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Evening Meeting.

Monday, April 7th, 1873.

REAR-ADMIRAL SIR ASTLEY COOPER KEY, K.C.B., F.R.S., in the
Chair.

NAMES of MEMBERS who joined the Institution between the 7th and
21st April, 1873.

LIFE.

Littledale, H. W. A., Lieutenant, R.N.
Jocelyn, W. H., Lieutenant, R.N.

ANNUAL.

Abbott, Saunders A., Major-General, late Bombay Army.	Burr, C. E. G., Lieut., 17th Regiment.
Bouverie, Henry H. P., Lieut. W. Som set Yeomanry Cavalry.	Shepherd, Henry, Captain, 2nd Kent Artillery Volunteers.
Barnett, Henry, Lieut.-Colonel, Oxford Yeomanry Cavalry.	Simpson, Frank, Staff Surgeon.
McPherson, Cecil, Capt., 17th Regt.	Todd, J. A., Lieutenant-Colonel, late 14th Hussars.
	Macpherson, J. C., Captain, R.E.

APPARATUS FOR AUTOMATICALLY RECORDING THE ROLLING OF A SHIP IN A SEA-WAY, AND THE CON- TEMPORANEOUS WAVE-SLOPES.

By WILLIAM FROUDE, Esq., M.A., F.R.S.

I PROPOSE to explain an automatic instrument I have made for recording some of the more important phenomena connected with the rolling of ships at sea. In order to explain the objects for which it is necessary that these phenomena should be recorded, it will be desirable that I should set forth in outline, some of the leading principles on which the rolling of ships depends. I would mention, however, at the outset, that the record given by the more special portion of this apparatus depends for its accuracy on simple mechanical principles, and is in no way dependent on the correctness of any theoretical assumption, yet its operation, as a whole, is specially relevant to the principles referred to, and its records will be specially suitable for testing the theory to which the principles seem to point. I need scarcely observe that the determination of the causes of the rolling of ships is an important and essential step towards the discovery of remedies for the evil.

In considering the rolling of ships, the primary question is, why does the passage of a wave cause the ship to roll? In what precise manner does the peculiar deformation of the surface of water called a wave, impress rolling motion upon a ship?

To consider, in detail, the various actions of each portion of the wave-

water, would involve complication; but these all resolve themselves into a total which is very simply expressed by the proposition that the ship tries to place itself upright to the slope of the wave, for the same reason and with the same energy as she tries to place herself upright to the level surface of smooth water. This may at first sight seem a little paradoxical; but it sounds more reasonable when it is suggested that the ship desires to place herself in this sloping position on the slope of a wave, in obedience to substantially the same set of conditions as those which cause the water itself to assume this sloping position. In fact it correctly expresses the effect of waves on bodies floating upon them, to say, that the apparent direction of the force of gravity, is square to the surface of the wave. This apparent alteration of the vertical direction of gravity, which causes the water to stand temporarily sloping instead of level, and causes bodies floating on its surface to try to become perpendicular to this slope instead of truly vertical, is due to the translatory sideways accelerations involved in the motion of the particles of which the wave consists.

This effect may be illustrated by a plumb-line. If its point of attachment is stationary, the plumb-line will hang vertically, but if the attachment be accelerated sideways, the plumb-line will hang askew as long as the acceleration is continued, the direction of the suspending line indicating what I have termed the apparent direction of gravity at the plumb-bob. If, for the plumb-bob, a cup of water is substituted, it will be seen that the surface of the water in it will be always square to this apparent direction of gravity, and the slope thus impressed on the water, is closely analogous to the sloping surface of the wave. For just as the acceleration sideways of the particles of water in the cup, causes the surface of the water in the cup to be sloping, so the acceleration sideways of the particles of water forming a

FIG 1.—EXPERIMENT OF FLOAT IN SUSPENDED CUP.

- a. Point of suspension being accelerated sideways from left to right.
- b. Suspending lines.
- c. Cup.
- d. Surface of water in cup perpendicular to the line of suspension, or the apparent direction of gravity.
- e. Float in the water.
- f. Short mast in the float carrying
- g. A small plumb-bob which hangs in conformity with the apparent direction of gravity, and thus at right angles to the surface of the water.

wave, causes the surface of the wave to be sloping. And the analogy may be carried further, by placing on the water in the cup, a float having a small mast from which a plumb-bob hangs; then when the cup is swung about as before, not only does the water in the cup stand perpendicular to the line of suspension, but the float continues upright to the surface of the water, as also does the plumb-line hanging within it (see Fig. 1). In trying this experiment it is essential that the float should be very stable and the plumb-bob one of "short period," so that they should instantaneously assume the varying inclinations impressed on them by the varying external forces we are examining.

This proposition that the apparent direction of gravity is square to the surface of the water, which thus simply and directly describes the origin of the rocking forces administered to a ship by the waves, is shown by the above experiment to be true of the water in the suspended vessel. That it also holds good in the case of a body floating

FIG. 2.—EXPERIMENT OF FLOAT IN WAVES.

- e. Float made of a cork ring.
- f. Short mast in the float carrying.
- g. A small plumb-bob which hangs in conformity with the apparent direction of gravity, and thus at right angles to the surface of the water.

FIG. 3.—EXPERIMENT OF FLOAT IN BREAKING WAVE.

- e. Float made of a cork ring.
- f. Short mast in the float, carrying
- g. A small plumb-bob which hangs in conformity with the apparent direction of gravity, and thus at right angles to the surface of the water.

on waves is demonstrable by theory, and I have also shown it by an experiment with the float and plumb-line. In this case the float consisted of a cork ring like a life-buoy, about four inches in diameter, and was placed among waves of various form (see Fig. 2). In one case I placed it in waves breaking on a beach, and I could see it on, or rather under, the overhanging crest of a breaking wave, and even in this case the plumb-line was pointing rigorously square to the surface of the water, although that surface was actually facing downwards (see Fig. 3).

The condition of things, then, under which it happens that a ship is set in rolling motion, and from which she derives that motion, when floating in wave-

disturbed water, as contrasted with that under which she remains at rest in undisturbed water, is the fact, that, whereas in still water, the attitude into which the pressure of the water perpetually tends to force her, is constant and unchanging, in wave-water, on the contrary, this attitude is continually varying. It is in the constantly defeated attempt to attain the attitude of rest, that the ship among waves is perpetually changing her attitude; and since this attitude is

the ship's position of momentary equilibrium, that is to say, her right position for the moment; and since it is the fact that when she deviates from it by a given amount, the force which urges her towards it, is the same as that which tends to right her, when similarly inclined in still water, it may be conveniently termed a righting force, and I shall speak of it thus, even though it may be occasionally urging the ship into an inclined position, as it must do when she is more upright than if, at right angles to the wave slope.

Were the ship perfectly rapid in her movements, if, that is to say, she possessed an infinitely powerful righting force when inclined in still water, or were her moment of inertia (that is to say her momentum as a fly-wheel) infinitely reduced, she would then at once attain this attitude of rest, not by being stationary, but by keeping pace precisely with the changing inclinations of the wave slope. But inasmuch as the force which urges her is necessarily finite, and her moment of inertia necessarily considerable, she is always involved in adventitious and additional movements, sometimes overrunning, sometimes falling short of the position she is seeking; and thus instead of following the motions of the waves like a raft, or, like the experimental floats, the ship "rolls."

The principle, then, out of which the rolling motion is developed, is profoundly simple; it is certain that it exists and must operate. It is what Herschel has called a "*vera causa*." Whether its operation is sufficient to account for and explain the rolling which a given ship will undergo when exposed to a given wave—whether in fact it is only a collateral cause, not *the* cause of the rolling, is a question which can only be answered by counting up its proper effects by the known laws of dynamics, and comparing them with the results which actually ensue.

It will be said that out of the multitudinous and confused inclinations which the surface of the sea presents, it is impossible to evoke order and uniformity enough to render the case amenable to treatment by ordinary mathematical processes. And this is undoubtedly true to a considerable extent, though to a less extent than might be *prima facie* supposed.

It is true that we cannot by any method, mathematical or other, predict, when a ship is despatched on a given voyage, precisely how much rolling she will experience, partly because we cannot predict what seas she will encounter; partly also because no doubt the seas she does encounter will be of such unknown irregularity, that we cannot hope to appreciate their absolute effects.

But this difficulty is at least a conclusive answer to those who, without having even a definite *locus standi* as a basis of thought, who, even discountenancing the existence of such a *locus standi*, are yet ready to advance definite and confident predictions about the behaviour of individual ships.

I do not advert to men of large and varied experience, who attempt cautiously to educe rules, by induction, from the facts which their experience has supplied to them. Yet even to them I would appeal not to rely in this case with too confident preference on conclusions which

experience, taken alone, and unguided by established laws, may seem to them to have justified.

For, confidence in conclusions which such experience induces men to form, is nowhere so ill-placed as where the surroundings of the question are multitudinous and complex; since in such cases it is almost impossible to select with certainty, by experience unenlightened by theory, which out of those surroundings is the real governing condition of the problem. And on this ground I invite even those whose experience may have least inclined them to agree with me, to join me in tracing from the *vera causa* which I have explained, the consequences which, with a very near approach to exactness, must follow from it, by applying to it the known laws of dynamics and the known laws of wave motion.

