work called "Sight and Touch," 1864, p. 57, collates the opinions of Heraclitus, Aristotle, and Cicero, who all assign to the moon's apparent diameter the length of a foot. He quotes, too, from Arthur Collier's *Clavis Universalis* the expression, "the moon which I see is a little figure of light, no bigger than a trencher." The old-fashioned trencher was about a foot in diameter. The late Sir William Rowan Hamilton pointed out to me this passage in Descartes' "Dioptrice," cap. VI. § xx. — "Abque hoc patet ex eo quod Luna et Sol, qui sunt e numero corporum remotissimorum, quæ contueamur, . . . pedales ut plurimum, vel ad summum bipedales nobis videantur," &c. Bishop Berkeley, as Philonous, asks Hylas, "Since, according to you, men judge of the reality of things by their senses, how can a man be mistaken in thinking the moon a plain, lucid surface, about a foot in diameter?" Many more such cases could be cited from ancients and moderns, all concurring in assigning a foot, or something between one foot and two feet, to the apparent diameter of the moon. Let me now cite two recent cases. A law-clerk, whose lay notions certainly owed very little to books, told me that the moon always appeared to him of the size of a door-handle. This would give, at most, a diameter of three inches. An eminent astronomer tried his daughter with the question. She replied that the moon looked to her about *half a degree*. He said, "Come, you learned that from astronomy; but answer as a girl of common sense." She now replied, "A small saucer." That would be some four or five inches in diameter.

I suppose I was too early spoiled by trigonometry to enter into the merits of this style of estimate. Look at the moon as I may, I cannot compare her to anything definite, as a door-handle, a saucer, or a trencher. Judging by the distance at which we ordinarily see such things in the use of them, they all seem to me to be enormously too large. Looking at the moon through my window, sitting three or four yards from it, I should guess that a wafer stuck on it would eclipse her. My own conviction is that in the ordinary estimate (or rather comparison), there is no reference, the most covert or subconscious, to any standard of distance. True, thirty-six or thirty-seven yards would be the distance at which Arthur Collier's trencher would subtend the same angle as the moon. But who thinks of this in connection with a trencher, which is usually under a man's nose or on the kitchen rack or shelf, at a distance not exceeding four or five yards? In another letter I will, if you will allow me, call attention to some of the *veræ causæ* which are probably concomitant in these popular estimates; and this I shall do in respect to the apparently augmented size of the moon's disc on the horizon. Meanwhile, let me ask as a preliminary to that inquiry, is it a matter of fact that *to the naked eye* the moon does subtend the same angle at the horizon as at positions near the zenith? I am unable to perform the measurement myself, not merely for want of a proper instrument, but by reason of the fact that I always see in the moon a multitude of discs partially overlapping each other, five of which I can distinctly count. It would be awkward to find that one was attempting to solve an imaginary problem, like the Royal Society over King Charles's fish.

Ilford, March 24

C. M. INGLEBY

THE "lurking idea" of Mr. G. C. Thompson, that the moon looks about the size of a fourpenny piece, seems to me to show that those views of it have made most impression on him which he has taken when standing a few feet from the window, when it would cover some such space: while others, with the one foot or two feet idea, have been more wrought upon by unconscious measurement of it against trees in the garden, or house chimneys along the street. I do not think we can get beyond this, in regard to a "personal equation." As to the apparent difference between the moon near the horizon and the moon in mid-sky, your correspondents have not yet referred to the theory that the felt degree of convergence of the eyes is one help toward measuring distance; which, however, soon ceases as the object is more remote, and the convergence insensible: and that, in looking at the moon along the earth's surface, we feel that she lies beyond this limit by comparison with the objects which intervene, while in looking up through free air there is no such gradation to guide us; that, therefore, we assign, unconsciously, a greater distance to her, *i.e.*, a greater "lurking idea" of estimated magnitude for the same apparent surface, in the former case than in the latter. I write from a dim recollection of one of Sir Sidney Smith's lectures on Moral Philosophy, but I suppose the notion is trite to experts. Is there anything in it?

J. R.

Concomitant Sounds and Colours.

THE investigation of the points of resemblance between two sciences, has its value and assists the development of both. Music gains by being thus raised from a mere sentimental recreation to the dignity of a science, but the science of colours may perhaps gain even more than music by the comparison, and this because the ear, in most persons, can distinguish with more precision a discord in sound, than the eye can in colour.

In the most ancient times it was well known that concomitant sounds produce a resultant whose vibrations are generated by the interference of the sonorous waves of the primaries. This physical fact was not only known but employed in the construction of Gregorian *Cantilenas*, whose succession of intervals shows a deep penetration of this truth.

