ART. III.—*An experimental proof of Ohm's Law: preceded by a short account of the discovery and subsequent verification of the law; by ALFRED M. MAYER.*

I PURPOSE giving in this paper a simple and direct experimental proof of Ohm's law, $\left(C = \frac{E}{R}\right)$. Generally a mere formal statement of this law with illustrations are given in text books on Physics, and the student is left to infer that its truth is shown by the cumulative evidence given by the immense number of quantitative relations in electrical actions which the law associates, and by the experience that deductions made on the basis of this law agree in measure with the results of experiments. The latter fact is certainly one of the best proofs of the truth of the law; but, nevertheless, the relations between C , E and R are not directly and simultaneously shown to be exactly expressed by $C = \frac{E}{R}$. It is true that some works

give experiments to show this relation but they are so difficult to perform by reason of the difficulty of maintaining constant C , E and R , that the results of the experiments only approximate to those required by the law.

Ohm was led to the conception of this law by *assuming* that the flow of electricity in a voltaic circuit is similar to the flow of heat by conduction in a rod of indefinite extent. Also, his assumptions that the actions of two electrified particles are directly as their distance and that the electricity is uniformly dense over each cross section of a conducting wire were directly opposed to the laws and facts well established by Coulomb for statical electricity. It is not surprising that scientific men were slow in adopting the views and theory of Ohm. In his memoir (*Die Galvanische Kette mathematisch bearbeitet* von Dr. G. S. Ohm: Berlin, 1827), he states: * "Three laws, of which the first expresses the mode of distribution of the electricity within one and the same body, the second the mode of dispersion of the electricity in the surrounding atmosphere, and the third the mode of appearance of the electricity at the place of contact of two heterogeneous bodies, form the basis of the entire memoir, and at the same time contain everything that does not lay claim to being completely established. The two latter are purely experimental laws; but the first, from its nature, is, at least partly theoretical.

* See translation, published in vol. ii, of Taylor's Scientific Memoirs, p. 402. London, 1841.

“With regard to this first law, I have started from the supposition that the communication of the electricity from one particle takes place directly only to the one next to it, so that no immediate transition from that particle to any other situate at a greater distance occurs. The magnitude of the transition between to adjacent particles, under otherwise exactly similar circumstances, I have assumed as being proportional to the difference of the electric forces existing in the two particles; just as in the theory of heat, the transition of caloric between two particles is regarded as proportional to the difference of their temperatures. It will thus be seen that I have deviated from the hitherto usual mode of considering molecular actions introduced by Laplace; and I trust the path I have struck into will recommend itself by its generality, simplicity, and clearness, as well as by the light it throws upon the character of former methods.

With respect to the dispersion of electricity in the atmosphere, I have retained the law deduced from experiments by Coulomb, according to which, the loss of electricity in a body surrounded by air, in a given time, is in proportion to the force of the electricity, and to a coefficient dependent on the nature of the atmosphere. A simple comparison of the circumstances under which Coulomb performed his experiments, with those at present known respecting the propagation of electricity, showed, however, that in galvanic phenomena the influence of the atmosphere may almost always be disregarded. In Coulomb's experiments, for instance, the electricity driven to the surface of the body was engaged in its entire expanse in the process of dispersion in the atmosphere; while in the galvanic circuit the electricity almost constantly passes through the interior of the bodies, and consequently only the smallest portion can enter into mutual action with the air; so that in this case, the dispersion can comparatively be but very inconsiderable. This consequence, deduced from the nature of the circumstances, is confirmed by experiment; in it lies the reason why the second law seldom comes into consideration.

The mode in which electricity makes its appearance at the place of contact of two different bodies, or the electrical tension of these bodies, I have thus expressed: when dissimilar bodies touch one another, they constantly maintain at the point of contact the same difference between their electroscopic forces [potentials].

With the help of these three fundamental positions, the conditions to which the propagation of electricity in bodies of any kind and form is subjected may be stated. The form and treatment of the differential equations thus obtained are so similar to those given for the propagation of heat by Fourier

and Poisson, that even if there existed no other reasons, we might with perfect justice draw the conclusion that there exists an intimate connection between both natural phenomena; and this relation of identity increases, the further we pursue it. These researches belong to the most difficult in mathematics, and on that account can only gradually obtain general admission; it is therefore a fortunate chance, that in a not unimportant part of the propagation of electricity, in consequence of its peculiar nature, those difficulties almost entirely disappear."