Our problem is this. Given the rate at which the wave on which the ship rests, broadside on, changes its inclination *in transitu*, and given the rate at which the ship will change her inclination when exposed to a righting or inclining force of given magnitude, what will be the rate of change and the total amount of motion which will ensue as wave after wave passes under her?

In the first place, the laws which govern the motion of regularly formed waves, are now very well known, and are reducible into tractable mathematical expressions.

The rate, in terms of time, at which the slope of a wave *in transitu* varies, is almost exactly the same as that at which the inclination of an ordinary pendulum varies—quite nearly enough to be used in the investigation of this problem.

The rate, in terms of time, at which the ship will change her position, also admits of being exactly calculated, given her form and the disposition of her weights on the one hand, and the actual force on the other.

This branch of the problem, however, is perhaps best elucidated by tracing the actions on which it depends as exhibited by a ship when set oscillating in still water.

Let us suppose the ship to be hove down artificially to the successive inclinations, $1\theta, 2\theta, 3\theta$, &c., by means of a succession of inclining forces acting on a lever of given length, and that the successive forces employed are $1F, 2F, 3F$, &c. Now if on a base line we mark off a series of spaces which, dating from the same zero point, represent to scale the successive inclinations, and plant at the end of each, an ordinate which represents to scale the corresponding force, the tops of the ordinates, in effect, constitute a line which is called the "curve of stability."

This curve represents in fact the variation of the righting or inclining force in terms of the inclination, and it inevitably follows from the principles on which the stability depends, that up to some considerable inclination the righting force is exactly proportioned to the inclination, and for the purposes of the present explanation it will be assumed that this law holds good up to any inclination that the ship can in fact reach. The assumption involves so little error under ordinary circumstances, that the results deduced by help of it may be regarded as typical.

Such then is the force which, in still water, tends to right a ship

when she is forcibly inclined. It has here been viewed as a simple fact determined by an experiment performed on the ship herself; but it may also be arrived at by calculations based on the ship's form and the distribution of her weights, and may be expressed in terms of the elementary conditions inherent in the ship's form and displacement, namely, her weight, and her metacentric height, that is to say, the distance between her metacentric and her centre of gravity.* We have next to see what are the rotational motions which this rotational force will impose on the ship under the various conditions under which it is free to operate, excluding for the present the collateral action of what is commonly termed "the resistance of the water," and we will consider these motions first, as they are exhibited and measured by the ship's behaviour in still water.

When the ship after having been hove down, as described in still water, is released and thus allowed to right herself, the rate at which she will do so is proportioned, directly, to the righting force at the instant, and inversely, to her moment of inertia, that is to say, her momentum regarding her as a fly-wheel; and she will proceed thus with accelerated speed till she reaches the upright position (which she will pass through with a maximum velocity), and travel onwards with retarded speed till she reaches an inclination opposite to that from which she departed, and indeed equal to it but for the surface-friction, keel-resistance, &c., she has encountered *in transitu*, and which for the present, as has been said, it will be convenient to treat as non-existent. Thus she will continue to oscillate like a frictionless pendulum.

The time expended in the complete transit will clearly be the greater for a given total arc traversed, in proportion as the force employed is weaker, and the moment of inertia is greater. But if we compare the times expended in the transit by a given ship, according as the arcs are large or small, the condition that the righting force is directly proportioned to the inclination, shows at once that the time will be the same whether the arc is large or small, because in traversing the larger arc, both the acceleration and the accumulated velocity will be greater exactly in proportion as the space to be traversed is greater.

This is in fact the essential condition of isochronism in oscillation, and in virtue of this it happens that with a given ship, as with an ordinary pendulum of given length, the time occupied in performing a given complete oscillation or single swing, is within practical limits the same, whether the arc of oscillation be large or small. It is here convenient to observe that the time occupied by a double oscillation is called the "period," because it virtually completes the circuit, bringing the oscillating body back to the position and conditions from which it took its departure; and the "period" is perhaps to be regarded as the fundamental or standard unit of designation in most investigations allied to this; but as the half-period, or time of a single swing, appears to

* This is commonly formulated as follows:— $F = WM\theta$, where F is the force, supposed to act with an unit of leverage, W the ship's weight, M her metacentric height, and θ the inclination.

me a more convenient unit in this particular investigation, I adopt it in preference, calling it the "time of oscillation."

It is easy to see that the rate of acceleration at any instant under a given righting force is completely expressible for a given ship in terms of her half period or "time of oscillation." And it is convenient to express it thus, because the time of oscillation is, *per se*, simply determinable by oscillating the ship in still water; though it must be added, that like the righting forces in still water, it may also be arrived at by the somewhat lengthy and laborious calculations* necessary for determining her curve of stability, and her moment of inertia, from the distribution of her weights.

Thus, then, as by inclining the ship forcibly in still water we may measure absolutely the force which at any given inclination urges her towards the upright position, we may by observing her time of oscillation when allowed to oscillate in still water, learn exactly at what rate she will acquire rotational velocity in any direction in terms of the rotational force which at the instant urges her in that direction; and we may proceed to import these considerations into the study of her behaviour when acted on by waves. I will again observe that the operation of resistance is still excluded from the enquiry.

Now in virtue of principles which have been already explained, the ship having for an instant a given inclination relative to the wave slope, will experience the same righting force tending to destroy that inclination, as she experiences when in still water she has the same inclination absolutely, or with reference to the horizon. And hence the rate and the direction of the acceleration which she experiences will be the same in both cases, in terms of the inclination to which it is due; and as the inclination relatively to the wave slope is the difference between the ship's absolute inclination and the slope of the wave, we are able to express the acceleration† at each instant, in very simple

* Assuming the moment of inertia and metacentric height to have been determined by the calculations referred to, then calling the time of oscillation T , the radius of gyration ρ , the metacentric height, as before, M , and the force of gravity, g , the formula for T is $T = \frac{\pi\rho}{\sqrt{gM}}$; but T may have been simply determined by the still water experiment, as described, and this result is *per se* available.

In any case, by the laws of isochronous oscillation, the equation of angular acceleration in still water is $-\frac{d^2\theta}{dt^2} = \frac{\pi^2}{T^2}\theta$, θ being the inclination at the instant. The negative sign of the differential corresponds with the circumstance that the acceleration is always tending to diminish θ .

† The equation of angular acceleration for the ship broadside to the waves is thus $-\frac{d^2\theta}{dt^2} = \frac{\pi^2}{T^2}(\theta - \theta')$, θ' being the inclination of the wave slope at the instant. Now, the wave curve may, with close approximation to truth, be regarded as a curve of sines, if we take the height to be as small compared with its length as it usually is in large waves; and under these conditions, if Θ' be the inclination of the steepest part of the wave slope, and T the half period of the wave, it follows that approximately $\theta' = \Theta' \sin \frac{\pi t}{T}$, so that the complete equation is $-\frac{d^2\theta}{dt^2} = \frac{\pi^2}{T^2} \left(\theta - \Theta' \sin \frac{\pi t}{T} \right)$

The integration is somewhat lengthy, nor need it or its results be formally given here. The object of giving the fundamental equation is simply to show in complete

terms, since, as has been pointed out, the slope of the wave at each instant is easily expressible in terms of time.

The working out of the conditions tells us what will be the ship's inclination and angular velocity at any future instant, given the steepness of the waves and the ratio of their period to the period of the ship, and given her inclination, her angular velocity, and her position on the wave, at the instant from which the lapse of time is counted.

The simplest form to which the solution can be reduced, arises under the limiting conditions that at this point of departure the ship is stationary and upright, and is either at the crest or hollow of the wave when the slope is zero; and the results of this form of the solution will be taken as typical, and their principal features will be presently traced.

It is, however, instructive and perhaps necessary, to explain that if the solution is taken without any limiting conditions (namely, on the assumption that the ship when started at the zero point of wave-slope, possessed any assignable inclination and angular velocity), the completely general results that it presents are connected by a very simple and interesting relation with those obtained under the simpler form of solution. For, the ship's initial inclination and angular velocity at the level starting point may be regarded as, potentially, part of a definite oscillation, which, if continued independently in still water, would have assigned to her a definite inclination and angular velocity at every future instant. Now the interesting aspect of the general solution which I refer to is this: the prospective movements which it assigns to the ship are the same as we should arrive at by combining those which are potentially due at each instant to the equivalent still water oscillation, with those due to the operation of the wave series under the operation of the limiting conditions which give rise to the simple form of the solution. The total result at any future instant consists of that which the wave-slopes would have imposed on the ship independently of her initial inclination or velocity, added to that which the initial inclination and velocity would have imposed on her in still water or independently of any wave-slope, hence it is sufficient to regard the results of the simple form of the solution as typical.

The principal results arrived at are entirely common-sense in their character, and such as may be readily apprehended by any mind well trained in dynamical reasoning.

