PHOTODYNAMIC VIRIALS IN THE SOLAR SYSTEM.

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The theory of the virial, or mean *vis viva* during stationary motion, enables us to co-ordinate all forms of cyclical motion, rotary, orbital and oscillatory. The grandest manifestations of the virial, which are given in cosmical motion, must be governed by the same laws as govern molecular movements. The complete development of the theory should, therefore, remove all the obscurity which still clings to the doctrine of radiodynamic unity.

A few cosmical illustrations will show the simplicity which considerations of mean energy introduce into the approximate solution of problems which would otherwise be exceedingly complicated. The illustrations are based upon the following postulates.

1. That cosmical masses represent internal energies, such as would be found if they were condensed from some primitive, tenuous, elastic form of matter.

2. That all chemical elements may have been condensed, in like manner, from a single primitive element, or æther.

3. That the velocity which enters into the primitive radial virial of the oscillating ætherial particles is the velocity of light (v_λ).

4. That the stationary motions of central inert masses, which represent the equal actions and reactions of primitive and derived virials, should continue until the velocity of the primitive virial has been alternately acquired and lost.

5. That all stationary motions which represent equal actions and reactions should be harmonic.

Solar or stellar centres of planetary systems, are central inert masses, which are endowed with velocities of stationary motion, tending to give velocities of stationary orbital revolution, sending forth æthereal oscillations with the velocity of light, and representing internal energies like those which would spring from nebular condensation.

Circular orbital velocity which is due to solar action, may be represented by the equation, $v_n = \sqrt{g_n r_n}$. 1

The limiting value of v_n , which it cannot exceed, is found at sun's surface (r_0), where g is a maximum. It may be represented by

$$[v_0] = \sqrt{g_0 r_0}. \quad 2$$

The 3d and 4th postulates lead to the equation

$$g_o t_o = v_\lambda \quad 3$$

This equation holds good for all values of r in an expanding or contracting nucleus, inasmuch as g varies as $\frac{1}{r^2}$, and the principle of conservation of areas requires that the time of rotation should vary as r^2 . The product of the two factors should, therefore, be constant.

The rotating æthereal tendency of stationary motion, which is limited by equations (2) and (3), gives the following value for the limiting radius of orbital and æthereal tendencies.

$$\rho_\lambda = \frac{v_\lambda}{[v_o]} r_o = v_\lambda \sqrt{\frac{r_o}{g_o}} \quad 4$$

Laplace's limit of equal rotary and orbital velocity (l) is given by the equation

$$l = \left(\frac{\rho_\lambda}{\pi r_o} \right)^{\frac{2}{3}} r_o = \left(\frac{\rho_\lambda r_o^{\frac{1}{3}}}{\pi} \right)^{\frac{2}{3}} \quad 5$$

The limit at which the equatorial rotary velocity of stationary motion would give $[v_o]$ is

$$\left(\frac{l}{r_o} \right)^{\frac{3}{2}} r_o = \left(\frac{l}{r_o^{\frac{1}{3}}} \right)^{\frac{3}{2}} = \frac{\rho_\lambda}{\pi} \quad 6$$

The limit at which the equatorial velocity of stationary motion would give v_λ , as deduced from (4) and (6), is

$$[l] = \frac{\rho_\lambda^2}{\pi r_o} \quad 7$$

The limit of a homogeneous elastic æthereal atmosphere, which would propagate undulations with the velocity of light, is

$$M = \pi [l] = \frac{\rho_\lambda^2}{r_o} \quad 8$$

The virials of rotating tendency must influence grosser inert particles or masses, as well as the æthereal atmosphere. Loci of important influence may be found at radii of mean æthereal momentum (ρ_a), of linear oscillation (ρ_β), of reciprocal linear oscillation (ρ_γ), of spherical oscillation (ρ_δ), and of reciprocal spherical oscillation (ρ_ϵ). Taking ρ_λ as the virial locus of these several oscillatory centres, we have

$$\rho_a = 2 \rho_\lambda \quad 9$$

$$\rho_\beta = 1.5 \rho_\lambda \quad 10$$

$$\rho_\gamma = 3 \rho_\lambda \quad 11$$

$$\rho_\delta = 2.5 \rho_\lambda \quad 12$$

$$\rho_\epsilon = \frac{5}{3} \rho_\lambda \quad 13$$

The tangential virial of an oscillating æthereal particle (μ_a), is $\frac{2}{3}$ of the radial virial of wave propagation for the same particle (μ_β). I first called attention to this relation in 1872.* Maxwell subsequently published it, apparently as an independent deduction, in 1877.†

$$\mu_a = 1.8 \mu_\beta \quad 14$$

An energy which is wholly transferred from one æthereal mass to another equivalent æthereal mass, must be accompanied by a like transfer of velocity, whether the transfer is through torsion (v_t), rotation (v_r), heat (v_θ), work (v_w), electricity (v_ϵ), or gravitation (v_γ).

