to remove inter-pair interference for SUs and a power control algorithm is employed to maximize the sum rate of the secondary network while ensuring no harmful interference is introduced to PUs. The work in [5] and [8] is not valid for the multicarrier systems.

The main contribution of this paper is to jointly optimize the power for transceivers and relay nodes, subcarrier pairing and relay assignment that achieves the best capacity with minimal interference to PUs in OFDM-based cognitive two-way relay network. An optimal solution based on the dual decomposition method is proposed. An efficient suboptimal scheme which achieves a near optimal performance with a drastic reduction in the computation complexity is also pre-

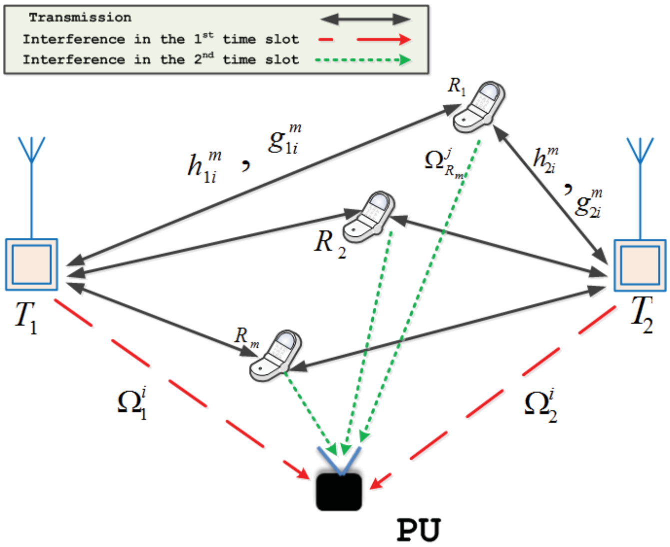

Fig. 1: System model of a two-way relaying OFDM CR network.

sented.

II. System Model and Problem Formulation

A CR relay system coexists with the primary system in the same geographical area is considered as shown in Fig. 1. an OFDM-based two-way relay CR network with multiple relays is investigated. Due to bad channel conditions, large distance and/or the existence of obstacles, there is no direct link between the two transceiver nodes T_1 and T_2. The transceivers try to exchange their information through M relay nodes. The network frequency spectrum is divided into N orthogonal subcarriers each having a Δf bandwidth. Perfect channel state information (CSI) of all links is available and the subcarriers and power can be feasibly allocated by a centralized scheduler or by one of the transceiver nodes. Moreover, all sub-channels are assumed to experience independent, frequency-selective fading. The CR system can employ the temporarily unused PU bands guaranteeing that the total interference introduced to the PUs does not exceed the maximum interference threshold, I_{th}, prescribed by the PU.

The relay nodes are assumed to be half-duplex, thus receiving and transmitting in two different time slots. To complete a full information exchange, two phases are considered, MA phase and BC phase. In the MA phase, T_1 and T_2 transmit their data simultaneously to the selected m^{th} relay using the same subcarrier. In the BC phase, the selected relay amplifies the received signals, and broadcasts them to the two transceiver nodes. Once received, T_1 and T_2 can extract the required information by canceling self-interference.

The relay node, R_m, receives the combined signal on subcarrier i in the MA phase and then amplifies and broadcasts it on another subcarrier j in the BC phase. The subcarrier-pairing scheme between the two phases is deployed, where subcarrier i in the first time slot and its corresponding subcarrier j in the second time slot form a subcarrier pair $\langle i, j\rangle$. Let h_{1i}^m and h_{2i}^m denote the channel coefficients over the i^{th} subcarrier from T_1 and T_2 to the relay m respectively. Similarly g_{1j}^m and g_{2j}^m denote the channel coefficients over the j^{th} subcarrier from the selected relay node to T_1 and T_2, respectively. $\Omega_k^i,\ k \in \{1, 2\}$ and $\Omega_{R_m}^j$ are the subcarrier gains between the PU and the transceivers and relay nodes, respectively. In order to avoid the interference among the relays, each subcarrier pair is only allowed to be allocated to one relay node, but not vice versa. Accordingly, more than one pair of subcarriers may be assigned to a relay node.