The most striking and the most important of them is the conclusion that when the waves which act on the ship, have the same period as the ship herself, the range of her oscillation will receive an increment of definite and constant magnitude, wave by wave, so that in the transit of a few waves she must be infallibly overset. I have myself produced this result by artificially generated waves of definite period, acting on a float so shaped as to oscillate with a minimum of resistance, and with the same period; it was invariably overset by the transit of four or

shape the circumstance that the whole mathematical theory of unresisted rolling can be worked out in reference to any individual ship on the basis of experiments which may be performed with her in still water. It will be seen farther on that the same mode of procedure is, with even greater appropriateness, available for the treatment of the difficulties which belong to the subject of resistance.

five waves. And this is what would, in fact, certainly happen to ships under the conditions named, if it were not that their oscillations are, in fact, performed in a resisting medium, and that their range is thus restricted by a growing force which will, in most cases, neutralise the increment before the fatal result ensues. But this branch of the subject has been, for the present, thrown out of the question, and will be noticed presently. At present it is enough to point out how thoroughly this conclusion, at least in its general form, accords with common experience, which tells us that if we wish to create the greatest amount of oscillation in a body capable of it, we must repeat the impulses, oscillation by oscillation, and in exact harmony with the phases of the motion. It will be in the experience of many Naval Officers especially, that even the largest ships may be worked up to an oscillation of very considerable range by running the crew from side to side, if only the runs are exactly timed to the recurring motion.*

The increment of range which, but for resistance, co-periodic waves would impose on the ship, wave slope by wave slope, or oscillation by oscillation, is $\frac{\pi}{2}$ (or about $1\frac{1}{2}$) times the inclination of the wave at its steepest part, so that on a wave, the steepest part of which has 6° of inclination, each oscillation would carry the ship 9° farther than its predecessor.

While this form of co-periodic oscillation is in progress, the ship's maximum inclination must always occur when she is at the top and bottom of the wave, and she is just at the middle of her swing when the mid-height of the wave passes her.

The result which is thus produced by co-periodic waves, is the most complete form of what is termed "cumulative rolling."

But cumulative rolling will also be produced by waves which are not co-periodic, though the amount of accumulation will be less, in proportion as the dissidence of the period or dis-periodicity (as it may be called) is great, since only part of each impulse can co-operate with the existing motion, and part must be antagonistic to it.

If the dis-periodicity is not very great, the transit of the first wave-slope will produce a result not far short of that which the first co-periodic wave would have produced; but, wave by wave, the want of harmony becomes more and more pronounced, the portion of the impulse which co-operates with the ship's roll becomes smaller, that which is antagonistic becomes larger, accumulation ceases and diminution commences, and thus the rolling, after attaining a maximum, dies out by just the same steps as those by which it had grown up.

Here again the results of the solution are strictly in accordance with common sense, and indeed with common experience.

A not uninteresting illustration of the action which has just been traced, is frequently furnished by the behaviour of a pocket-watch if divested of its chain and hung by its pendent on a nail, which often serves roughly as a knife-edge, and allows it to swing freely, as a

* This process is often adopted as a means of scouring a ship's bilges, a certain amount of water having been admitted to them that it may be thus forced to rush from side to side.

pendulum of very short period. In this condition, its period is generally not very far from that of the balance which is oscillating within it, and the alternate extensions and compressions of the balance-spring impress on it a series of alternating impulses of small but appreciable magnitude, and it will frequently be seen that the watch, regarded as a pendulum, will acquire a considerable series of oscillations, which, owing to the want of exact co-periodicity of impulse, grow up to a maximum, die out again, and recommence in an exactly reproduced order.

It is instructive to observe that the greatest increase of range which the transit of any single wave can, under any circumstances whatever, produce on the ship during a given roll, is when the phases of her rolling have become so related to those of the wave that the intervals of antagonism between the wave slope and the ship's motion, happen where the wave slope is near its minimum, that is to say, at or near the hollow and crest of the wave, the ship being inclined in such a direction that the approaching wave slope becomes piled up (so to say) on her inclination, and intensifies the righting force throughout the roll, the steepest part of the wave co-operating with the motion, and supplying an effective relative angle even when the absolute angle is near zero; and the relation which plainly best satisfies these conditions is that, whether the period of the ship be greater or less than that of the wave, the commencement and termination of the roll shall be about equidistant in point of time from the hollow and crest respectively. The whole effort of the individual wave is thus applied with its greatest possible advantage towards the augmentation of the roll, and does the least possible towards the alteration of the period.

There remains to be mentioned a third type of unresisted rolling, which, however, grows out of the solution only in its most general shape, by assigning to the ship an initial inclination and angular velocity which have a special relation to the ratio of the period and the steepness of the waves.

This is what is called constant rolling, and it consists of a state of things in which the operation of the successive waves obliges the ship to roll to an invariable range, her period being forced into agreement with that of the waves, though these are in fact dis-periodic.

This result, it will be seen, is in marked contrast to that type of cumulative rolling which has been traced out as deducible from the simpler form of solution when applied under these conditions, namely when a ship is engaged with dis-periodic waves. It is, however, one which may be even quantitatively arrived at without any technical mathematical reasoning, and it appears more instructive to trace it out thus than to refer to it merely as a consequence of the solution.

If we consider what is happening when a ship is oscillating at her natural period in still water, we must perceive that the period of the roll is determined by the circumstance that the righting force is throughout not merely in direct proportion to the inclination at the instant, but that the numerical term which expresses the proportion, is exactly of such magnitude as to bring out the result which does, in fact, arise, namely, such as will oblige the ship to complete the oscillation in that particular

time. But it is easy to see that if the proportion had been other than it is, whether greater or less, the speed of the ship's motion at each instant would have been increased or lessened accordingly, so that the oscillation would have been completed in a less or greater time; and a little reflection shows that the relation which subsists between the increase of force and the decrease of period, or *vice versa*, is that the period will be inversely as the square root of the intensity of the force. It is as the square root of the intensity, not as the intensity simply, because, when the force is, for instance, increased, and the velocity thus increased also, the time during which the force acts is thereby proportionally lessened, so that, for instance, it requires a force of quadrupled intensity to produce in a given swing a doubled velocity, or a period twice as short.

Now a scale of force thus uniformly intensified throughout, or *vice versa*, may by hypothesis be exactly supplied to the ship while oscillating in still water, if we suppose a wave of appropriate steepness and period to be (so to say) administered to her as she rolls, if the relation of its phases as it operates on her be duly related to the phases of her rolling. This relation must, in fact, be such that the wave-phase which makes the wave-force zero, namely, the transit of its hollow or crest, shall operate on the ship at the moment when she is upright and is moving with her maximum velocity, as happens with the force naturally delivered during the still-water oscillation; and further, that (again in accordance with that naturally delivered force) the maximum wave-slope or phase of maximum wave-force shall operate on her when she is at the extreme range of her oscillation: and so on, throughout. If these two conditions hold, it will be found that a due apportionment of uniformly increased or diminished force has been secured, corresponding with the alteration of the period.

If, for instance, we desire to assign the wave-condition under which the ship will perform an oscillation of given range, in half her natural period, when, as was just now explained, the actuating force must be throughout quadrupled, we may see that the necessary conditions will be fulfilled, if the wave we assume has in the first place half the period of the ship, and, in the second place, a maximum inclination treble the ship's maximum inclination; for this trebled inclination, combined with the ship's own inclination, supplies, in the whole, the quadrupled force. It will be readily seen that the general proposition which this instance has illustrated, must include the condition, that when the ship's period is thus to be shortened, the advancing wave slope must face in opposition to the ship's roll, so as to intensify the righting force. When it is to be lengthened the wave slope must face in accordance with the roll so as to moderate the righting force.*

* It is readily seen in the light of this reasoning, that the constant relation which must subsist between the wave-slope, the ship's inclination, and the period-ratio, is (retaining the previously used notation) $(\theta + \theta') = \frac{T^2}{T'^2}\theta$, or $\theta' = \left(1 - \frac{T^2}{T'^2}\right)\theta$.

This of course includes the relation between the maximum wave-slope and the range of the ship's roll, which would be written $\theta' = \left(1 - \frac{T^2}{T'^2}\right)\theta$.

This arrangement thus worked out would keep the ship rolling to a constant inclination, and with a constant but modified period, for an indefinite time. This then is what has been called "constant rolling."

It is instructive to contrast the conditions under which the passing wave slope simply alters the period of the ship's roll, without either increasing or diminishing the range of the roll, with those under which it leaves the period of the roll unaltered, but produces the greatest augmentation in its range which, as a single wave, it is capable of producing. The characteristic feature of the contrast is that at the mid-height of the wave when the wave slope is steepest, in the former case the ship's angular motion is zero, and the full range is attained—in the latter the ship's angular motion is greatest and she is in the middle of the roll. Thus the whole effect of the wave in the former case is devoted to the alteration of the period; in the latter to the augmentation of the range.

It is now time to turn to the essential but (theoretically speaking) somewhat obscure subject of Resistance; that force or set of forces, namely, which is called into existence in the surrounding fluid by the ship's motion, always acting in opposition to her motion, and tending to bring her to a state of rest.

It is not proposed here to deal with the subject in its theoretical and obscure aspect, but rather to trace the action as measured by experimentally ascertainable results, and to explain how these may be used to correct, or rather to complete and reduce into a practical form the abstract propositions in which the laws of unresisted rolling have been expressed.

Here also the behaviour of a ship when set oscillating in still water throws much light on the operation of the force we are investigating.