We have, therefore, for limiting velocities,

$$v_\lambda = v_t = v_r = v_\theta = v_w = v_\epsilon = v_\gamma \quad 15$$

In cyclical movements which are due to virial transfers, these several equivalents may be indicated by equations analogous to (3).

In order to test the foregoing equations, let us take sun's semi-diameter (r_s) as the unit of length, and the British Nautical Almanac estimate of sun's apparent semi-diameter ($961'' \cdot 83$) as the parallactic unit. We then find, for earth's semi-axis major,

$$\rho_3 = 214.45 r_s. \quad 16$$

Earth's mean orbital velocity (1) may be found by dividing $2 \pi \rho_3$ by the number of seconds in a year (31558149). This gives

$$v_3 = .000000199099 \rho_3 \quad 17$$

This value varies slightly with varying orbital eccentricity, but the greatest secular range of variation is less than $\frac{1}{8}$ of one per cent.

Circular orbital velocity varying inversely as the square root of the radius vector, we find (2) and (17)

* Proc. Amer. Phil. Soc., xii, 394. † P. Mag., iii, 453; iv, 209.

$$[v_o] = \cdot 00000291562 \rho_3 = \cdot 000625255 r_o \quad 18$$

$$g_o = \cdot 0000003909445 r_o \quad 19$$

Struve's constant of aberration gives, by (3) and (19)

$$v_\lambda = g_o t_o = 214\cdot45 r_o \div 497\cdot827 = \cdot 430772 r_o \quad 20$$

$$t_o = 1101876 \text{ sec.} = 12\cdot753 \text{ days} \quad 21$$

This gives for a double oscillation, or complete rotation of sun, 25·506 days. Laplace's estimate was 25·5 days. The motion of sun-spots near the equator is accelerated, by centrifugal force, tendencies to orbital velocity, "repulsion," or some other unknown influence. Spörer's formula gives 24·62 days for the equatorial period.

From (4), (18) and (20) we find

$$\rho_\lambda = 688\cdot954 r_o = 3\cdot212654 \rho_3 \quad 22$$

From (5) $l = 36\cdot366 r_o \quad 23$

From (6) $\rho_\lambda \div \pi = 219\cdot301 r_o \quad 24$

From (7) $[l] = 151088\cdot1 r_o = 704\cdot538 \rho_3 \quad 25$

From (8) $M = 474657\cdot3 r_o = 2213\cdot37 \rho_3 \quad 26$

From (14) and (22) we find the following regular series of approximations to planetary loci. The subscripts, 1, 2, 3, denote, respectively, secular perihelion, mean, and secular aphelion.

$$1\cdot8^{-4} \rho_\lambda = \cdot 3060 \rho_3 \text{ Mercury}_1 = \cdot 2974 \rho_3 \quad 27$$

$$1\cdot8^{-3} \rho_\lambda = \cdot 5509 \rho_3 \text{ Venus}_1 = \cdot 6722 \rho_3 \quad 28$$

$$1\cdot8^{-2} \rho_\lambda = \cdot 9916 \rho_3 \text{ Earth}_2 = 1\cdot0000 \rho_3 \quad 29$$

$$1\cdot8^{-1} \rho_\lambda = 1\cdot7848 \rho_3 \text{ Mars}_3 = 1\cdot7365 \rho_3 \quad 30$$

$$1\cdot8^0 \rho_\lambda = 3\cdot2127 \rho_3 \text{ Asteroid 108} = 3\cdot2120 \rho_3 \quad 31$$

$$1\cdot8^1 \rho_\lambda = 5\cdot7828 \rho_3 \text{ Jupiter}_3 = 5\cdot5193 \rho_3 \quad 32$$

$$1\cdot8^2 \rho_\lambda = 10\cdot4090 \rho_3 \text{ Saturn}_3 = 10\cdot3433 \rho_3 \quad 33$$

$$1\cdot8^3 \rho_\lambda = 18\cdot7362 \rho_3 \text{ Uranus}_2 = 19\cdot1836 \rho_3 \quad 34$$

$$1\cdot8^4 \rho_\lambda = 33\cdot7252 \rho_3 \text{ Neptune}_3 = 30\cdot4696 \rho_3 \quad 35$$

$$\text{Geom'l Mean} = 3\cdot2127 \rho_3 \quad \text{Geom'l Mean} = 3\cdot2200 \rho_3 \quad 36$$

All of these approximations represent loci of belt-condensation, for the respective planets, which are in accordance with the nebular hypothesis. The geometrical means differ by less than $\frac{1}{4}$ of one per cent. The photodynamic mean represents the semiaxis major of Asteroid 108; the planetary mean, the semiaxis major of Asteroid 122. The second photodynamic locus ($.5509 \rho_3$) is, within less than one per cent., the arithmetical mean between the semiaxes major of Mercury and Venus ($.5552$).