In the MA phase, the received signal Y_{mi} at the m^{th} relay over subcarrier i can be expressed as

$$Y_{mi} = h_{1i}^m \sqrt{p_{1i}^m} X_{1i} + h_{2i}^m \sqrt{p_{2i}^m} X_{2i} + Z_{im}, \tag{1}$$

where X_{ki} $(k \in \{1, 2\})$ is the unit power transmitted symbol of the terminal node T_k over subcarrier i, p_{ki}^m is the average transmission power, and Z_{im} is the independent complex Gaussian noise with zero mean and variance σ_{im}^2. By application of the central limit theorem, the interference induced by the PU to the CR system is assumed to be included in Z_{im}.

The received signal by the m^{th} relay is multiplied by the amplification factor $D = 1/\sqrt{p_{1i}^m |h_{1i}^m| + p_{2i}^m |h_{2i}^m| + \sigma^2}$ and broadcasted to the transceivers. Once the signals are received, the transceiver nodes extract the desired signals by canceling self-interference. The received signals at the terminal nodes T_1 and T_2 over subcarrier j in the BC phase are given by

$$Y_{1j} = D g_{1j}^m \sqrt{p_{Rj}^m} Y_{mi} + Z_{1j} \tag{2}$$

$$Y_{2j} = D g_{2j}^m \sqrt{p_{Rj}^m} Y_{mi} + Z_{2j}, \tag{3}$$

where p_{Rj}^m denote the average transmission power of the relay node R_m over subcarrier j and Z_{nj} is the independent complex Gaussian noise with zero mean and variance σ_{nj}^2 $(n \in \{1, 2\})$. Without loss of generality, the noise variance is assumed to be constant for all subcarriers, i.e., $\sigma_{im}^2 = \sigma_{nj}^2 = \sigma^2$.

The received end-to-end signal-to-noise ratio (SNR) at T_1 and T_2 through R_m over the subcarrier pair $\langle i, j\rangle$ can be expressed as [3]

$$SNR_1 = \frac{(p_{2i}^m p_{Rj}^m |h_{2i} g_{1j}|^2)}{\sigma^2 (p_{Rj}^m |g_{1j}^2| + p_{1i}^m |h_{1i}^2| + p_{2j}^m |h_{2i}^2| + \sigma^2)}. \tag{4}$$

$$SNR_2 = \frac{(p_{1i}^m p_{Rj}^m |h_{1i} g_{2j}|^2)}{\sigma^2 (p_{Rj}^m |g_{2j}^2| + p_{1i}^m |h_{1i}^2| + p_{2j}^m |h_{2i}^2| + \sigma^2)}. \tag{5}$$

The end-to-end AF data rate on a given subcarrier pair $\langle i, j\rangle$ that is allocated to the m^{th} relay, $R_{AF}^{m,i,j}$, is expressed as [5]

$$R_{AF}^{m,i,j} = \frac{1}{2} \log_2 (1 + SNR_1) + \frac{1}{2} \log_2 (1 + SNR_2). \tag{6}$$

The pre-log factor of $(1/2)$ in equation (6) above, is due to the fact that two time-slots are required for the complete transmission process.

Let $\mathbf{p} \triangleq (p_{1i}^m, p_{2i}^m, p_{Rj}^m)$ represent the transmission power of nodes T_1, T_2 and the selected relay R_m, respectively. All channel gains for the network can be adopted by convectional channel estimation approaches already used in non-CR systems. $\psi_{m,i,j} \in \{0, 1\}$ is the relay selection indicator with $\psi_{m,i,j} = 1$ when the subcarrier pair $\langle i, j\rangle$ is allocated to the relay R_m. Moreover, $\theta_{i,j} \in \{0, 1\}$ is the subcarrier-pairing indicator, that is, if subcarrier i in the first time slot is paired with subcarrier j in the second time slot, then $\theta_{i,j} = 1$ and otherwise $\theta_{i,j} = 0$.