If, when by running men from side to side, or by other means, we have set the ship rolling to a considerable range, we cease the impulse and leave her to carry on the oscillation without interference, we shall see that, roll by roll, the range of oscillation becomes extinguished, and after a time the ship will come absolutely to rest. This extinction is obviously both the consequence, and the natural exponent, of the resistance experienced by the individual ship.

Supposing that by exact observations both of time and range, or what is better, by automatic means, we have obtained a record of the *rate* of extinction, we have thus in effect secured a measure of resistance in a shape which directly serves our purpose.

For while in the former part of the investigation we learned how much motion the transit of a given wave will communicate to the ship during the performance of a given roll, we learn in the ascertained rate of extinction, how much motion the resistance acting during the performance of the roll, will abstract from her. It is true that the solution thus obtained must be regarded as only an approximate one, but its *rationale* is simple and intelligible, and plainly takes us very far in the right direction, and on the other hand, it is found that practically the results which follow from it are very near the truth. It is true also,

that we cannot readily initiate artificial rolling to as great an inclination as to include the deepest rolls which possible waves will impose on the ship; but within the limits of easy experiment we can attain sufficient range to determine with considerable exactness, the law which governs the rate of extinction in terms of the range of the roll, and are thus enabled to calculate it pretty exactly for much deeper rolls. In fact, if after having performed the experiment and recorded its results, we mark off, on a base line, successive equal intervals, representing the count or numeration of the rolls, and duly plant on the base thus marked, a series of ordinates representing, to scale, their successive ranges, the ends of the ordinates will constitute a curve, which is a portion of what may be termed the "curve of extinction," and which may be extended by calculation as proposed.*

Speaking broadly, we may say that just as the theory of unresisted rolling has informed us what is the roll-originating power exerted by the waves, roll by roll, so this curve tells us what is the roll-extinguishing power exerted by the resistance, roll by roll; and it is plain that accumulation must cease when the rolling has become so deep that the two forces, roll by roll, are equal. And clearly this state of things indicates that, under the operation of resistance, a form of constant rolling tends to become established, resembling, but differing of course in some characteristic peculiarities, from that already described.

The simplest case in which the application of this principle can be traced is that of rolling in co-periodic waves.

* It appears to me, both by experiment and by a rational appreciation of the data, that the rate of extinction as governed by the range, is expressible by the sum of two terms, of which one is simply as the range, the other as the square of the range, so that, if in mathematical language we put θ as the range of the roll, and n as the number or count of the roll (*e.g.*, first, second, third, &c.), the rate of extinction or

loss of range per roll will be $-\frac{\Delta\theta}{\Delta n}$, and we have $-\frac{\Delta\theta}{\Delta n} = (a\theta + b\theta^2)$, and the ex-

pression, if regarded as the differential equation of the curve of extinction, is readily integrated; and on a careful analysis of the numerous specimens of these curves which I have treated, it has proved in every case that such values of (*a*) and (*b*) might be introduced into the integral of the equation, that the calculated curve will fit the experimentally determined curve with remarkable exactness. Hence it is presumable that it would be found to accord pretty nearly with what would have been the result of experiment, if this could have been extended so as to include rolls of as great range as are ever encountered.

It is also probable that the terms which thus accurately express the resistance in the case of still-water rolling, will bring out an approximately correct result when applied to rolling in a sea-way, for it is presumable that the resistance the ship will experience in performing a roll of given range will, in the aggregate, be approximately the same, whether it is performed in still water or in a sea-way.

Only when the conditions are such that the ship is forced to perform the roll in a lengthened or shortened period, a correction must be made in accordance with the effect which the difference in velocity will have had on the resistance. But the correction may be readily made in virtue of the condition which rationalises the separate existence of the two terms of the equation involving θ and θ^2 , the condition, namely, that the resistance is made up of two elements which are respectively proportioned to the velocity and to the square of the velocity with which the roll is performed.

Here, as we have seen, the transit of each successive wave slope would, but for resistance, be capable of adding, roll by roll, an increment of range $\frac{\pi}{2}$ (say $1\frac{1}{2}$) times, the steepest part of the wave, whatever be the range already attained; and this may be termed the potential increment. The actual increment which takes place in each roll will, therefore, fall short of this, by a quantity which corresponds with the extinguishing effect of the resistance during the roll, and will become zero therefore, when the range is so great that the rate of extinction equals the potential increment appropriate to the particular wave series.

Now, on reference to the complete curve of extinction, it is easy to see what part of the curve exhibits, as due to a single roll, a decrement of ordinate equal to the potential increment, and the ordinate at the middle point of this portion of the curve indicates the maximum range of roll which the particular wave series can impose on the particular ship.

When this state of things has been arrived at, so that the ship will, wave by wave, perform an oscillation of definite magnitude, in which on the whole, the impulsive force of each wave slope is balanced by the resistance experienced during the corresponding roll, the state of things is somewhat analogous to that of a clock pendulum which is kept going by the "maintaining power." In both cases alike the aggregate "work" communicated by the impulse, equals that abstracted by resistance, only that in the case of the clock the impulse is momentary, and is administered solely at the middle point of the swing, while with the ship engaged in this particular type of rolling, the phases of the wave and of the roll are so related, that the impulse is delivered continuously, and *pari passu* with the resistance, indeed throughout in tolerably exact proportion to it, for, as has already been explained in relation to unresisted co-periodic rolling, the instants of maximum inclination of the ship, when her angular velocity is zero, are at the top and bottom of the wave when the slope is zero, and the instant of her maximum velocity is almost precisely when the mid-height of the wave passes her, and the wave-force is at its maximum; and as the resistance increases and decreases with the velocity, a general equilibrium is thus approximately maintained.

The analogy between the maintaining power which sustains the swing of the clock pendulum, and the wave impulse which maintains the ship's oscillation, supplies a point of view from which we may see our way to what is virtually the conclusion which has already been reached, but in a somewhat different, and in effect, a more comprehensive form; as an answer, namely, to the question, what must be the steepness of a wave series of given period to keep the ship rolling with a given range, instead of to the question, what will be the range of oscillation imposed on the ship by a wave series of given period and given steepness.

If we trace out the answer thus in relation first to a co-periodic series, we shall see our way to the solution in a more general form.

If we assume the ship to be set rolling up to a given range in still

water, then if the medium in which she rolls were incapable of offering resistance, the rolling would continue for ever without any abatement, just as would happen with a frictionless pendulum oscillating *in vacuo*. If we now suppose the resistance of the water to begin to operate, we may propose to ourselves the enquiry, what system of surface disturbance must we at the same time impose on the water, in order to neutralise the extinguishing power of the resistance, and maintain the rolling with unchanged period and with unabated range.

Clearly the disturbance must be of the nature of a co-periodic wave series passing under the ship, with its inclination so apportioned to the ship's angular velocity as to be assisting the ship's motion, just as the resistance obstructing it.

Were it possible to create a wave series, the steepness of which should follow an arbitrary order in terms of time, we might make this order such as precisely to neutralise the resistance at each instant; but in fact we are not at liberty to assign this order precisely, since when the steepness of the wave series has been defined at any one particular point, the order of its steepness at other points is defined by inherent dynamical laws; yet, as has been pointed out, these inherent laws do in fact so define it, as to correspond very nearly with the order in which the resistance is delivered, and if we assume a wave series of such mean steepness, that its impulsive effect will be in the aggregate equal to the aggregate extinguishing effect of the resistance, the range of the oscillation will be conserved; only the intermediate variations of the ship's inclination as she rolls, will experience some almost infinitesimal modifications of alternate accelerations and retardations, during the progress of each roll.

Now the curve of extinction tells us how much of the assumed range would be lost by resistance in the single roll of the assigned mean range; on the other hand, the laws of unresisted oscillation tell us that the transit of a single co-periodic wave slope of given maximum inclination is capable of adding to the range of the ship's previous roll an increment of range, which is a little over $1\frac{1}{2}$ times that inclination, or (shaping the proposition suitably for our present purpose) to add a given increment of range per roll by the intervention of a co-periodic wave, we must make the maximum wave slope barely $\frac{2}{3}$ of that increment, which is in effect identical with the conclusion already reached.

But having now gained the idea of a wave series as a maintaining power, we can apply it to the case of what has been called "disperiodic" rolling, and by help of this idea we may proceed to determine what is the wave series which will keep the ship rolling with a forced period, either longer or shorter than her natural period, and with a range unimpaired by resistance.

In the first place, we have already seen how, in the theory of unresisted rolling, the introduction of an auxiliary wave series, with its steepness and its phases definitely related to those of the ship's oscillation, will maintain in the ship the definite range of oscillation, but with the period altered in any assigned degree.

It is true that the alteration of period involves an altered scale of

velocity in the performance of a given range, and, therefore, an altered scale of resistance; so that the data presented by the curve of extinction are not nakedly applicable; but, if we know the law of resistance in terms of velocity, we can deduce from the curve of extinction the extinguishing power of the resistance operating during any given roll, if performed by the ship, in either a lengthened or a shortened period; that is to say, if we know the obstructive "work" done by resistance during the roll of given range if performed in the ship's natural period, we can determine it when the same roll is performed in the altered period, and can also determine the corresponding new decrement of range.