From (9), (10), (11), (12), (13), and (22) we get the following approximations:

$$\rho_a = 6.4253 \rho_3 \quad \text{Cardinal centre} = 6.4451 \rho_3 \quad 37$$

$$\rho_\beta = 4.8190 \rho_3 \quad \text{Jupiter}_1 = 4.8863 \rho_3 \quad 38$$

$$\rho_\gamma = 9.6380 \rho_3 \quad \text{Saturn}_2 = 9.5389 \rho_3 \quad 39$$

$$\rho_\delta = 8.0318 \rho_3 \cdot 4 \text{ Uranus}_3 = 8.2717 \rho_3 \quad 40$$

$$\rho_i = 5.3545 \rho_3 \quad \text{Jupiter}_2 = 5.2028 \rho_3 \quad 41$$

$$\text{Ar. Mean} = 6.8537 \rho_3 \quad \text{Ar. Mean} = 6.8690 \rho_3 \quad 42$$

$$\text{Ge. Mean} = 6.6325 \rho_3 \quad \text{Ge. Mean} = 6.6421 \rho_3 \quad 43$$

It will be seen from (40) that the second locus of spherical rotary projection from ρ_λ , ($2.5 \times 2.5 \rho_\lambda = 20.0795 \rho_3$), is within the secular

orbital range of Uranus. The cardinal centre (37) is the centre of gravity, at conjunction, of Saturn₂ and Jupiter₃. It represents, therefore, the locus of mean rotary momentum for their combined masses, at the time of Jupiter's incipient rupturing subsidence, according to Herschel's modification of the nebular hypothesis. It also represents important relations to the following additional virial loci.

$$\rho_\xi = \sqrt{\Sigma m \rho^2 \div \Sigma m} = 9.2443 \rho_3 \quad 44$$

$$\rho_\eta = \Sigma m \rho \div \Sigma m = 7.5228 \rho_3 \quad 45$$

$$\rho_\theta = \frac{1}{2} (\text{Saturn}_2 + \text{Jupiter}_3) = 7.5291 \rho_3 \quad 46$$

$$\rho_i = \frac{1}{2} (6.4451 + 8.2717) = 7.3584 \rho_3 \quad 47$$

$$\rho_\pi = \frac{1}{4} \text{Neptune}_2 = 7.5084 \rho_3 \quad 48$$

The locus of mean planetary nebular inertia (44) is in Saturn's orbit, where the rings, the satellite system and the specific gravity bear witness to the results of nebular condensation. The locus of mean planetary nebular momentum (45) approximates closely to the arithmetical mean between Saturn₂ and Jupiter₃ (46), to the arithmetical mean between the cardinal centre and the incipient virial locus of spherical rotation for Uranus (47), and to the virial locus for the mean linear momentum of Neptune's semiaxis major (48).

The virial radius of mean momentum not only determines the centre of gravity of the two chief planetary masses (9), (37), but it also determines the relative masses of Sun (m_o) and Jupiter (m_s) at initial nebular rupture (secular perihelion). We find, accordingly,

$$m_o r_o = m_s \rho_{51} \quad 49$$

Stockwell's estimate of Jupiter's secular eccentricity is .0608274. This gives $\rho_{51} = .9391726 \times 5.202798 \times 214.45 = 1047.872 r_o$. Therefore, (49)

$$m_o = 1047.872 m_s \quad 50$$

Bessel's estimate is 1047.879. This harmony is the more significant because Jupiter's nebular locus of incipient rupture (4.8863) is central between the loci of incipient subsidence of Uranus (20.6792) and Neptune (30.4696) at opposition,

$$\rho_{51} = \frac{1}{2} (\rho_{73} + \rho_{63}) \quad 51$$

While Jupiter thus traverses the primitive nebular centre, Earth traverses the centre of the belt of greatest condensation.

$$\frac{1}{2} (\rho_{11} + \rho_{43}) = \rho_3 \quad 52$$

Stockwell's estimates for the secular limits of the dense belt (Mercury₁ and Mars₃) are, $\rho_{11} = .2974$; $\rho_{43} = 1.7365$. This gives for (52) $1.0169 \rho_3$.

