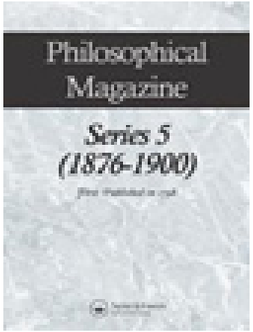

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XXXVIII. Determination of the acceleration of gravity for Tokio, Japan

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XXXVIII. *Determination of the Acceleration of Gravity for Tokio, Japan.* By W. E. Ayrton and John Perry*.

As no experiments had, as far as we were aware, been made to determine the value of g in Japan, it appeared to us desirable that the value should be accurately measured, at any rate for our own college in the capital, Tokio. Consequently in 1877 an elaborate series of experiments was carried out by some of the students under our supervision. The method first employed consisted in experimentally finding two parallel axes in a pendulum on opposite sides of the centre of gravity, and in a plane with it, such that the times of oscillation about either axis would be the same. The distance, then, between these axes experimentally found would, as is well known, be the length of the equivalent simple pendulum, from which g could be calculated by the formula

$$t=\pi\sqrt{\frac{l}{g}}.$$

Two Kater's pendulums were employed—one made by Messrs. Elliott, and the other by Messrs. Negretti and Zambra. Borda's method of coincidence was employed; that is, one of the Kater's pendulums was suspended exactly in front of the pendulum of a clock, consisting of a wooden rod carrying a brass bob and beating approximately seconds. One observer watched the two pendulums, vibrating one in front of the other, through a telescope some ten feet away; and at the instant a pointer attached to one of the pendulums exactly coincided with a line drawn on the other (the coincidence taking place in the axis of the telescope as observed by the cross-wires), a signal was given, and the time of the clock noted by two independent observers. A very large number of successive coincidences was in this way observed, then the Kater's pendulum inverted, swung on the other knife-edge, and the same thing repeated. From these experiments the exact time of vibration of the Kater's pendulum about either knife-edge was ascertained, and one or both knife-edges moved to diminish the difference in the time of vibration, and the whole experiment repeated.

But although these observations, first with one of the Kater's pendulums and then with the other, were continued for some months, many thousands of vibrations being observed by the students, who worked at these experiments in their usual most praiseworthy way, the results were always unsatisfactory. For an approximate value of g for any part of the

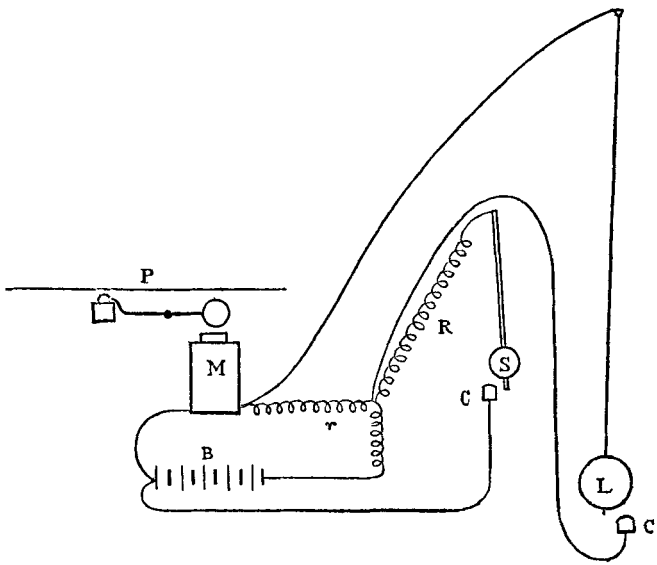
* Communicated by the Physical Society.

earth's surface can be calculated by a formula developed by Clairault, who from pendulum-experiments made at a variety of latitudes on the earth's surface has shown that, approximately, for any latitude λ and any height h centimetres above the level of the sea,

$$g = 980{\cdot}6056 - 2{\cdot}5028 \cos \lambda - 0{\cdot}000003 h.$$

As the latitude of the Imperial College of Engineering, Tokio, is about 35° 39′, Clairault's formula gives for g the value 979·7 centimetres per second per second. But the value obtained by the series of experiments made with the Kater's pendulums differed too much from this and from one another to allow of our trusting them.