Hence, proceeding by exactly the same method as before, we can define in terms of this newly determined decrement, the steepness of the auxiliary wave series which is required as a maintaining power to neutralise the effect of resistance operating under the altered period.

Thus, to maintain the rolling of given range in a dis-periodic wave series, the combined operation of two distinct auxiliary wave-series is required—one of which may be called the "period-governing" series, the other the "maintaining" series; but for reasons which have been already explained, the relation between the wave phases and those of the ship's oscillation, are characteristically different for the two auxiliary wave series.

The instant of the maximum wave slope in the "period-governing" series, concurs with that of the ship's maximum inclination and the zero of her angular motion; in the "maintaining" series, it concurs with the zero of her inclination and the maximum of her angular velocity.

The combination, or superposition as it is called, of these two auxiliary waves, creates the total wave by which the roll of given period and range will be maintained in the ship. Perhaps, however, those to whom the idea of a wave, consisting of several waves in superposition, is for the first time presented, may desire some explanation.

The figure or profile of a wave-series is represented with sufficient exactness by a continued repetition of what is called the "curve of sines," a curve the nature of which may be sufficiently described to those who are not conversant with it mathematically, as follows:—Imagine a clock pointer to travel once round the dial, while the wave passes from crest to crest, and let the length of the hand be half the height of the wave from hollow to crest, when drawn to the required scale; draw a horizontal diameter across the dial, and draw a base line, spacing it off with intervals representing either the length, or the period, of the wave from crest to crest, then subdivide each of these intervals into twelve equal spaces, corresponding with the time intervals at which the pointer arrives at the successive hour-marks on the dial, and at each station thus fixed on the base, draw an ordinate, the length of which is the vertical distance of the end of the pointer, above or below the diameter; the length of this ordinate represents the height or level of the wave surface at that instant.

The following well-known and interesting property of this kind of curve, illustrates what is called the superposition of waves:—

Draw any two such curves on the same base, making them alike only

record, first of the angle which at each instant of time the ship's mast makes with a truly vertical line; and, secondly, of the angle which at each instant of time the ship's mast makes with a line drawn normal, that is to say, at right angles to that portion of the wave-surface on which the ship is at the instant of time referred to. The first of these angles, namely the angle between the ship's mast and a truly vertical line, may be conveniently called the ship's *absolute angle*, and the second angle, or the angle between the ship's mast and the normal, may be called the ship's *relative angle*, as being taken relatively to the wave-surface (see Fig. 4). The *absolute angle* is the angle

FIG. 4.—EXPLANATION OF THE TERMS “ABSOLUTE ANGLE” AND “RELATIVE ANGLE.”

usually referred to in all ordinary records of ships rolling; thus, when it is stated that a ship rolled 10 degrees to port, it means that her mast made an angle of 10 degrees on that side with a truly vertical line. It will be evident, however, from the principles set forth in considering the causes of the rolling of ships, that it is important not only to record the *absolute angles* of a ship's rolling, but also her angles *relatively* to the wave-surface, it being borne in mind that the righting tendency of the ship tends to place her mast at right angles to the wave-surface. It must also be remembered, that the apparent direction of gravity to a body floating on a wave, is at right angles to the wave-surface, as was indicated in the experiment of the short plumb-line on the float, either in the suspended cup or in the wave-water (see pp. 859, 860, and Figs. 1, 2, and 3).

A vessel may to all intents and purposes be considered to roll about her centre of gravity, and thus the centre of gravity while partaking

of the vertical and translatory motions imparted to the vessel by the waves, will be free from the rotatory motion experienced by the other parts of the vessel, and a pendulum hung freely at the centre of gravity of a ship will not be disturbed by the rolling of the ship, and will thus be under conditions similar to those of the plumb-line in the small experimental floats before referred to.

But for the reasons I have before pointed out, even a pendulum hung at the centre of gravity of the vessel in a seaway, will not, as on shore or in still water, hang constantly in a vertical line, but it will point in a direction at right angles to the wave-surface, so that its record is fallacious if taken to indicate the *absolute angle* of a ship's rolling. Still more misleading is the record of a clinometer placed at any part of a ship other than the centre of gravity, for then it will be subject to the rotatory motions of the vessel. The extent of the error thus introduced is exhibited by the larger inclinations assumed by lamps hung, even with much friction, under the upper deck of a lofty ship, or the extent to which in walking the upper deck of a vessel rolling, one must, in order to counteract its sideways sway, lean over to an angle far greater than the angle actually rolled by the ship. For instance, a pendulum-clinometer hung at the upper deck-level of an old line-of-battle ship would often swing to an angle at least half as much again as the true angle of the vessel's roll on the wave. Moreover, as the clinometers placed on board ships are of some length and freely suspended, they, when subject to the rotating movements of the vessel, set up a considerable swing of their own, and thus the record is still further vitiated. There can be no doubt that many most exaggerated statements as to the angles to which vessels have rolled, are due to the angles having been measured by the clinometer.

But though fallacious as a record of the *absolute angles* rolled by a ship, a pendulum gives a valuable record if hung at the centre of gravity of the ship. Here, as I have said, it will be free from adventitious influences, and will tend to point constantly in a direction at right angles to that portion of the surface of the wave on which the ship is for the movement. If, then, we can keep a continuous record of the positions which at each instant of time the pendulum assumes—assumes, that is to say, in relation to the mast of the ship, we shall have a record of the *relative angle*, that is to say the angle which the ship herself makes at each instant of time with the wave-surface. I will here point out that, as I have said, the tendency of the fluid pressures is to set the ship at right angles to the wave-surface, and as this tendency is measured in terms of the angle the ship makes with that wave-surface, it follows that the record given by the pendulum, forming, as I have said, a continuous record of the *relative angle* the ship makes with the wave-surface, supplies also a continuous record of the forces of the water acting on the vessel to cause it to rotate in the one direction or the other.

The way in which the successive positions of the pendulum are continuously recorded, will be familiar to most of my hearers, being similar to the self-recording operations of indicator diagrams, tide gauges, &c. As the pendulum swings in the ship, it moves a hori-

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TIME SCALE.

SCALE OF DEGREES.

DIRECTION OF TRAVEL OF PAPER.

Direction of swing of pendulum, and of motion of recording pen of pendulum.

- In this diagram, the ordinates of the curve, as $c d$, indicate, by the scale of degrees, the "relative angle," or the angle made by the ship's mast, with the normal to the wave surface at the instant of time represented in the diagram by the distance along the line b of the point d .

The pendulum in this apparatus is made very heavy, and has a very short period. In appearance it is very unlike what is usually understood by a pendulum. Instead of a long bar with a big weight

at the end of it, it is a horizontal heavy cylinder suspended by its upper edge (see Fig. 3, plate XLVIII*). Thus, while very heavy and powerful, the weight is very near the point of suspension, and consequently the time of each swing is very short. A pendulum like this, of short period, while it instantaneously obeys the forces impelling it to set itself at right angles to the wave surface, is much less liable than a longer pendulum to be influenced by any adventitious causes, and is less likely to set up, so to speak, a swing of its own. The pendulum is delicately hung on knife edges in a special manner, which I shall have occasion hereafter to refer to.

I have now, I think, explained how a pendulum, though misleading as a record of the *absolute angle* of rolling of a ship, may be usefully employed to furnish a continuous record of the relative angle, which, let me remind you, measures at each instant the righting force acting on the ship. Before leaving this branch of the subject, I should mention that this method of recording the relative angle had been successfully adopted by M. Bertin, an able French naval architect of Cherbourg, before it independently occurred to me.

I will now proceed to consider the methods for observing the absolute angle of a vessel's rolling, that is to say, the angle which her mast makes with the vertical. By far the readiest point of departure for this purpose, when it is available, is an observation of the horizon. To the uninitiated I may explain that an observer standing up in a ship sufficiently high to observe the horizon clear of the waves, sees the horizon apparently rise and fall up and down the rigging as the vessel rolls, and by using the spaces between the ratlines of the rigging as a scale, a very correct appreciation of the angles to which the ship rolls, may be obtained. To employ the same principle more exactly, a graduated staff or *batten* is fixed upright against the bulwark of the ship, with degrees of inclination marked on it. The observer, with his eye at a proper height and distance from the staff, notes the angles to which the ship's side is depressed and elevated in reference to the horizon, or, in other words, the *absolute angles* of rolling of the ship. But this observer cannot do more than note the extreme angle reached by the ship in each roll, or at most, with the aid of an assistant with a watch, he can note the extreme angle, and the time at which each roll is made.

It is, however, desirable to obtain a continuous record of the angle assumed by the ship at each instant of time, a record, in fact, analogous to that which I have already described as given by the pendulum. If we had in the ship a bar endowed with the property of remaining truly vertical, or indeed in any fixed position in a vertical plane, it is clear that this bar might, like the pendulum, be made to actuate a pen which pen should trace a line recording the angles which at each instant of time the ship makes with the fixed bar, or in other words with a truly vertical line. It is to supply this fixed bar that the special portion of the apparatus I have to describe has been designed.

In some of the experiments I have lately conducted on the rolling of ships at sea, the function of this fixed bar was supplied by a rod or

* The Institution is indebted to the Institution of Naval Architects for Plates XLVII and XLVIII.—ED.