Our objective is to jointly optimize the power, relay assignment and subcarrier pairing in order to maximize the throughput of the multi-relay two-way OFDM CR system and guarantee that the instantaneous interference introduced to the primary system is below the maximum tolerable threshold I_{th}. The optimization problem can be formulated as follows

$$\begin{aligned}
&\max_{\psi_{m,i,j},\theta_{i,j},\mathbf{p}>0} \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{m=1}^{M} \theta_{i,j}\psi_{m,i,j} R_{AF}^{m,i,j} \\
&\text{- (C1): } \sum_{i=1}^{N} \theta_{i,j} = 1, \forall j; \text{ and } \sum_{j=1}^{N} \theta_{i,j} = 1, \forall i \\
&\text{- (C2): } \sum_{m=1}^{M} \psi_{m,i,j} = 1 \quad \forall i, j \\
&\text{- (C3): } \sum_{i=1}^{N}\sum_{m=1}^{M} P_{ki}^{m} \leq P_k, \quad k \in \{1,2\} \\
&\text{- (C4): } \sum_{j=1}^{N} P_{Rj}^{m} \leq P_R, \quad \forall m \\
&\text{- (C5): } \sum_{i=1}^{N}\sum_{m=1}^{M} \left(\Omega_1^i p_{1i}^m + \Omega_2^i p_{2i}^m\right) \leq I_{th} \\
&\text{- (C6): } \sum_{m=1}^{M}\sum_{j=1}^{N} \Omega_{R_m}^{j} p_{Rj}^{m} \leq I_{th}.
\end{aligned} \tag{7}$$

(C1) expresses the subcarrier allocation constrain. It implies that each subcarrier in the MA phase is paired with one, and only one subcarrier in the BC phase. (C2) represents the relay selection constraint which indicates that each subcarrier pair can be assigned to one relay only. (C3) and (C4) express the individual power constraints in the transceivers and the different relays, respectively. I_{th} is the maximum tolerable interference to the primary users expressed by constraints (C5) and (C6).

III. Optimal Resource Allocation Based on Dual Decomposition

The optimization problem satisfies the time sharing condition described in [9]. Hence, the duality gap of the problem approaches zero as the number of subcarriers is sufficiently large regardless of the problem convexity. Thus, in this section the dual decomposition technique is used to find the optimal solution of the optimization problem. The dual problem is expressed as

$$\min_{\boldsymbol{\lambda}>0} D(\boldsymbol{\lambda}). \tag{8}$$

with $\boldsymbol{\lambda} = [\lambda_{T_1}, \lambda_{T_2}, \lambda_{R_1}, \cdots, \lambda_{R_M}, \lambda_{I_1}, \lambda_{I_2}]$, where $(\lambda_{T_1}, \lambda_{T_2})$ and $(\lambda_{R_1}, \cdots, \lambda_{R_M})$ are non-negative dual variables associated with individual power constraints (C3) and (C4). Moreover, the dual variables $(\lambda_{I_1}, \lambda_{I_2})$ are associated with the tolerated interference to the PU constraints (C5) and (C6). The dual function $D(\boldsymbol{\lambda})$ is defined as

$$\begin{aligned} D(\boldsymbol{\lambda}) \triangleq &\max_{\psi_{m,i,j},\theta_{i,j},\mathbf{p}>0} \mathcal{L} \\ \text{s.t.} \quad &(C1), (C2) \end{aligned} \tag{9}$$

where the Lagrangian $\mathcal{L}$ is given by

$$\begin{aligned}
\mathcal{L} = &- \sum_{m=1}^{M}\sum_{i=1}^{N}\sum_{j=1}^{N} \theta_{i,j}\psi_{m,i,j} R_{AF}^{m,i,j} + \sum_{k=1}^{2} \lambda_{T_k}\left(P_k - \sum_{i=1}^{N}\sum_{m=1}^{M} P_{ki}^{m}\right) \\
&+ \sum_{m=1}^{M} \lambda_{R_m}\left(P_R - \sum_{j=1}^{N} P_{Rj}^{m}\right) + \lambda_{I_2}\left(I_{th} - \left(\sum_{m=1}^{M}\sum_{j=1}^{N} \Omega_{R_m}^{j} p_{Rj}^{m}\right)\right) \\
&+ \lambda_{I_1}\left(I_{th} - \left(\sum_{i=1}^{N}\sum_{m=1}^{M} \left(\Omega_1^i p_{1i}^m + \Omega_2^i p_{2i}^m\right)\right)\right).
\end{aligned} \tag{10}$$