The law of combination of the vibrations of concomitant sounds may be stated thus:—The resultant of two sounds has, as its number of vibrations, the difference between those of its primaries. Also any number of sounds combined two and two together, the 1st with the 2nd, the 2nd with the 3rd and so on, will form a series of resultants, which similarly combined two and two together form a second series of resultants; so that (continuing this process) we finally arrive at a single resultant which is that of the original combination. This law has been tested experimentally by Hallström and Scheibler. I considered that it might be useful to express this law by a general formula, so I will give it in this place.

If we have n sounds whose vibrations are $x_1, x_2, x_3, \dots, x_n$, all in ascending order as to *pitch*, then the resultant will be

$$R = (x - 1)_{n-2} (x_2 - x),$$

where the suffixes must be treated as indices, and $(x - 1)_{n-2}$ expanded according to the binomial theorem.

R is not, however, a resultant in the strict sense requiring the vanishing of the primary sounds; it might, perhaps, be better called the *Residuant* of the combination. It is thus the measure of the imperfection of the combination which is, more or less, a discord according as R is less or more nearly related to the primaries.

If we apply this formula we shall easily see that the tonic and subdominant generate a note two octaves below the subdominant:

for example, C and F generate $\frac{F}{4}$. Also CEG , CGC^2 , and CDE

and all similar combinations in which the vibrations stand in arithmetical progression generate no residuant, hence, combinations of this class are perfectly consonant and are called by Boethius *equisonal concords*. (The combination CDE is discordant enough on a modern instrument, but I mean CDE tuned perfectly without temperament.)

Supposing then that an impression is made upon the retina by two or more colours in juxtaposition, analogous to that produced on the auditory nerve by two or more simultaneous sounds. We shall perceive that two complementary colours placed side by side ought to increase in intensity that one whose vibrations are the most rapid. Red and green, for instance, should give intensity

to the green, since D and G generate $\frac{G}{4}$. Moreover, the colours corresponding to the *equisonal concords* ought to give us the most harmonious combinations; these are they:—

Violet placed between two yellows,		
Red	"	greens,
Orange	"	blues,
Yellow	"	indigo-blues,
Green	"	violets,
Blue	"	reds.

Yellow and indigo-violet ought always to be discordant, as they correspond with the discord $F B$ or *tritone*. Again—

Violet, orange, green	} All correspond to equisonal concords and should form perfectly harmonious combinations.
Yellow, blue, violet	
Indigo-blue, red, yellow	
Violet, red, orange	
Yellow, green, blue	
Indigo-blue, violet, red.	

It will be noticed that these tints must be precisely of the same shade as those in Newton's image; the slightest variation would destroy the harmony of colour. I have no doubt if pigments were made of tints identical with the ring-colours, the beauty of these combinations would be appreciated by all who used them.

It will be noticed also that D, F, A generate $\frac{Bb}{16}$, or red, yellow, blue generate indigo, blue. A, C, E generate $\frac{F}{16}$, or blue, violet, orange generate yellow. I need not increase the length of my paper by more examples, but leave the field open to all who choose to test the above formula as regards its application to combinations of colour.

In conclusion, I wish to make the following suggestion.

Should it be admitted that the musical scale, in its perfect division into intervals under the law of harmonical progression, finds its counterpart in Newton's rings, rather than in the prismatic spectrum, would not a spectroscope, constructed so as to give the image of these rings, be a more perfect instrument for the comparison of colours than that in present use? We might also have a double spectroscope, capable of giving the images of the secondary rings produced by the refraction of homogeneous light, the cube roots of whose diameters give the series which corresponds to that of the musical scale.

We should in this way be able to know the melody which corresponds to the light of any particular star, provided that the light be strong enough to produce the images of the secondary rings.

W. S. OKELY

Rome, February 18

Analogy of Colour and Music

AT the close of my short article on the Analogy of Colour and Music, published in your journal of January 13, I ventured to ask for the opinion of physicists on the subject. Accordingly I, for one, am much indebted to the many able contributors who thereupon addressed you. The correspondence having apparently ceased, I will now ask your permission to say a few words.

Although I do not attach too much importance to the closely approximate ratios, given in my paper, between the wave-lengths of colour and the notes of the diatonic scale, yet I think nothing said by your correspondents seriously affects my main argument.

The most important objection is that urged by Mr. Monro (NATURE, No. 14), who regards the correspondence of the two ratios as a mere coincidence, depending on the mode by which Prof. Listing obtained his scale of wave-lengths of the colours. By an ingenious calculation, Mr. Monro shows that Listing most probably "divided his spectrum into seven equal parts upon some scale which varies inversely as the wave-lengths; . . . so that it nearly corresponds with the ratios of the musical scale because these approximately form a harmonic progression." When I wrote my article I had not read Listing's paper, but, as stated, quoted his numbers from a recent memoir by Thalen. The perusal of his original paper shows me that Listing obtained his numbers in the following manner, which I think Mr. Monro will see confirms his calculation, but overrides his criticism:—