To illustrate Mr. Froude's Paper on an Instrument for automatically recording the rolling of Ships.

*From a photograph of the apparatus as mounted on a rocking platform representing
the deck of a Ship rolling in a seaway, and for the instant greatly inclined.*

Engraved by G. S. Ingram.

To illustrate M. Froude's Paper on an Instrument for automatically recording the rolling of Ships.

Fig. 1. General arrangement of Apparatus.

Fig. 2. Mode of mounting frictionless Wheel. Scale 1 inch = 1 foot.

Fig. 3. Knife edge arrangement for suspending Pendulum.

Elevation.

Scale 2 inches = 1 foot.

End view.

pointer, which an observer on the upper deck of the vessel kept level by pointing it continually at the horizon. The motion relatively to the ship, of this really motionless pointer, was conveyed to its recording pen by strings led down through the deck. It is evident, however, that a method of this sort, depending on the skill of an observer, and which can be only employed in daylight, should, if possible, be replaced by some self-acting piece of mechanism.

Admiral Paris has suggested and applied the gyroscope for this purpose, but the difficulties of maintaining continuous rotation and of rendering the suspension of the gyroscope sufficiently free from friction, have proved too great for its useful adoption.

I have accomplished the result by paying due attention to the far simpler dynamical principle, that a mass of matter which is at rest will remain at rest, except so far as it is put in motion by external force, and thus if, while we push it about, the force by which we push it is such as will not communicate rotation, no rotation will ensue. Now, any mass of matter suspended by its centre of gravity is, theoretically, in this condition, and a balanced fly-wheel is in this condition if held only by the centre points on which it balances, assuming this suspension to be frictionless. If the wheel thus mounted were placed on the deck of a ship, the ship might be swayed about by the waves and might oscillate incessantly beneath the wheel without imparting to it any rotary motion. It would remain thus at rest, simply because there was no cause for its assuming rotation. But to utilise this method in practice, it is necessary to adopt some method of suspension most carefully freed from friction.

The use of a knife-edge suspension, such as that of a delicate scale-beam, would be a ready way of securing the result; but, unfortunately, a knife edge, to work with exactness, requires a level, or nearly level, plate to rest on, which could not exist in a rolling ship; while a notch deep enough to prevent the knife edge sliding sideways, under deep inclinations of the ship, would be fatal to its proper action. The well-known expedient of a cylindrical axle running on friction rollers, of course suggests itself as an alternative, and it is seen this may be simplified, when it is recollected that the angles through which the ship rolls are limited, and an absolute rotation of 60 degrees on either side of the position of rest is more than sufficient for all the duty that could legitimately have to be thrown on the axle, by the oscillation of the ship. Hence but a very small portion of the circumference of the friction rollers can be called into play; and it is thus sufficient to provide only a small segment of the circumference of the roller, instead of a complete wheel, and the radius may be thus greatly enlarged without introducing cumbrous dimensions in the apparatus as a whole. In fact, this arrangement has been successfully employed as a frictionless method of suspending clock pendulums.

Assuming that a heavy wheel may thus be provided with a practically frictionless suspension, there remains, however, the necessity of obliging the relative motion of the ship and the wheel, to produce a continuous record on a travelling sheet of paper, and it is obvious that this process can hardly be accomplished without the introduction of

some adventitious friction, involved in the process of marking the line, or in the motion of the parts which carry the marker. This adventitious friction, however, is of course independent of the weight of the wheel; and, by making this weight sufficiently large, the friction, per pound of weight of wheel, may be reduced so far as to be immaterial.

Thus far the wheel has been spoken of as absolutely balanced on the axis of support; but it is plain that under these circumstances the slightest motion, if once imparted to it by any accidental cause, such as a touch from the hand or even the clothes of the attendant, would continue without abatement, and that the wheel would ultimately overrun the range of the friction rollers, nor would it be possible for a bystander in the moving ship so to touch the wheel as to set it at rest in its proper position. This difficulty may be surmounted by giving the wheel a slight preponderance on one side, making its centre of gravity slightly eccentric—thus making it, in fact, a pendulum with a very long period of oscillation. It is true that it becomes thus subject to the disturbing causes which necessarily operate on a pendulum suspended in an oscillating ship. But if the preponderance is very small, and, consequently, the period of the pendulum is extremely long, the time during which a disturbing cause operates in one direction is relatively so short that the amount of disturbance actually created during its operation, may be rendered inappreciable, and will be obliterated during the counter oscillation. In fact, bearing in mind that, on the average, the sea is level and the ship is upright, the wheel with its slight preponderance, if set in motion, will simply oscillate very slowly, and will gradually come to rest in its proper mean position, the motion becoming gradually extinguished by the resistance of the air and other retarding causes. Having thus generally characterised the conditions which must be adhered to to give effect to the suggested method, I will point out how they have been carried into effect in the apparatus shown in Plates XLVII and XLVIII.

The wheel (*a*) weighs 200 pounds, and is 3 feet in diameter; it rests on an axle of steel, coated with hard steel collars (*b*), 1 inch in diameter. The centre of gravity is about $\cdot 006$ of an inch out of the centre of the axle, and the time of a single swing from right to left is about 34 seconds. Each friction roller arm (*c*) has a radius of 18 inches, and consists of a stout casting, with a counterbalance (*d*) below its pivots; the segment-facings (*e*) are of hard steel, $1\frac{1}{4}$ inches broad. It will be seen that if these were rigidly fixed to the radii, the axle would almost inevitably take its bearing on one or other edge of each segment. To prevent this they are mounted on pivots (*f*), so as to square themselves into perfect contact with the axis. The delicacy of the suspension is attested in this particular instrument by the smallness of the rate of extinction of its own oscillation when put in motion, and by the fact that when at rest, a breath on the circumference will move it perceptibly.

The axis carries a wooden semi-circle (*g*) having a circumference $\frac{1}{8}$ inch to a degree, and by a connecting-rod (*h*), which takes the motion of the circumference lineally, the angles are transferred to and become marked on the travelling sheet of paper (*k*), tracing a continuous curve (similar to that already described as

traced by the pendulum) the ordinates of which are the absolute inclinations of the ship at each instant, and the abscissæ the intervals of time. The motion of the paper is nearly isochronous. The paper, however, has a time scale marked on it by a piece of clock-work (*l*), as it travels. Each line is traced, not by a pencil—for this requires pressure to secure its marking—but by a pen consisting of a light vulcanite cylinder 0.1 of an inch in internal diameter, which delivers the ink through a conical metal point, having an aperture 0.005 of an inch; the pen and the arm that carries it (*m*) are counter-balanced, so that its point barely touches the paper, while the ink is delivered by its capillary adhesion to the paper. The friction is thus practically inappreciable. Should it happen, as it occasionally does, that the wheel is put into motion by some accidental external impulse, the oscillation thus instituted is performed with the long periodic motion which the wheel independently possesses, and thus it scarcely even disfigures the curve which shows the ship's oscillation. The result simply is that the curve is thus referred to a base-line of small definite curvature, instead of to one absolutely straight. The angles indicated by the pendulum, are similarly traced on the same paper and to the same scale.

Before leaving the subject of the details of the instrument, I should mention that I propose to make trial of a somewhat different arrangement for the frictionless support of the wheel. Although the arrangement as it stands at present, has answered very well, it requires great attention, as a very small speck of dust getting between the segments and the axis of the wheel, is sufficient to impede the free motion of the parts, and to set the wheel oscillating. I said before that a knife edge suspension for the wheel would not answer, as a knife edge to work properly must rest on a level, or nearly level plate, which could not, of course, be obtained in a ship, where, if the plate was tilted, the knife edge would slide off. However, after this instrument was made, it occurred to me that the difficulty of employing knife edges might be got over by placing, at each end of the axis, two knife edges at right angles to one another, with their edges in the same straight line, in prolongation of the centre line of the axis of the wheel, one knife edge being nearer to the wheel than the other, but both of them being rigidly connected with the wheel. These knife edges would rest on flat plates, which would be placed at right angles to one another. Thus the tendency of each knife edge to slide down over the inclined surface of the plate it was resting on would be prevented by the other knife edge pressing against its plate. To prevent any rubbing of the knife edges on the plates, and to allow for any errors of workmanship in fixing the knife edges in their exact position, the plates on which the knife edges rest, will be made so as to rock on short radius arms, in a similar manner to the segments on which the wheel axis now rests. Not only will this arrangement of knife edges be simpler and more compact than the present arrangement, but the small bearing surfaces of the knife edges will be able to crush and obliterate particles of dust, which would have done mischief between the circular axis and the segments. This arrangement of knife edges is adopted for the bearings of the

heavy short period pendulum already described, and is shown on Fig. 3, Plate XLVIII.

To return to the objects of the apparatus now before us, and having described the manner in which we obtain two curved lines drawn on paper which supply us with the information we require as to the *absolute angle* and the *relative angle* of the vessel's mast at each instant of time (see Fig. 6), I wish to point out that from these two records we

FIG. 6.—EXAMPLE OF DIAGRAM MADE BY INSTRUMENT, SHOWING RESULT OF ANALYSIS OF IT.