From these premises, and guided by results of experiments made by him and by Ritter, Erman, Jäger, Davy and Becquerel he arrived at the following conditions as existing in a voltaic circuit.

1. In a homogeneous conductor, forming part of a voltaic circuit, the difference of the electric tensions at any two points of the conductor is proportional to their distance.

2. In different conductors forming part of a circuit, the difference of tensions at two points separated by an interval equal to the unit of length is in the inverse ratio of the section of the conductor and of its coefficient of conductivity. Hence, in different conductors, equal differences of tension correspond to lengths whose electric resistance is the same.

3. At the point of contact of two different conductors, there is a sudden variation of electric tension.

4. If A equals the sum of the electro-motive forces, L the resistances, λ the resistance reckoned from a point m of the circuit to a point p when the tension is zero, the tension at the point m is given by the formula

$$u = A \frac{\lambda}{L}.$$

Ohm eventually arrives at the formula $S = \frac{A}{L}$, which expresses

what is generally known as his law. Which formula, he says, "is generally true, and already reveals the equality of the force of the current at all points of the circuit; in other words it may be thus expressed: The force of the current in a galvanic circuit is directly as the sum of all the tensions, and inversely as the entire reduced length of the circuit, bearing in mind that at present by reduced length is understood the sum of all the quotients obtained by dividing the actual lengths corresponding to the homogeneous parts by the product of the corresponding conductivities and sections."

The words "tension" (*Spannung*) and "electromotive force" used by Ohm are the equivalent of the word *potential*. He was the first to introduce this conception into the theory of

the voltaic circuit and to the above words and to *current* and *resistance* he attached precise meanings and showed the relations existing between those quantities. The clear definitions Ohm gave of these terms marked a transition from vague ideas of "quantity" and "intensity" to the clear conceptions of potential, electromotive force, current and resistance. The word *energy* he also used with clear and accurate meaning as is shown in the following statement: "that the decomposing force of the circuit is in direct proportion to the energy of the current, and moreover, that it depends on a coefficient, to be derived from the nature of the constituent parts and their chemical equivalents." This was published in 1827, six years before Faraday's researches on electrolysis.

Neither Ohm nor his contemporaries were able to test the truth of the four statements given above as embodying Ohm's theory. It was reserved for Kohlrausch in 1849 to show by very ingenious and accurate experiments that Ohm's statements were true in mode and in measure. Kirchhoff* and Quincke† applied with success Ohm's theory to the flow of electricity in thin conducting plates, or bodies of two dimensions, and the same was done by Smaasen‡ not only in a plane but in bodies of three demensions. The most remarkable confirmation of Ohm's law was made in 1876§ by experiments, suggested by Maxwell and performed by Chrystal in the Cavendish Laboratory, Cambridge, "in which the testing of this law seems to have been carried to the limit of experimental resources."

Though Ohm's law has thus received such ample verification that it ranks with the best established laws of nature, yet, as Maxwell says, "Ohm's law must, at least at present, be considered a purely empirical one. No attempt to deduce it from pure dynamical principles has as yet been successful. . . . The conduction of electricity through a resisting medium is a process in which part of the energy of an electric current, flowing in a definite direction, is spent in imparting to the molecules of the medium that irregular agitation which we call heat. To calculate from any hypothesis as to the molecular constitution of the medium at what rate the energy of a given current would be spent in this way, would require a far more perfect knowledge of the dynamical theory of bodies than we at present possess. It is only by experiments that we can determine the laws of processes of which we do not understand the dynamical theory."

* Pogg. Ann., t. lxiv, 1845, and t. lxvii, 1846.

† Pogg. Ann., t. xcvi, 1856.

‡ Pogg. Ann., t. lxiv, and t. lxxii.

§ Brit. Assoc. Rept., 1876, p. 36.

Surely if an experiment, that is easily made, shows the truth of a law of such theoretical and practical importance as that of Ohm, even if it is one restricted in its range of C , E and R , but shows within its limitations the relations $C = \frac{E}{R}$, then it should be made by all teachers of Physics so that clear physical conceptions of those relations may be given to students. As those who have seen these experiments have deemed them worthy of being more generally known, I now publish an account of them.