The dual function in (9) can be rewritten as follows

$$\begin{aligned}
D(\boldsymbol{\lambda}) = \max_{\psi_{m,i,j},\theta_{i,j},p>0} \Bigg[&- \sum_{m=1}^{M}\sum_{i=1}^{N}\sum_{j=1}^{N} \theta_{i,j}\psi_{m,i,j}\mathcal{W}_{m,i,j} + \\
&\sum_{k=1}^{2} \lambda_{T_k} P_k + \sum_{m=1}^{M} \lambda_{R_m} P_R + I_{th}\left(\lambda_{I_1} + \lambda_{I_2}\right) \Bigg],
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
\mathcal{W}_{m,i,j} = R_{AF}^{m,i,j} - \lambda_{T_k} P_{ki}^{m} - \lambda_{R_m} P_{Rj}^{m} - \\
\lambda_{I_1}\left(\Omega_1^i p_{1i}^m + \Omega_2^i p_{2i}^m\right) - \lambda_{I_2}\Omega_{R_m}^{j} p_{Rj}^{m}.
\end{aligned} \tag{12}$$

A two phase solution of the dual problem is adopted. First, the resources variables $\{\psi_{m,i,j}, \theta_{i,j}, \mathbf{p}\}$ are optimized for a given feasible dual variables vector $\boldsymbol{\lambda}$ and then a sub-gradient method is applied to optimize $\boldsymbol{\lambda}$ where each of $\{\psi_{m,i,j}, \theta_{i,j}, \mathbf{p}\}$ is refined at each iteration. Therefore, starting by assuming initial values for the dual variables and assuming that the subcarriers $\langle i, j\rangle$ are already matched and allocated to the m^{th} relay, the optimal power allocation can be determined by solving the following sub-problem for every (m, i, j) assignment

$$\max_{p_{1i}^m, p_{2i}^m, p_{Rj}^m} \mathcal{W}_{m,i,j} \qquad \text{s.t.} \qquad p_{1i}^m, p_{2i}^m, p_{Rj}^m \geq 0. \tag{13}$$

Achieving the optimal power allocation is not a trivial problem because of the different radio link conditions that are faced by the multicast users, that share the same resources. Therefore, different possibilities have to be considered in performing the power allocation, in multicast networks, such as the best or worst user within each subchannel or the requirements of the individual users [10]. Based on that, the optimal power allocation can be obtained via searching over the power of T_1, T_2 and R_m and considering that each takes discrete values over a number of power levels L and that the interference constraint is not violated.

By substituting the solution of (13) into (12), the power variable can be evaluated and the best relay assignment can be determined for every $\langle i, j\rangle$ pair by solving the following optimization problem

$$\max_{\psi_{m,i,j}} \mathcal{W}_{m,i,j} \qquad \text{s.t} \qquad (C2). \tag{14}$$

Therefore, the optimal allocation strategy is achieved by assigning each $\langle i, j\rangle$ pair to the relay which maximizes $W_{m,i,j}$. Accordingly, $\psi_{m,i,j} = 1$ if $m = \arg\max_{m} \mathcal{W}_{m,i,j}$ and zero otherwise.

Once the power levels as well as the best relay are determined for all the subcarrier pairs, the optimal subcarrier pairs is determined by solving the following problem

$$\max_{\theta_{i,j}} \mathcal{W}_{m,i,j} \qquad \text{s.t} \qquad (C1). \tag{15}$$

The problem in (15) is a linear assignment problem which can be efficiently solved by the Hungarian algorithm with a complexity of $\mathcal{O}(N^3)$ [11].

The sub-gradient method can be used to solve the dual problem with guaranteed convergence [12] since a dual function is always convex. Based on initially selected dual variables vector, the different dual variables can be updated at the $(i+1)^{th}$ iteration. With the updated values of the dual variables, the different optimization variables are evaluated again. The iterations are repeated until convergence.