- a. Time scale marked by clock on the paper as it travels.
- b. Curvilinear line drawn by wheel, and indicating by its ordinates, to scale, the "absolute angle" of the ship at the instants of time indicated by the time scale.
- c. Curvilinear line, drawn in like manner by pendulum, and indicating "relative angle" of ship at each instant of time.
- d. Zero line to which the ordinates are referred.
- n. Curvilinear line referred to a zero line, p , deduced from the curvilinear lines, b and c , by taking the differences, as e, g, h, i, k, l, m , between the ordinates of these two curves, b and c , for the ordinates $e', g', h', i', k', l', m'$, of the line n . Thus the curvilinear line n represents by its ordinates the difference at each instant of time between the absolute angle and the relative angle; and this difference (see Fig. 4, page 875), is the wave slope.

NOTE.—In the actual diagrams as taken by the instrument, the lines b and c are not placed, as here shown, in their correct juxtaposition, since it is necessary in order to allow the pens to pass one another that they should work in different positions on the paper, hence before analysing the diagrams, one of the curves has to be "plotted" again in its correct position. Also, the curvilinear line drawn by the wheel has to be corrected for any curvature in its zero line due to long oscillations of the wheel. Both these corrections have been made in the diagram here shown, which is taken from an actual diagram produced by the apparatus.

can at once obtain a measure of the wave slope on which the ship is at each instant of time.

For this wave slope will clearly be the difference between the absolute angle and the relative angle (see Fig. 4, page 18), and so the angle

of the wave slope at each instant of time, is represented on the diagram by the distance between the two curved lines. Now by knowing the wave slope at each instant of time, we can graphically describe the form of the wave by plotting on paper successively the different inclinations of wave surface indicated by the diagram. But although we can thus draw the form of the wave, we do not know its actual dimensions. We have a correct drawing, but we do not know the scale on which it is drawn. We can determine this, however, in the following way: if the interval of time between the passage of two successive waves is known, the received theory of waves tells us what is the distance between the crests of those waves. Now a reference to the diagram tells us the interval of time between the passage of two of the wave crests we have drawn, and thus we can find the distance between the wave crests, and so we get the scale of the drawing of the wave.

Here then we have by the pendulum and wheel diagrams, a means of recording the waves which act on a vessel at sea. We have by the pendulum diagram, a record of the forces tending to make the vessel roll, and we have by the stationary wheel diagram a record of the extent to which the ship actually does roll. Thus a great help is given to our power of treating theoretically the rolling of ships, for we have the means of verifying our theory by practice, seeing in what way they differ, and determining the cause of that difference.

For instance, we have determined for a given ship, by rolling her in still water, her scale of resistance to rolling, and from this we are able to predict what will be her behaviour in any given waves. We try the same ship at sea, and from the diagrams we obtain a record of the waves she experienced. We then apply our information as to the still-water resistance, and calculate what, according to theory, should be the ship's behaviour, namely to what angle she should roll at each point of the waves in question. But we also have an exact record of what she actually did do in those very waves, and we thus have a crucial test of the accuracy of our calculations.

I may mention that in some experiments conducted for the Admiralty, this test has actually been applied in the case of two ships of over 1,000 tons, one of them having large bilge keels, and it appeared that the actual oscillations in the seaway went precisely through the phases which the theory prescribed; but as regards the amount of roll, the angles actually reached were throughout slightly less than those theoretically deduced, and this indicated that either the resistance experienced by the ships while rolling in the seaway was greater than that indicated by the still-water trials, or that the effective wave slope was throughout somewhat less than that indicated by the diagrams.

Also by the accurate records given by this instrument, we can determine the merits of different forms of ships so far as their qualities affect their rolling, for we can measure their behaviour under strictly comparable conditions. It is impossible to get the sea exactly alike on different days, and thus great differences of opinion exist as to the relative merits of various ships. But with this apparatus we can tell what were the conditions under which a vessel exhibited exceptional behaviour and in what respect those conditions or that behaviour differed from

those experienced or exhibited by the same or other ships at any other time. We are able thus to judge of the value of the various devices tending to modify the rolling; the effect of alterations of stowage of the additional of bilge keels, and of the steadying effect of canvas can be exactly recorded and measured. The direction in which to search for further remedies may thus be indicated, and important improvements may be confidently anticipated.

The CHAIRMAN: I do not know whether any one will venture to discuss this paper, but if any gentleman wishes to ask any question, or to have any further explanation, I have no doubt Mr. Froude will be very glad to afford it.

Captain COLOMB, R.N.: A *sotto voce* remark fell from the Chairman, that he thought nobody would dare to make any comments upon the paper, and I feel his difficulty, the condition of things being this, that whereas an audience in this theatre listening to a paper is usually in the position of a number of critics upon the performance of the reader of the paper; our position to-night is that of a number of disciples sitting at the feet of a master. I should think that nobody in the Navy, and I might possibly go further and say, nobody in Europe, would attempt to *criticise* such a paper as we have heard to-night,—most elaborate, and yet so clear, that even I suspect the thickest of our heads (and I number mine amongst them), have been able to follow it to a very great extent. I do not suppose that it is possible to offer that sort of criticism which Mr. Froude, in common with other great minds, would wish for. But what one would like to do would be to try to convey to a mind such as his, some of the difficulties that ordinary minds such as ours feel in following him. My difficulty, and I only mean to advert to one, is that purely elementary one upon which the theory of the wave rests. I understand, so far as I have read the papers on the now received theory of the formation of waves, that any particle on the surface of an advancing wave slope describes a circle in a direction opposite to that in which the wave is travelling; that it reaches the top of the circle at the moment the crest of the wave reaches it, and that, consequently, it finds itself just where it started when the wave has completely passed, supposing that it started from the bottom of the wave slope, and that the slope passed it, and then that it found itself at the bottom of the next hollow. Now my difficulty is to understand that translation of the wave particle. Waves we know are of very great height, and the diameter of the circle described by a particle on the surface of a wave may reach, I suppose, 30 feet. Now it is a difficulty to my mind to get the theoretical fact to coincide with one's personal observation. I have not myself, except in breaking waves, observed that particular translation. And there is one observation that I have made which increases the difficulty to my mind. When a perpendicular surface, such as the side of a breakwater, is offered to advancing waves, any small object lying within a foot or two of the perpendicular surface does not appear to an observer to alter its distance from it. Its motion appears to be distinctly vertical; as the wave rises, so the object rises; as it falls, so the object falls. But you cannot by that sort of observation detect a difference in the slope of the wave, nor is there in such a case any break or disturbance of the surface of the wave itself. It is of course quite different in a breaking wave, such as we see in a surf. There we have what is described a distinct motion of the particles which we can detect with our eye. We know that if you place a small object on a rolling surface, you observe that it has that motion of translation which I understand it is supposed that every particle of a wave has, even in deep water. We have that translation in a breaking wave very distinctly brought under our observation in every rolling sea. We are accustomed to note the commencement of the breaking of a wave, and after the wave has completely spent itself and broken, we can trace a long drift of foam where the breaking wave has passed over. That, Mr. Froude, is my difficulty. I hope you do not think I am a doubter. I am trying to be a learner, and no more, and I think if you can descend to our minds, I should just like to give one piece of confirmation to some points you have stated which are to those who have studied the question quite accepted, but I believe are not so accepted generally in the Navy; one is the isochronous vibration of ships. I made it my

business last time I was at sea, at any spare moment, whatever the state of the sea was, even in comparatively close harbours, to take that sort of rough observation which was possible, and I found in the ship I was in, no matter what the state of the sea was, in the heaviest sea and when the motion was hardly perceptible, the motion of the ship never seemed to be under 11 rolls, and never over 13.

Again, another point you adverted to was that cycle of rolling; that is how, in certain seas, you arrive at a maximum arc, and then that your roll became less and less down to a certain point, at which it began again to increase. I made observations upon that point, and I have tracings of the rolls, where I noticed that most distinctly; but I did not at that time know the cause, which has been explained to us to-night.

Dr. HIRST: Since the efficacy of the beautiful piece of mechanism which Mr. Froude has exhibited to us this evening, depends essentially upon the perpendicularity to the surface of the wave passing under the ship, of the direction indicated by the short pendulum of his apparatus, I should like to ask him to what extent this perpendicularity may be actually relied upon. The diagram on the right represents an extreme case, the circumstances of which might be roughly imitated by rigidly connecting the point of suspension of a short pendulum with the circumference of a large wheel, and on the concave side thereof. Now if the wheel rotated with sufficient velocity, centrifugal force would undoubtedly preponderate to such an extent as to maintain the perpendicularity of the string of the little pendulum to the immediately adjacent part of the circumference, which here represents the profile of the wave. But at smaller velocities of rotation, this would certainly not be the case, and the question arises whether some similar departure from perpendicularity may not present itself in the experiments of Mr. Froude.