We therefore decided on employing a totally different method and a much less laborious one:—A brass ball 2352·2 grammes in weight was suspended by a long steel wire 0·45 millimetre in thickness, and in the earlier experiments 978·7 centimetres in length. The wire was supported from a steel knife-edge resting on a brass plate. Both the brass ball and the bob of the seconds-pendulum of the standard clock were fitted with fine pieces of platinum wire, either of which dipped into a small cup of mercury when the pendulum to which it was attached was vertical. The mercury-cups &c. were then joined up with a battery and resistance-coils to a quick-running Morse instrument, as seen in the figure, in which M is



the Morse instrument, B the battery, P the paper, S the seconds-pendulum, L the long pendulum, C, C the mercury-cups, R a resistance small compared with that of M plus that of the coil r, but large compared with that of M alone. The whole constituted what is known as a "break-circuit chronograph;" that is, a continuous ink mark was made on the paper run out by clockwork, broken by a very small gap each time the wire attached to the bob of the seconds-pendulum passed through the mercury. These breaks, then, in the ink line indicated seconds; if, however, both pendulums were simultaneously in the vertical line, no break was made. Hence the absence of a break in the line at the end of any special second indicated coincidence of the two pendulums; and in this way the times of a large number of coincidences could be automatically registered.

During this set of experiments we could not measure the length of the long fine steel wire with as much accuracy as was desired, since, although we had two or three brass scales, the makers had omitted to record on them at what temperatures they were correct. However, assuming that one of them was accurate at 0° C., then a rather large number of experiments gave, as the value of g, 978·8 centimetres per second as a first rough approximation.

Subsequently we obtained from the Finance Department of Japan the loan of two very beautiful standard brass scales, by Deleuil of Paris, and guaranteed correct at 0° C. One was graduated in millimetres; the other consisted of a brass rod with two pieces at its ends at right angles to the rod, and the distance between the two planes of the inner surfaces of the pieces was exactly a metre at 0° C. We now had, then, the means of making a far more complete series of experiments than before; but as our trial pendulum was nearly ten times as long as the seconds-pendulum of our clock, the method of coincidences was an inconvenient one; and so we merely adopted the following:—The long pendulum alone controlled the "break-circuit chronograph;" so that the number of breaks in the line during any time indicated the number of vibrations of the long pendulum in that time. At the commencement of the experiment, after the pendulum had been set swinging and the paper was running out at a fairly uniform speed, a mark was made on it by tapping sharply the armature up with the finger when a chronometer, lying beside the Morse instrument, indicated a certain time; and after an hour or so, the paper being kept running all the time, a second mark was sharply made on the paper when the chronometer indicated a certain other noted time. So much paper had then run out

in the interval of time shown by the chronometer; and the breaks in the line, counted carefully afterwards by two independent students, gave the whole number of vibrations of the pendulum in that time. The fraction of a vibration could also, of course, be ascertained by comparing with the length of the lines in the neighbourhood the length of the first line made after the first break had been produced, on tapping the armature, and repeating the same process at the end of the paper. The experiment is, of course, independent of the rate at which the paper runs out, provided, of course, it is never allowed to run so slowly that there is any difficulty in distinguishing the different breaks electrically made by the long vibrating pendulum. The mean temperature of the wire was carefully taken at each experiment. A sample of one or two of the many experiments recorded in the students' laboratory notebook follows.

Number of Experiment 26.—25*th of January*, 1878.

Number of vibrations obtained from counting on the paper	1015¾
Time taken—observed on the chronometer .	0 h. 52 m. 0 s.
Mean temperature at the beginning	10°·5 C.
" " " end	11°·25 C.

Time of vibration = 3·0744 seconds.

Number of Experiment 45.—15*th of February*, 1878.

Number of vibrations obtained from counting on the paper	1385·5
Time taken—observed on the chronometer .	1 h. 11 m. 0 s.
Mean temperature at the beginning	9°·25 C.
" " " end	12°·25 C.

Time of a vibration = 3·0745 seconds.

Number of Experiment 53.—21*st of February*, 1878.

Number of vibrations obtained from counting on the paper	1288·5
Time taken—observed on the chronometer .	1 h. 6 m. 0 s.
Mean temperature at the beginning	8°·5 C.
" " " end	12°·25 C.

Time of a vibration = 3·0741 seconds.

