THE EARTH'S RESIDUAL MAGNETIC FIELD.

A. TANAKADATE; L. A. BAUER.

A. Tanakadate:¹

“ Dr. L. A. Bauer has constructed the map of residual magnetic field after deducing the first harmonics in Gaussian expansion from the observed values of each component, and has shown in a marked

way several centers of attractions and repulsions on the Earth's surface. The question naturally arises, What will be the residual field after deducing all the harmonics from the observed values? The following table and map [reproduced on reduced scale], constructed from the differences in Tables III and VIII in Prof. Ad. Schmidt's Paper in the XXI Jahrgang of *Aus dem Archiv der Deutschen Seewarte*, furnishes the answer. The map was originally constructed on a globe of 30 inches diameter by two of my assistants, Messrs. T. Kato and T. Yasuda."

¹ Extracted from Professor Tanakadate's recent paper entitled "Terrestrial Residual Magnetic Field," reprinted from *Denkigakukwai Zasshi*, No. 220.

"Those residuals, though still presenting certain aspect of continuity, the relation between the horizontal and vertical components does not in general show centers of attractions or repulsions, except in few cases which may be real or accidental. Considering the way in which the data (the Beobachtete Werthe at regular intervals of longitude and latitude), these residuals would seem to have but little physical sense, as is duly remarked by Prof. Schmidt himself."

"The residuals exceed .03 C. G. S. units in some places, so that we may consider the second figure of the magnetic elements as yet uncertain in most places, especially in high latitudes. In this respect the value of the great undertaking of the Carnegie Institution will require no comment. The maximum residual when only the first harmonic was taken is 0.16, so that we may roughly say that the expansion to 7th harmonic reduces the residuals to about $\frac{1}{5}$. From the nature of the problem, the convergency of the series can not be predetermined unless we assume the mode of distribution of the magnetism."

"Upon the want of data at present, if we take the strength and depth of the Asiatic focus of disturbance as deduced from the magnetic survey of Japan² to be 1.41×10^{13} C. G. S. and 1010 kilometers, the field at the epicenter is .138 C. G. S. Instead of the direct calculation, however, if we expand this in spherical harmonics, the sum of the first 20 harmonics amounts to .117, giving a residual of still .021. We see from this that if there is disturbance whose effect can be represented by such an image the convergency of the Gaussian expansion becomes hopelessly slow, an enormous number of constants being required to exhaust even the second figures in observed values."

"On the other hand, the first approximation of the Earth's mean magnetic moment can be obtained, within about 10%, from observations at any *single point*. . . .

"The difficulty of reducing residuals by extending higher harmonics is apparent at a glance on the map at the end of the paper; this suggests to us the comparative shallowness of the seats of the disturbing field, although no valid deduction can mathematically be drawn as to the internal mode of distribution. Whatever may it be, if we try to account for it by a distribution not far from the Earth's center, an unprobably strong and complex magnetization would have to be supposed."

² Journal of the Science College, Imperial University, Tokyo.

"In conclusion, permit me to briefly sketch the general scheme I am following in the analytical discussion of the magnetic conditions prevailing in the United States as exhibited by these maps. [Iso-magnetic maps of the United States of North America for 1905]. Some of you are familiar with the general analytical treatment by spherical harmonics of the Earth's magnetic condition, at any stated period, initiated by Gauss. You will doubtless recall that he carried the expansion of the potential to terms of the fourth degree, inclusive, thus involving twenty-four coefficients to be determined by a least square treatment of the available magnetic data. The later analysts have extended the expansion to terms of the sixth order, inclusive, hence determining forty-eight coefficients. However, in spite of the increase in the number of terms and coefficients, the results as computed from the formula differ from the observed quantities to such an extent that no *practical* use, whatsoever, can be made of them. For example, in the United States the formula would give magnetic declinations differing at *least* 1° to 3° from observed values."

"Gauss's formula has only a certain theoretical significance, but, as said, no practical value as an interpolation formula. Why is this? Is it due to imperfection or incompleteness of the underlying data? Yes, to a certain extent. For example, all of the various analyses to date have had to be based on maps depending largely on land data, *i. e.*, on data covering but a fourth of the area of the globe. Nothing was known as to the reliability of the map data over the remaining three-fourths—the ocean areas. Thus the Carnegie Institution work has shown that the magnetic map data over the Pacific Ocean are erroneous 1° to 3° in declination and dip, and about one twenty-fifth part in horizontal intensity. So we may confidently expect some improvement with the later more accurate data and it is possible that a new formula as based upon these data will give results sufficiently close in agreement with observation over the ocean areas for *practical* purposes if not for *scientific* purposes."

"When we come to land areas, however, it is questionable whether any general terrestrial formula can be established which

³ Concluding remarks of a paper on "Some Results of the Magnetic Survey of the United States," presented at the meeting of the National Academy of Sciences, Washington, D. C., April 22, 1908, and printed in full in "Science." N. S. Vol. XXVII, No. 699, pp. 812-816 (May 22, 1908).

will represent the land observations sufficiently closely, even if the expansion be carried farther than it has already been, viz., beyond the sixth degree. The prime reason for this has been made evident from our study of the magnetic field of the United States. We find, namely, that we must deal with irregularities in magnetic distribution of various gradations covering areas of varied extent, a locality, county, state or an entire continent. Thus we may have the magnetic forces observed at any given point in the United States as the resultant effects of (1) a general or terrestrial magnetic field due to the general magnetic condition of the Earth, (2) a general, terrestrial disturbing cause which distorts the general magnetic condition of the Earth at the place of observation, (3) a continental disturbing effect, due largely to that portion of the North American continent above the general ocean bed, (4) a regional disturbance due to lowlying magnetized masses or Earth currents covering a certain region, and (5), a local disturbance due to the magnetized masses or currents in the immediate vicinity."

"Now a formula based upon the entire Earth can, of course, not include also disturbances of such restricted areas as embraced under the last two heads, viz., regional and local disturbances, but it appears that the present formulae do not even adequately represent the *continental* effects. In these formulae we have the various terms advancing and recurring by trigonometric functions of multiples of the longitude and the latitude. Thus a term involving six times the longitude is supposed to continue around the Earth six times in unaltered manner. However, such a term arising from the North American continental shelf which in the United States has a width in longitude of about 60° , would not continue around the Earth, but would in all probability die out before the European or the Asiatic continent is reached. In other words, it must be recognized that with the recurrent or continuous terms embraced in the higher harmonics of Gauss's formula, we can at the best but "counterfeit" the facts of nature but never get at the actual truth."

"Here we are brought face to face with the question: How far is it really worth while to go in the establishment of the formula? The geodesist contents himself with the determination of at most two constants for the figure of the Earth. No one, to my knowledge, has succeeded in establishing a formula which represents the actual shape of the Earth and exhibits at least the most marked of the Earth's manifest irregularities of surface. Professor Love, I believe, has established a formula representing the irregularities of

the first, second and third degrees, but has not deemed it worth while to go beyond this extent. And so it would appear to me that the time has come to halt in the establishment of a complex formula involving forty-eight unknowns or more which at the very best can give but an inadequate representation of the actual facts of the Earth's magnetism. It would seem more logical to stop with a certain finite number of terms involving a limited number of unknowns which represent, from a physical standpoint, the chief and principal facts of the magnetic condition of the Earth. The magnetization resulting from this limited expression we should call our "normal field" or "field of reference" as the geodesist calls his adopted figure, the *spheroid of reference*. The residuals from this field of reference would then receive separate or special treatment in accordance with their extent and their character."