Independently of the extreme case I have just alluded to, we have been informed by Mr. Froude that the shortness of the length of his pendulum is essential to the accuracy of the indications of his instrument. He has told us, moreover, that the centre of gravity of the large wheel, which turns on friction rollers, is slightly below the axis on which it turns; so that the wheel itself represents a slowly oscillating pendulum, in other words, a simple pendulum of great length; and the machine, unless I have misunderstood his description, may be said to record automatically the variations which occur in the inclination of these two pendulums when the wave is passing under the ship. Hence, if both pendulums alike placed themselves perpendicular to the wave surface, the instrument would obviously not serve the purpose for which it was designed. Its efficacy, therefore, may be said to depend materially upon the difference between the virtual length of the one pendulum, and the actual length of the other. I should be glad to know if Mr. Froude has had an opportunity of ascertaining the effect of altering the relative lengths of these pendulums. I have given frank utterance, Sir, to the thoughts which occurred to me as I listened to Mr. Froude's interesting paper. Those now present who, like myself, have not made such questions a subject of special study, may be glad to have the difficulties I have raised removed. To Mr. Froude this will be an easy task, for we all know with what experimental skill and large experience he has worked for a long time upon this and other kindred portions of a very important but difficult subject, a subject which up to the present time has remained, I believe, almost inaccessible to rigorous mathematical treatment.

Captain SELWYN, R.N.: Allow me to ask a question or two for my own instruction. Would Mr. Froude be kind enough to give us the explanation of the cause he assigns for the rolling of ships where it occurs in a comparatively smooth sea, and where no waves are perceptible—a case which constantly arises; and also how far the velocity of the translatory motion, whether that arises during the roll of a ship, or in such a case as the advancing surf in the drawing, how far the velocity of the translatory motion tends by the law of the composition of forces to make the pendulum assume a direction which it would not otherwise have taken. Also with regard to the cup full of water retaining a plane parallel to the wave slope, how far that would be true of a vessel of water if set on the deck of a ship—what would be the action of water inside a vessel set on the deck of a ship as compared with that which he attributes to the cup swinging in the hand which he has shown us?

Mr. FROUDE: With reference to the translatory motion of a wave, I think if you

were to place an upright post in moderately deep water traversed by a series of moderate waves, and to place a piece of cork on the water close to it, you would see actually that cork—at least I have never found any difficulty in seeing it—the cork move forward hollow at the crest and backward in the wave. I cannot say you ever see waves regular enough to show the absolutely circular motion, but you do simply see that the motion is of that character. You must recollect the translation of the profile of a wave itself is vastly greater than the translation of the particles of which the profile at the moment consists. Take the condition of an Atlantic wave, 500 feet in length, the transit of which, from crest to crest, will take ten seconds, making 50 feet per second as the speed of the translation of the profile. Now suppose the wave to be 20 feet high, so that the cork float at the surface is describing a circular orbit 20 feet in diameter, of which the circumference is, therefore, in round numbers, 60 feet, or a speed relatively so insignificant that an observer would certainly be unable to appreciate it correctly in the open sea when masked by the eightfold greater speed of the wave-profile; yet if, when the wave was passing him, he had a fixed object by which to observe its motion, he would be satisfied that the motion is at least 3 or 4 knots, as it really is, and that the top of the wave is moving forward, and the bottom of the wave is moving backward with about that speed, I won't say he could exactly count the speed by a watch, but he would be satisfied as to the character of the movement. With regard to the action of waves moving near a vertical quay wall, when the motion is *along* the wall, you do see the very thing happen. The case when the wave is approaching the wall no doubt may at first sight seem perplexing, but the fact is, that the existence of the wall converts the translatory motion of the contiguous particles, wholly, and of those nearly contiguous, partially, into vertical motion, and the increased rapidity of the up and down motion serves to disguise to a great extent the lateral translation which remains; but that it exists is quite unquestionable. If they are calculated, you will find that in all cases the actual translations are such as to correspond with the deflection of the pendulum, in virtue of the composition of forces, no less in the very flat waves which Captain Selwyn referred to, than in those of larger elevation. The composition of forces governs the result wholly. The forces to be compounded are gravity, which, strictly speaking, must include the vertical translatory forces due to such up and down motion as there may be, and the horizontal translatory forces due to such lateral motion as there may be. If you place a bucket of water on the top of a piston rod, and make the piston ascend and descend, then while the bucket is being accelerated upwards, there is an intensification of gravity in the water; when the bucket is being accelerated downwards, there is a diminution of gravity in the water, and you might make the acceleration so great that the water would be left behind altogether, and in that condition when gravity is neutralised by the downward acceleration, the water particles are in a perfectly neutral condition so far as mutual pressure is concerned. The water rising and falling up and down the face of a quay wall is very nearly in that position. It is thrown up, it is losing upward velocity and gaining downward velocity as fast as gravity can urge it, and therefore a very small lateral translation satisfies the condition that the surface of the water shall be at right angles to the compound of gravity and translatory force, even when the surface appears almost perpendicular. I do not know whether this explanation seems sufficient, but if you watch the phenomena by light of the explanation, I think they may be more correctly apprehended, and, as I have said, if you plant a small post in the water for the waves to wash past, you will see the very thing happening.

I think there is a little misconception in supposing that the mechanically rigid surface of that wheel can be taken to represent the living curvature of the hollow of a wave. Even were the wave-profile circular, that circle would not be the orbit of the particles. Even if the wheel were rotating, the pendulum would not hang normal to its circumference, nor if you were to put a drop of water on the circumference, would the water stay there. The pendulum might become horizontal in some part of the rotation, were the velocity sufficient, and the same circumstances would keep the water-surface in an upright condition, which would keep the pendulum in the horizontal condition. The length of the pendulum is important only because a very short pendulum is capable of at once assuming that position which is for the moment its position of equilibrium; a long pendulum, if not in that position, would be so long

assuming it that it would set up an independent oscillation. The short pendulum so instantly gravitates in the direction which is its momentary position of rest, that no time is lost. If you put a spirit level on a raft, if you put a marble in a floating saucer, and put them on regular shaped waves, the marble should not roll about; the bubble of the spirit level should remain at rest; the fluid in the spirit level is but as the fluid which surrounds the spirit level. Imagine a raft consisting of bamboo canes on a wave slope, you would be surprised to see the water running down through the canes; in fact, if a tube floats like a spar, pointing up a wave slope, the water should surely remain in it as inert as it appears to be in the rest of the wave-surface; yet the water is not really inert in the wave slope or in the tube, for the steady slope bears witness to the forces by which it is being urged.

The CHAIRMAN: I am sure you will wish me to thank Mr. Froude in your name most cordially for having so clearly explained a very difficult subject, for every Naval Architect and every seaman must feel it is one of the most important subjects that can be investigated. It is only during a very few years that the question of the theory of wave-oscillation has been scientifically and experimentally entertained at all. I suppose we could count on our fingers the men in Europe engaged in this investigation, and of these Mr. Froude stands almost foremost. We know the great difficulty there is in obtaining accurate records of a ship rolling. It is a matter of the greatest practical and scientific importance to the Naval Architect and the seaman in determining the disposition of the weights in a ship. We all know that it is practically a very great difficulty to ensure that the oscillations of a ship in a sea-way are truly recorded, and if this instrument assists us in accomplishing that object, and I fully believe it will do so, we cannot overrate its value, for it will enable our Naval Architects to deduce laws from the facts thus recorded, that will be of the greatest value to them. As regards the determination of the wave-slopes, I share with Captain Colomb the feeling that I am only learning this evening, and I cannot say I thoroughly have understood all we have heard, although Mr. Froude has made it so clear, but it appears to me one of the great values of ascertaining the wave-slope in connection with the angle of heel, is that you make your curve of stability of great importance in enabling you to ascertain the righting force acting on a ship when rolling, which the curve of stability in ordinary cases enables them to do when inclined in still water; but without knowing the angle of the wave-surface, the righting force cannot be ascertained. I have shared the difficulty that I heard expressed by Dr. Hirst as to the question of the inclined position of the pendulum, but I am ready to put faith in Mr. Froude, and only hope that he will be enabled by taking his instrument to sea to carry out his experiments further, and to prove to our satisfaction that what he says on that point, really exists. I may make one remark on it which perhaps may show how little I do understand the question. I cannot recognise that the pendulum can remain in the position of a normal to the wave-slope, except with some definite velocity of translation of the ship and a definite length of the pendulum. Experiment may satisfactorily prove that I am mistaken; but I should imagine the velocity of the translation of the point of suspension, and the length of the pendulum, were elements that must be taken into consideration.

Mr. FROUDE: It is because the ship is being translated by the wave at a certain rate, or rather because its translation is being altered at a certain rate, that the pendulum is deflected. The proposition is that there is inherent identity between the causes which govern the slope of the water-surface and those which govern the direction of the pendulum.

The CHAIRMAN: There is another point that I have no doubt experiment will prove. It is this; that that representation of the float partakes entirely of the motion of the surface, whereas a ship drawing 4 or 5 feet of water cannot entirely partake of the motion of the surface.

Mr. FROUDE: The diminution of the motion in the underlying water no doubt somewhat lessens the effective wave slope, which is thus different from the surface wave slope; the action is a compound action. I speak of the surface generally, merely by way of expressing the general laws which are in action without going into the complexity of the internal mechanism of the wave; this alters slightly the measure of the results, but does not alter their character.

The CHAIRMAN: I beg to offer Mr. Froude the most cordial thanks of the Council and audience for his most interesting paper.