IV. Proposed Suboptimal Algorithms

The optimal solution has a high computational complexity. In order to solve the problem efficiently, two low complexity suboptimal algorithm is proposed. It should be noted that most of the complexity resides in the power allocation and the Hungarian algorithms. Focusing on these two parts, the proposed algorithm tries to simplify the computational complexity and get an efficient algorithm. The proposed algorithm starts by distributing the power of the relays and transceivers uniformly over the subcarriers and assumes that the interference introduced to the PU by every subcarrier is uniform. Therefore, the power allocation in the transceivers can be found using the following relation

$$p_{ki}^{m} = \min\left(\frac{P_k}{N}, \frac{I_{th}}{\Omega_k^i N}\right); k \in \{1,2\}. \quad (16)$$

while that of the relays can be found according to the following formula

$$p_{Rj}^{m} = \min\left(\frac{P_R}{N}, \frac{I_{th}}{\Omega_{R_m}^j N}\right). \quad (17)$$

The two proposed sub-optimal schemes can be summarized as follows:

1) **Scheme 1**: the powers are allocated according to (16) and (17), while the relay assignment and subcarrier paring are performed by solving equations (14) and (15), respectively.
2) **Scheme 2**: the powers are allocated according to (16) and (17), while the relay assignment is found by solving (14). The subcarrier used for the transmission in the MA phase is fixed and used again for the BC phase.

V. Simulation Results

The simulations are performed under the scenario given in Fig. 1. An OFDM system of $N = 16$ subcarriers, which is sufficient to have a zero duality gap [9] and $M = 6$ relays are assumed. The noise variance is assumed to be $\sigma^2 = 5 \times 10^{-6}$ and the channel gains are outcomes of independent Rayleigh distributed random variables with unity mean. All the results have been averaged over 1000 iterations.

Fig. 2 depicts the achieved capacity of the optimal and suboptimal schemes versus the transceivers and relays power constraints when the interference threshold is fixed to -10 dBm. It can be noted that the capacity of all schemes increases with the power constraint. This increase in the capacity continues until a certain value of the power constraints, after which the capacity becomes constant with the power constraint. This is because the induced interference reaches the prescribed threshold and the system cannot use more power. Additionally, the dual decomposition-based solution has the highest performance. This is an asymptotically optimal solution and works as an upper bound for the rest of the suboptimal schemes. The closest performance to the optimal solution is achieved by the *Scheme 1* algorithm.

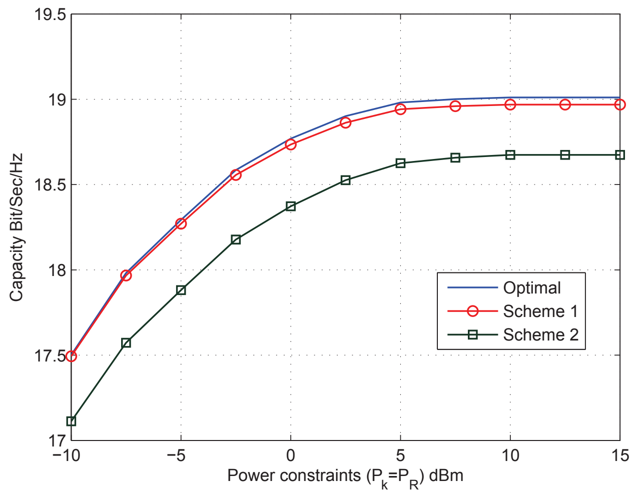

Fig. 2: Achieved capacity vs. power constraint.

VI. Conclusion

In this paper, a joint resource allocation problem in AF OFDM based two-way multiple relay cognitive radio network is considered, where two transceiver nodes exchange information via a relay node. The full transmission happens in two phases: MA phase and the BC phase. Considering individual power and interference constraints, the power allocation, subcarrier pairing and relay selection are jointly optimized in order to maximize the sum-rate. The dual decomposition technique is applied to obtain the optimal solution. Additionally, two sub-optimal algorithms are proposed to get rid of the high computational complexity of the proposed scheme.

Acknowledgment

This work is funded by the French National research Agency (ANR) research project PROFIL with grant agreement code: ANR- 13-INFR-0007-03, the Spanish Ministry of Economy and Competitiveness (Ministerio de Economiay Competitividad) under project TEC2011-29006-C03-02 (GRE3N-LINKMAC), and from the Catalan Government (2009SGR0891).

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