In the diagram the parts of the apparatus are shown, but not at their relative distances apart or in the proper proportions as to size. G is a low-resistance Thomson-galvanometer. At L is the condensing lens of a lime-light lantern, which is covered with a cap having a rectangular opening in it. Across the middle of this slit is a vertical wire. The scale of the galvanometer is at C , distance 165^{cms} from the mirror of the galvanometer. The width of the divisions on this scale are 2.5^{cms} , and the lines are drawn 2.5^{mms} in breadth, or $\frac{1}{10}$ the distance apart of the centers of the lines forming a unit of the scale. The scale is at such distance from the galvanometer-mirror that the image of the vertical slit just fits in the space of a scale unit, while the breadth of the image of the vertical wire is exactly equal to the breadth of a scale line. This arrangement gives the means of observing a deflection of the beam of light to $\frac{1}{10}$ and $\frac{1}{20}$ of a unit with quickness and accuracy.

The image of the slit is so bright and that of the wire so distinct that this method of observing deflections of the galvanometer may be used in broad day light and the deflections may be read throughout the room.

An incandescent electric lamp with a part of its surface (behind the plane of its filament) silvered may replace the lime-light. Thanks to this arrangement, I have been able during many years to make before my class electrical measurements, and to measure the radiation, reflection, refraction, diathermancy and polarization of radiant heat.

At M is a magnet 25^{cms} long and $1\frac{1}{2}^{\text{cms}}$ in diameter. On this magnet slides a wooden disc. At R is box containing 1, 2 and 3 Ohms of resistance, made of coils of copper wire.

An insulated copper wire wound at its middle in a circle of one coil, or in a spiral of any number of coils is placed over the magnet and rests on the top of the wooden disc. The figure

shows, (one-half size) how this circle of one coil is made. It is bent around a wooden cylinder $3\frac{1}{2}^{\text{cms}}$ in diameter, and then the free ends of wire are bent one half turn on each other. The free lengths of the wire are then lashed to a light square rod of wood as shown in figure. The wire and rod are then coated with shellac to cement them firmly together. Rings of spirals of 2, 3, 4, 5 and 6 coils are also made in the same manner, but the coils are in a spiral, i. e. in one plane, and are then cemented together with shellac between rings of thin card-board.

The length of wire forming each of these rings of spiral coils with the portion on its handle is one meter long.

The resistance of this length of wire added to the resistance of the lengths between it and G and R, together with the resistance of the galvanometer is (for convenience) made one ohm.

It may be well here to speak of the adjustment of the galvanometer before describing the experiments, for I have noticed in some laboratories and lecture rooms galvanometers which are used not as they should be. I have noticed that the damping-magnet formed a considerable angle with the plane of the coil. This was either because the median plane of the coil was not in the magnetic meridian or because there was considerable torsion in the suspending thread.

In these galvanometers, or, at least, in mine, the median plane of the coil is placed parallel to the faces of the drum of the instrument. The plane of one of these faces is brought in the magnetic meridian of the room, which has been carefully

drawn on the table under the vertical center line of the galvanometer coil, by means of a long magnetic needle mounted like those used on plane-tables. A line at right angles to this meridian is now drawn so that its point of intersection with the meridian line shall be exactly under the suspending thread of the mirror. In the vertical plane of the line, drawn at right angles to the meridian, is placed the vertical wire in the slit of the lantern, L, and also the zero line of the scale C. The scale is parallel to the magnetic meridian. The galvanometer is now placed in the position given above and the "directing magnet" removed to a distance. The image of the vertical wire at L will now be found on the zero of the scale if there is no torsion in the suspending thread. If it does not come to zero then the head of the rod to which the thread is attached is turned till image of wire coincides with zero of scale, and then the instrument is in adjustment, and it will give deflections as the tangents of the strength of current, or, in other words, the current strength will be directly as the readings on the scale. The magnet M is now placed so that it causes no movement of beam from the zero of the scale. The directing magnet, above the coil, is now so adjusted that the time of an oscillation of the magnets of the galvanometers is above 5 seconds.