Employing pure spectra, and using every precaution, Listing experimentally determines the transition places and the central region of each colour, Fraunhofer's lines being used as landmarks. The observations are repeated upon the normal spectrum obtained by diffraction, and are checked by the independent observations of others, and by repetitions at different times. In this way the remarkable fact is disclosed that the numbers of vibrations at the transition spots form an *arithmetical progression* throughout the entire series of colours. For reasons given he adopts the following scale of colours—brown, red, orange, yellow, green, cyanogen, indigo, and lavender, and states as a law that this series can be physically expressed by an arithmetical progression of eight numbers, in which the last is the double of the first. He then proceeds to discover the constant factor by which this series can be turned into absolute values. After considerable care, and upon grounds fully detailed, he selects $48\frac{1}{2}$ billions as the number of vibrations per second expressing the range of each colour. The possible error he shows to be ± 0.038 —taking billions as unity—and this, though apparently a large error, is actually less than $\frac{1}{4}$ th of the interval between the two D lines.

The number of vibrations corresponding to the extreme limit of colour at the red end, he fixes, upon Helmholtz's and Angstrom's authority, at 363.9 billions per second, or a wave-length of 819.8 millionths of a millimetre. By adding to the former number half the colour interval—namely, $24\frac{1}{2}$ billions—the normal centre of the first colour is obtained; $48\frac{1}{2}$ billions added to that gives the centre of the next colour, and so on. These,

and also the limits of each colour, are tabulated along with the corresponding wave-lengths.

Listing closes his paper with the statement of a general law, that while the successive vibrations of the series of colours in the spectrum form an arithmetical progression, the same is also true of the logarithms of the vibrations corresponding to each musical note in the so-called chromatic scale. Hence he concludes that although physiologically and psychologically there may be differences, yet there is an indisputable *physical* basis for the analogy between tones and colours. From this very imperfect outline it will be seen that the entire memoir is a remarkable one, and I am surprised no translation of it has appeared. It is certainly the most important contribution to the analogy that I have met with, and renders my little paper on the subject quite unnecessary.

Mr. Okely, writing to the next number of NATURE, gives some additional evidence in favour of the analogy, but thinks my process of taking the mean of the limiting wave-lengths of each colour, in order to obtain the average wave-length, is "very rough." Mr. Okely does not tell us what he would do in such a case, but turns aside to become the champion of the widths of Newton's rings, charging me with having treated too summarily this old and famous ally of the analogy. But to this, my next critic, Mr. Sedley Taylor, replies, although other considerations also influenced me in neglecting this analogy.

Mr. Taylor, however, believes that he has deprived my comparison of any serious importance, for the following reason:—In the musical scale, he observes, "a very slight departure from accurate pitch in any concord provokes a harsh dissonance;" but "any part of any one colour-division produces an equally harmonious effect on the eye," because "in the spectrum there is very little, if any, change of tint except close to the extremities of any one colour." Whilst it would be certainly unwise to push the analogy too far, I think Mr. Taylor is here mistaken. There is a very material difference of tint in different parts of any one colour in the spectrum. Regarded alone, any region of the spectrum, like any single musical note, is, of course, equally agreeable; but it is not the case that an equally harmonious effect on the eye is produced by the combination of *any* part of any one colour-division with some other colour.

Mr. Stuart, in an interesting letter, points out a close relationship, discovered by Prof. Mossotti, between the intensities of the light in different parts of a normal spectrum and the notes of the diatonic scale. Finally, Dr. Chaumont, in an early part of this discussion, showed, what indeed had been noticed elsewhere, that if the ratios I give be accepted, then the once-called primary colours, red, yellow, and blue, correspond to the notes of the common chord; whilst the modern triad, red, green, and blue, correspond to the tonic, sub-dominant and dominant, that is to say, to the three notes which in music constitute the fundamental base of the scale.

In addition to what has been brought forward in this correspondence, there are some valuable remarks on the analogy in one of Dr. Thomas Young's memoirs, "Philosophical Transactions, 1802"; in Chevreul's work on the "Principles of Harmony of Colour"; in a recent brochure by Dr. Macdonald on "Sound and Colour"; and, lastly and chiefly, in §19 of Helmholtz's "Physiological Optics." In this last a list is given of authorities who have written on the subject since the time of Newton.

Reviewing what has been done in this matter, there are therefore, I believe, many good grounds for asserting the existence of a physical basis for the analogy between colour and tone. Opposed, it is true, are many mental differences: such, for example, as that of the judgment, which is far more prompt and correct in determining a colour than a note; then also colour primarily involves only the conception of *space*, music the conception of *time*. Nevertheless against all this we may place the facts that the source of harmony in colour, as in music, is purely a question of *relative* impressions; and a painting and a melody evoke a succession of ideas that have a remarkable similarity.

Woodlands, Isleworth, March 13

W. F. BARRETT

THE METROPOLITAN MAIN DRAINAGE

THE magnitude of the underground works of London is scarcely understood by the public in general. They occasionally hear of this or that sewer or pumping station being completed, but as the greater portion of