While the rotation of the chief nucleal centre (Sun) is determined by the velocity of light (3), the rotations of the secondary centres of nebulosity (Jupiter) and condensation (Earth) are determined, respectively, by circular orbital velocities at Sun's surface [v_o] and at the mean centre of gravity of Sun and Jupiter [v_a].

$$g_s t_s = [v_o] = \sqrt{g_o r_o} \quad 53$$

$$g_s t_s = [v_a] = \sqrt{g_a r_a} \quad 54$$

The data for the solution of (54) have been more accurately and satisfactorily determined than for (53).

$$g_3 t_3 = \frac{32.088}{5280} \times \frac{86164.08}{2} = 261.821 \text{ miles.} \quad 55$$

Circular orbital velocity varying inversely as $\sqrt{r}$, we find (49), (50), (54), (55)

$$g_3 t_0 = [v_0] = g_3 t_3 \div \sqrt{.9391726} = 270.167 \text{ miles.} \quad 56$$

$$[v_3] = [v_0] \div \sqrt{214.45} = 18.449 \text{ miles.} \quad 57$$

$$\rho_3 = 31,558,149 [v_3] \div 2\pi = 92,662,000 \text{ miles.} \quad 58$$

$$r_0 = \rho_3 \div 214.45 = 432,090 \text{ miles.} \quad 59$$

At Earth's surface, $\sqrt{gr} = 4.9073$. It varies as $\sqrt{\frac{m}{r}}$. Therefore,

$$(57) \quad \frac{m_0}{\rho_3} : \frac{m_3}{r_3} :: 18.449^2 : 4.9073^2 \quad 60$$

$$m_0 : m_3 :: 330482 : 1 \quad 61$$

All of the results which have been drawn from (3), (53), and (54) involve the principle of persistency of vibrations, by which waves tend to propagate themselves indefinitely, with the velocity which is due to their locus of origination.

The influence of Jupiter's locus of incipient subsidence on the comparative masses of Jupiter and Saturn, finds some analogy in the two chief planets of the dense belt, Earth and Venus.

$$m_2 \rho_{23} = m_3 \rho_3 \quad 62$$

Substituting Stockwell's estimate of the secular aphelion of Venus ($\rho_{23} = .7744234 \rho_3$) in (61) (62),

$$m_0 = 426750 m_2 \quad 63$$

Hill's estimate is 427240, which differs from (63) by less than $\frac{1}{8}$ of one per cent.

The general equation of fundamental velocity (15) rests on Laplace's principle of periodicity, "that the state of a system of bodies becomes periodic when the effort of primitive conditions of movement has disappeared by the action of resistances." Hence (3) (20),

$$v_\lambda = v_\gamma \quad 64$$

The investigations of Weber, Kohlrausch, Thomson, Maxwell, Ayrton, and Perry have shown that

$$v_\lambda = v \quad 65$$

In Coulomb's formula of torsional elasticity, if we substitute $\frac{m}{2}$ for f , $gt^2 = M$ (26) and

$$gt = v_\lambda = v_t \quad 66$$

In throwing a ball into the air, the thermal equivalent of the projectile force is equivalent to the product of the mass by the sum of the retarding resistances. In solar superficial radiation, the gravitating reaction, as we have seen (64), is exhausted in a half rotation. The corresponding projectile velocity may be regarded as representing, at pleasure, heat, work or rotation, giving

$$v_\lambda = v_\theta = v_w = v_r \quad 67$$

Combining (64), (65), (66), (67), we find a practical confirmation of (15).

A NOTE UPON THE WORKING OF SULPHURIC ACID CHAMBERS.

By HENRY PEMBERTON, JR.

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During several years subsequent to the panic of 1873 a set of acid chambers, the property of a company of this city, was worked under my supervision. Owing to the depressed state of trade and consequent stock of goods on hand, the chambers were often run below their full capacity. They were run without a Gay Lussac absorbing tower, and burned best unmixed seconds brimstone, 4,000 pounds being the normal charge in 24 hours. Good results, approaching very nearly to the theoretical figures were obtained from such a charge when 10 per cent. nitre was used. Burning 3,600 pounds per day, 9 per cent. nitre was required, and with 3,000 pounds, 8 per cent. As the cubic capacity of the set was 107,200 cubic feet, the following represents the pounds sulphur, per cent. nitre, and cubic feet per pound sulphur.

Pounds sulphur.	Per cent. nitre.		Cubic ft. per pound sulphur.
4,000	10	=	26.8
3,600	9	=	29.8
3,000	8	=	35.7

Several years after this the capacity was nearly doubled by the addition of two more chambers, the whole being run in one set, with a