The coil, E, over the magnet is put in the circuit of G and R. The wires between E and G and R are twisted and tied together so that no induced current from the earth's magnetism may be caused by the motions of this part of the circuit. The image of wire is on zero of scale. Now on rapidly lifting the coil from around the magnet a deflection is produced by the magneto-electric current thus generated. It is sufficient to know that the cause of this current is the quick lifting of the ring with one coil. If we replace this by a ring of two coils we get twice the deflection, and rings of 3, 4, 5, and 6 coils gives 3, 4, 5, and 6 times the deflection given by the ring with one coil. Adopting the conception of the lines of magnetic force, we say that the ring with one coil cuts a certain number of these lines, this cutting of the lines causes the current, and is the *electromotive force*. The ring with two coils makes two cuts of these same lines, or, cuts double the number of lines, the rings of 3, 4, 5 and 6 coils cut 3, 4, 5 and 6 times the number of lines and hence give 3, 4, 5 and 6 times the electromotive force.

In these experiments the resistance of the circuit has remained constant. Now take the ring with 5 or 6 coils and let us have one ohm as resistance of circuit. On lifting ring from magnet we get a certain deflection, which we may make exactly equal to a whole number of the units of the scale by

sliding up or down the disc on the magnet. We now take out plug of resistance box and make resistance of circuit two ohms. The deflection of the galvanometer magnet now becomes one half of that of previous experiment, and successively making the circuit with resistances of 3, 4, 5, 6, and 7 ohms we get, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{2}{5}$, $\frac{1}{3}$, and $\frac{2}{7}$ of the deflection we got with one ohm in circuit.

When these experiments are made with the galvanometer in perfect adjustment, and with the precautions indicated below, the deflections arrive one after the other exactly as the law requires. Thus showing with sufficient precision for a lecture experiment that the current is directly as the electromotive force and inversely as the resistance. Indeed generally the closest scrutiny does not detect in the scale reading any departure from the law.

Certain precautions are, however, necessary in these experiments. The resistance outside the galvanometer must be of copper wire, for such is the wire of the galvanometer. Also, the whole of the apparatus must be put together the day before we make the experiments, and the room maintained at as constant a temperature as possible, so that the temperature of all parts of the apparatus is the same. The deflections should not exceed 15 divisions of the scale. Thus, if we start with 15 divisions of deflection for a resistance of one ohm we will get 7.5; 5; 3.75; 3; 2.5; and 2.143 deflections for resistances of circuit of 2, 3, 4, 5, 6 and 7 ohms; and if with a constant resistance we obtain a deflection of 2 divisions of scale with a ring of one coil, we will get deflections of 4, 6, 8, 10, and 12, with rings having 2, 3, 4, 5 and 6 coils.

It is necessary that the coils should be removed from the magnet *very quickly*, otherwise the deflections will not be as the law requires. In other words, the currents produced should be as instantaneous as can be obtained. Instead of rapidly removing the coils by the hand, I have sometimes lashed the coil and their handles to a spring board with a hole in it which went over the magnet. By a trigger this spring-board is released. We thus get the same velocity in lifting the coil in each experiment. We have found, however, that the hand of a good experimenter gives precise results. Sometimes I have sent the coil from the magnet by the blow of a stick delivered on the under side of the handle of the coil at its center of percussion. There is no doubt some departure from the law in these experiments, for it is not possible in such experiments to obtain what is understood by instantaneous currents; and the damping of the magnet by the mirror acting on the air must come into play. Yet I have never seen any

but insignificant and barely discernable departures from deflections required by the law. This follows from the small angles of deflections and low velocity of the motion of the galvanometer magnet in the experiments. It is also to be noticed that with a good magnet of the size stated, and with the galvanometer making one vibration in about 5 seconds, the coil with 5 turns passes over only 2 cms. or less, of end of magnet in order that it shall give a deflection of 15 divisions of scale. It is evident that in these conditions a very short time is occupied in cutting the lines of force. If the maximum deflection used is 15 divisions of the scale, the actual angular deflection of the magnets and mirror amounts to only $6^{\circ}29'$. Yet 15 divisions are quite a length on the scale, being equal to 37.5 cms. But these experiments may be as readily made with a ballistic galvanometer. Then the magnets and coils have to be of larger dimensions.

Experiments similar to those given have served to graduate galvanometers. We have here the means of sending definite amounts of currents through an ordinary galvanometer and we may thus graduate its angular readings into their relative values in current. The damping of the galvanometer has, however, to be applied to the readings, and then the results may best be put in the form of a curve.

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