Frequent sun-observations were made to check the rate of the chronometer, which is comparatively easy in Japan, as during the winter there the sun is seen almost daily from sunrise to sunset.

The next point was to measure accurately the length of the

wire. As it was impossible to do this satisfactorily with the wire hanging up, it was taken down without disconnecting either the knife-edge carrying it or the ball at the other end. The knife-edge was then fixed at one end of a horizontal rail, and the other end of the wire close to the ball hung over a wheel with very little friction. By this arrangement the wire in a horizontal position was, of course, stretched as much as it was in the vertical position, as far as the effect of the weight of the ball was concerned. A correction had, however, to be made for the weight of the wire itself, which of course caused the tension to be a little less at the bottom than at the top when the pendulum was hanging up vertically. A few centimetres of similar fine steel wire being weighed, a simple integration gave the small additional weight necessary to be added. This being done, the final result obtained was that the length of the pendulum equalled 939·09 centimetres at 0° C.; and the consequent value of g in air for Tokio, Japan, calculated from the result of about eighty thousand vibrations of the long pendulum, would be 980·06 centimetres per second per second, if the pendulum could be regarded as a simple mathematical pendulum.

Correcting Factors.

1. The two most obvious corrections to apply to this result are the corrections for infinitely small arcs and for the air-friction—neither of which were found of any practical consequence, on account of the very small angle through which the pendulum usually swung, and that the decrement of the amplitude of the vibrations was imperceptible even after many swings. Although, however, such a pendulum as we were using approaches very nearly a perfect simple pendulum, there are certain causes of possible error arising from its flexibility and slight elasticity which would not affect a rigid compound pendulum. To estimate the practical effect of these possible errors, it is necessary to solve generally the complete problem of a heavy ball supported by an elastic wire, one end of which is soldered to the ball and the other end to a steel knife-edge. When a suspended ball is swinging in the arc of a circle, we know that near the end of a swing the attachments of the ball have to resist a tendency for the ball to turn. For since the ball has been turned in passing from its lowest to its highest position, it would continue to turn were it not stopped by the wire itself. At the end of every swing, then, there must be a slight kick; so that in fact the ball will make minor swings about its point of attachment all the time of the motion. To make this kick less perceptible, we must make the fastening

of the wire to the ball capable of resisting the tendency of the ball to continue its turning motion. If we do this by soldering the wire, a smaller kick will result, and will be due to the bending-moment of the wire resisting the turning action. If there were no difficulty of construction, it might be better to get rid of this kick difficulty by making the bob capable of rotating in the plane of swinging about an axis through its centre of gravity.

The investigation of the general problem of the swinging of a heavy ball soldered to an elastic wire, the upper end of which is attached to a knife-edge, may take somewhat the following form:—Let A be a ball, of mass m and radius a, suspended from a free hinge at B by a chain of $n-1$ links, each of length a and hinged to one another, the last hinge being on the surface of the ball. Suppose at any hinge where two adjacent links make an angle θ with one another, equal and opposite couples act in them of moment $c\theta$ tending to bring them into the same straight line. As n is made greater and greater, we approximate more and more nearly to our actual case of an elastic wire. Let n be very great, and $\phi_1, \phi_2, \ldots \phi_{n-1}$ the inclinations of the 1st, 2nd, ... $(n-1)$th link to the vertical, the nth link being the radius of the ball up to the hinge, and its inclination ϕ_n. If now we know the mass of the links per unit of length, it is easy to state the values of $\psi_1, \psi_2, \ldots \psi_s, \ldots \psi_n$, the couples acting on each respective link: thus

$$\psi_n = mga \sin \phi_n + c(\phi_n - \phi_{n-1}),$$

care being taken to remember that the form of ψ_s is different from that of ψ_1 or of ψ_n.

If the inclination ϕ is everywhere very small, we find, if v_s is the velocity of the end of link s, that

$$v_s^2 = a^2\{(\dot{\phi}_1)^2 + 2\dot{\phi}_1\dot{\phi}_s \cos(\phi_s - \phi_1) + (2s-3)(\dot{\phi}_s)^2\},$$

where $\dot{\phi}_s$ means $\frac{d\phi_s}{dt}$.

