emanating from the lenticular drum are therefore depressed by simply raising the most luminous portion of the flame somewhat above the focus, while the rays coming from the catadioptric rings are depressed by giving these latter a corresponding inclination.

The greatest range in our immediate vicinity is found at the lights on Navesink Heights; they are elevated 248 feet above the sea, and in clear weather are visible about twenty-two nautical miles. The lights of Absecon and Cape May are elevated 167 feet and are visible about nineteen miles. On the Pacific slope, much higher elevations are found; the light of Cape Mendocino, Cal., for instance is elevated 423 feet, and is visible twenty-eight miles. One of the highest light-house elevations in the world is that of a light at the Cape of Good Hope, which is 810 feet above the level of the sea, and can be seen at a distance of thirty-six miles.

A DEDUCTION FROM THE PRINCIPLE "THE MOMENT OF THE MOMENTUM" IN THE CASE OF TURBINES.

BY PROFESSOR DE VOLSON WOOD.

Some time since I made the following transformation of the "moment of momentum," to see if it would shed any light upon the action of the water in a turbine. This principle being subject to independent proof, any deduction from it will be valid.

In the case of turbines, the moment of momentum of the water flowing through the wheel into the angular velocity of the wheel equals the rate of doing work; that is,

$$\left. \frac{du}{dt} = M V \rho \omega \right\} \begin{array}{l} \text{variable limit.} \\ \text{initial limit.} \end{array} \quad (1)$$

in which

M = the mass of water flowing through the wheel in a unit of time;

V = the circumferential velocity of the water;

ρ = the radius-vector from the centre of the wheel to any point in the buckets;

ω = the constant angular velocity of the wheel;

t = the time considered;

u = the work done in the time t .

Since the work is done at a uniform rate, it is only necessary to consider the work done in a unit of time, and that unit may be one second, hence integrating, and making $t = 1$, we have

$$u = \omega \left[M V \rho \right]_{\text{Initial lim.}}^{\text{gen. lim.}} \quad (2)$$

for the work done per second. The quantity within the parenthesis is the "moment of the momentum," and between the limits the expression becomes — the work done by the water from its entrance into the wheel to any point (or arc) in the wheel equals the angular velocity of the wheel into the difference of the moment of the momentum of the water at entrance and at a distance ρ in the wheel.

Let

δ = the density of a cubic foot of water = $62.5 \div 32.2$;

Q = the cubic feet of water flowing past an arc $ba = \rho \theta$;

θ = the angular part of the wheel through which Q flows;

$\rho = Oa$,

v_1 = the circumferential velocity, in reference to the earth, of the water passing ab ;

v = the velocity of the water along the bucket at a , in reference to the bucket;

γ = the angle between a normal to the bucket at a , and the radius through a prolonged;

then

$$v_1 = v \cos \gamma,$$

$$M = \delta Q,$$

$$V = v_1 - \omega \rho$$

$$= v \cos \gamma - \omega \rho,$$

where $\omega \rho$ is the circumferential velocity of the wheel along the arc ab ; and equation (2) becomes

$$u = \delta Q \omega \left[(v \cos \gamma - \omega \rho) \rho \right] \begin{matrix} \text{gen. lim.} \\ \text{initial lim.} \end{matrix} \quad (3)$$

The moment of the momentum of the radial component of v will be zero, since the direction of its action will be through the centre of the wheel; hence the last expression is the total moment into the angular velocity.

Differentiating, observing that in the operation the inferior limits disappear, we have,

$$\frac{du}{\omega} = \delta Q \left[-2\omega \rho d\rho + v \cos \gamma d\rho - \rho v \sin \gamma d\gamma + \rho \cos \gamma dv \right].$$

Let x = the depth of the buckets at distance ρ from the centre;

$m = \delta x \rho d\theta d\rho$ = the mass of an elementary prism at a ;

then

$$dQ = \delta x \rho \cdot v \sin \gamma \cdot \theta = \frac{m}{d\theta d\rho} \cdot v \sin \gamma \cdot \theta,$$

substituting which in the preceding, and differentiating again in reference to θ , gives us

$$\frac{d^2 u}{\omega} = \rho \sin \gamma \cdot m \left(-2\omega v + \frac{v^2 \cos \gamma}{\rho} - v \sin \gamma \frac{d\gamma}{d\rho} + \cos \gamma \frac{dv}{d\rho} \right)$$

The $d^2 u$ is an element of the work, and an examination of the process by which it has been obtained shows, from the last differentiation, that it is the work of an elementary prism, whose mass is m , and from the first differentiation that it is the work produced by this mass on account of its change of position and direction of motion in reference to a change in the variables ρ , v and γ . In the last equation $\rho \sin \gamma$ is the perpendicular from the centre of the wheel, O , upon the normal to the bucket at a ; and hence we infer that the quantities in the parenthesis multiplied by m are pressures normal to the bucket at a , due to the several changes in motion of the mass m with the wheel and along the bucket. Some of the terms in the parenthesis are readily interpreted, but the signification of others is not at once apparent, and although it may be a good exercise to determine their meaning from this standpoint, yet it is unnecessary to do so since it is virtually done in the solution of the problem of the turbine by Mr. Woodbridge, in the November number of this JOURNAL.