So that the kinetic energy T of the whole system may at once be written out in terms of the coordinates $\phi_1, \phi_2, \ldots$ &c., $\dot{\phi}_1, \dot{\phi}_2, \ldots$ &c.

We can therefore find the partial differential coefficients

$$\frac{d\mathrm{T}}{d\dot{\phi}_1}, \ \frac{d\mathrm{T}}{d\dot{\phi}_2}, \text{ \&c., and } \frac{d\mathrm{T}}{d\phi_1}, \ \frac{d\mathrm{T}}{d\phi_2}, \ldots \text{ \&c.},$$

so as to use Lagrange's equation

$$\frac{d}{dt}\left(\frac{d\mathrm{T}}{d\dot{\phi}}\right) - \frac{d\mathrm{T}}{d\phi} = -\psi.$$

In this way we have obtained the n differential equations connecting $\phi_1, \phi_2, \ldots \phi_n$, and their first and second differential coefficients with respect to t. As, however, these equations can only be regarded as true when n is infinite, and as the labour of solution is very great when n is great, it seems useless proceeding further with the solution.

If we regard the motion of the ball as a harmonic motion of period P, determined by assuming the connexions as rigid, combined with motions of much shorter periods $P_1, P_2, P_3, \ldots$ &c., there will be some little difficulty in finding the motion of shortest period P_1, namely the *kick* above mentioned; but we know that when the wire is, as in our experiments, very thin, the kick cannot be much less than the time of a complete vibration of the ball when freely suspended by a point on its surface, or

$$2\pi\sqrt{\frac{12a}{5g}},$$

where a is the radius of the ball. But this periodic time is 0·528 seconds, or about one twelfth of that of the pendulum moving as a whole, which is about 6 seconds.

Since the tendency of the ball to add this quick vibration to its motion is due to its rotational energy, it may be diminished by lessening the moment of inertia of the ball (that is, by making the ball small), or by diminishing the angular velocity of the pendulum (that is, by making the pendulum as long and its swing as small as possible). We may regard, then, the motion of the ball as compounded of a pure harmonic motion with an amplitude of about 30 centimetres and a periodic time of 6 seconds, with another motion having a very small amplitude and with a period of about half a second. But we have proved in the paper on our seismograph*, that in such a case the compound motion would differ very slightly from that of a pure harmonic motion, even if there were no internal friction in the substance of the wire (supposing the pendulum started without shock); but as internal friction, of course, exists in the wire, this error becomes exceedingly small.

2. Next, with regard to the stretching of the wire arising from variations in the centrifugal force of the ball while swinging. Since the time of a complete vibration of our pendulum was nearly 6 seconds and the arc about 30 centimetres,

* "On a Neglected Principle that may be employed in Earthquake Measurements," Trans. Asiatic Soc. of Japan, vol. v. part 1, p. 181; reprinted in Phil. Mag. July 1879.

the velocity at the middle of its path was

$$\frac{30 \times \pi}{6}, \text{ or } 15{\cdot}7 \text{ centimetres per second};$$

hence the pull on the wire, which at the end of the swing was equal to the weight of the wire, or 2352·2 grammes, was increased by

$$\frac{2352{\cdot}2 \times (15{\cdot}7)^2}{939 \times 979{\cdot}7} \text{ grammes}$$

at the middle of the swing. But this is less than a gramme, so that no practical extension of the wire arose from centrifugal force.

3. Shortening of the length of the wire, due to its curvature, arising from the resistance of the air making it concave in the direction of motion. It is easy to see that the shortening of the pendulum due to this cause is excessively small, and is of the same order as the lengthening arising from the centrifugal force; so that these two very small errors may be regarded as balancing one another.

Also, since it may be calculated that the period of transverse vibration of the wire is less than one fortieth of the periodic time of the pendulum, the resistance of the air cannot tend to cause amplification of the lateral vibrations in the wire itself.

We may therefore assume that our pendulum vibrated like a rigid body, consisting of a ball of brass, a straight steel wire, and a triangular steel prism of which the edge was the fixed axis.

Calculation of g.

The complete formula is, of course,

$$t = \pi \sqrt{\frac{\Sigma(mr^2)}{l \,.\, mg}},$$

l being the distance from the axis of rotation to the centre of gravity of the pendulum.

The steel knife-edge had a length of about 4 centimetres, a breadth of about 1 centimetre, and a depth of $\frac{1}{2}$ a centimetre; hence its weight was about 7·8 grammes, its moment of inertia about the axis of rotation 0·98 (gramme, centimetre), and the distance of its centre of gravity from the axis of rotation 0·33

centimetre. The weight of the wire was 11·6 grammes, and its length 934·99 centimetres at 0° C. Its moment of inertia was therefore $3{\cdot}3803 \times 10^6$ (gramme, centimetre), and the distance of its centre of gravity from the axis of rotation 467·49 centimetres. The weight of the brass ball was 2352·2 grammes, its moment of inertia about the axis of rotation $2{\cdot}0744 \times 10^9$, and the distance of its centre of gravity 939·09 centimetres at 0° C. Of the whole system, then, the weight was 2371·6 grammes, the moment of inertia about the axis of rotation $2{\cdot}0778 \times 10^9$ (gramme, centimetre), and the distance of its centre of gravity $2{\cdot}2144 \times 10^6$. Consequently

$$g = \left(\frac{\pi}{t}\right)^2 \frac{2{\cdot}0778 \times 10^9}{2{\cdot}2144 \times 10^6},$$

and

$$t = 3{\cdot}0748 \text{ seconds};$$

or

$$g = 979{\cdot}58 \text{ centimetres per second per second in air,}$$

or

$g = 979{\cdot}74$ *centimetres per second per second* in vacuo *for the Imperial College of Engineering, Japan*—

a result agreeing extremely closely with the number 979·7 obtained above from Clairault's formula.

In beginning this series of observations we expected to find g to be greater than what Clairault's formula gives it. Our reason was this:—Clairault's formula assumes a circular equator; Capt. Clarke has found that the equator is elliptical, one extremity of its major axis being in 15° 34′ E. longitude; and therefore Tokio is in longitude nearer a minor axis than a major one. We find, however, a reason why g satisfies so well Clairault's formula, in spite of this excentricity of the equator. The greatest depression of the earth's surface is only a few hundred miles to the east of Japan; and probably the diminution in g produced by this cause just counterbalances the increase of g produced by ellipticity of the equator. As for local perturbations, it is to be remarked that Tokio is situated on a very large plane, there being no hills of any magnitude within eighty miles. We think that the geodesy of Japan is of special interest on account of the great Pacific depression, and on account of the very gradual slope of the earth's surface from Japan to China, which causes Japan to be a sort of ridge.

Probably the best method of determining the value of g

would be to use a *rigid* homogeneous pendulum, of such a shape that its moment of inertia could be easily calculated with accuracy from its linear dimensions measured at any temperature—for example, a sphere at the end of a cylindrical rod with a knife-edge, all cast in one piece and turned true in the lathe, or a cylindrical bob cast at the end of the cylindrical rod. The only possible objection to this method would be the possible want of homogeneity in the metal. This might be allowed for in the following way :—Instead of casting the cylindrical bob in one piece, make it consist of a number of concentric tightly fitting cylindrical shells accurately turned. Experiments would then be made first with the cylinders all in one position, then with some of them twisted slightly round, and so on until in the mean result the errors of eccentricity of mass would probably be eliminated.

Another, and perhaps the best of all methods, would be for rigid compound pendulums to be accurately timed experimentally at Greenwich at a number of different temperatures, and sold with a scale of temperature-corrections for the time of vibration attached.

One set of experiments, then, with one of these pendulums anywhere would at once give the value of g; and such a pendulum would undoubtedly be the most suitable for surveys and expeditions in foreign countries. The fact is, the mathematical beauty of the principle involved in the Kater's pendulum has, in our opinion, caused far too much importance to be attached to it as a practical instrument for determining experimentally the value of the acceleration of gravity.

We have to thank several of our late students, and especially Messrs. Honda, Kikkawa, A. Kasai, J. Nakahara, and H. Nobechi for assistance rendered us during this investigation. And it may here be mentioned that this investigation, like the many others we have been enabled to carry out during the last few years, has resulted from the plan we have followed of teaching the laboratory students not, as is customary in Colleges, to repeat well-known experiments, but to endeavour in their investigations to advance, in some small degree at any rate, the bounds of existing knowledge. And this system of enlisting the assistance of even quite young students in original research we have found to create an enthusiasm in experimental work otherwise unproducible.