Terrestrial Magnetism *and* *Atmospheric Electricity*

VOLUME XXV

DECEMBER, 1920

NUMBER 4

ON VERTICAL ELECTRIC CURRENTS AND THE RELATION BETWEEN TERRESTRIAL MAGNETISM AND ATMOSPHERIC ELECTRICITY.¹

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1. To what extent the magnetic forces as observed on the surface of the Earth can be referred to a potential, is a subject of paramount interest. The solution of the problem is of great importance, both as regards the constitution of the so-called permanent magnetic field of the Earth and the systems giving rise to the manifold variations to which the terrestrial magnetic field is continually subject.

2. Any electric currents circulating above or below the Earth's surface in concentric layers, i. e., parallel to the surface, will give rise to magnetic forces which may be represented by a potential. Electric currents, on the other hand, cutting the Earth's surface, produce, in general, a mixed magnetic system: the horizontal components of such currents give rise to a magnetic potential, whereas, the vertical components cause magnetic forces which cannot be referred to a potential. As is well known, the test of the existence of a potential is the vanishing of the line-integral of the magnetic force taken around a closed curve, or circuit, on the Earth's surface. If the line-integral does not vanish and its departure from zero cannot be explained by error of observation, or local magnetic disturbances in the region of the circuit, then the existence of a non-potential is revealed; from the magnitude and sign of the integral we may then determine the strength and direction of the electric currents passing perpendicularly through the surface of the region enclosed by the circuit.

¹Presented at the Chicago meeting of the American Physical Society, December 30, 1920.

THE EARTH'S PERMANENT MAGNETIC FIELD.

3. Gauss was the first to apply a test for determining whether the Earth's permanent magnetic field is caused by a potential. He computed the line-integral around a triangular area, at the vertices of which the magnetic elements had been observed, and found that it approximately vanished.² He thence made his well-known spherical harmonic analysis of the Earth's magnetic field, as based on the potential theory, and obtained a fairly close representation of the magnetic data available to him at the time. From Gauss's analysis it appeared that the Earth's field could be regarded as arising practically entirely from magnetic and electric systems below the Earth's surface.

4. The most complete mathematical analysis we have had in recent times was that by Adolf Schmidt on the basis of the Neumayer magnetic charts of 1885. His analysis was based on the most general assumptions possible, namely, that the Earth's total permanent magnetic field is to be ascribed to the following: (1) a magnetic potential, V_i , caused by the systems below the Earth's surface; (2) a magnetic potential, V_e , caused by systems above the Earth's surface; and (3) a non-potential part, N . Schmidt found that V_i contributed about 95 percent of the total field; V_e and N together, produced the remainder. He further found that the average current-density of the vertical currents giving rise to the portion N was, on the average, for the entire Earth's surface, regardless of direction, one-sixth of an ampere per sq. km.; such vertical currents would account for about 2 or 3 percent of the Earth's total magnetic field.

5. Shortly after the announcement of Schmidt's results, Rücker³ computed the line-integrals for certain circuits in Great Britain on the basis of data resulting from the Rücker and Thorpe Magnetic Survey, but failed to find vertical currents of the strength and direction indicated by Schmidt's analysis. Schmidt's currents were about 50,000 times stronger than the vertical currents measured generally with atmospheric-electric interuments. Owing to these difficulties, Schmidt was finally led to doubt the validity of his general conclusion and to express the opinion that possibly his result was to be ascribed to systematic errors in the magnetic charts.⁴

²Allgemeine Theorie des Erdmagnetismus, p. 13; *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1838*, Leipzig, 1839.

³*Terr. Mag.* vol. 2, 1896, p. 84.

⁴Der magnetische Zustand der Erde, 1885, p. 27. See also "Mitteilungen über eine neue Berechnung des erdmagnetischen Potentials," *Abh. d. II. Cl. d. k. Muenchen, Ak. d. Wiss.* XIX Bd., I. Abth.

6. Since in evaluating N it is necessary in the analysis to introduce some assumptions, I decided to investigate the question as to the existence of vertical currents by the direct method of Gauss, namely, of computing line-integrals, which method can be immediately applied to the available magnetic data without first making a laborious mathematical analysis of the total field. In future analyses it may be found advantageous to attempt evaluating first possible vertical currents, and then correct the magnetic data for their effect before undertaking the mathematical analysis.

7. My first investigation consisted in computing the line-integrals, taking as circuits of the Earth parallels of latitude and using approximate data scaled from Neumayer's published magnetic charts for 1885; a brief summary was submitted to the committee on grants of the American Association for the Advancement of Science at the meeting in 1895. With the aid of a small grant received from the committee, I was enabled to obtain, through Schmidt's courtesy, a transcript of the Neumayer data used by him in his analysis and thus to extend my investigation, the results of which were published in 1897.⁵ The method of computation was such as to eliminate, or reduce to a minimum, the effect of systematic errors in the charts from which the magnetic data had been derived. A rather interesting and suggestive distribution of vertical electric currents was found for the region investigated from parallel 60° North to parallel 60° South. The average intensity of the vertical currents per sq. km. turned out to be about one-tenth of an ampere, which, while somewhat smaller than Schmidt's value, was still about 35,000 times that obtained from usual atmospheric-electric measurements.

8. In 1904, I published the results of a second investigation⁶ concerning vertical electric currents, the prime purpose of which was to obtain some idea as to the magnitude of the magnetic quantities involved in order that plans for tests to determine the existence of a potential might be included in the contemplated magnetic survey of the Earth under the auspices of the Carnegie Institution of Washington. Use was made of all the available magnetic charts to date: Sabine's of 1840-45, Creak's of 1880, and Neumayer's of 1885. Again latitudinal circuits were made from 60°N and 60°S. The combined results from the three charts were practically the same as those derived in my 1897 paper on the

⁵*Terr. Mag.* vol. 2, 1897, pp. 11-22.

⁶*Terr. Mag.* vol. 9, 1904, pp. 116-129.

basis of the Neumayer charts alone; the average current-density for the region 45°N to 45°S, over which the results appeared most certain, was one-thirtieth of an ampere per sq. km.—still about 11,000 times that found in atmospheric electricity.

9. It was shown in the 1904 paper how results concerning vertical currents could be derived from declination charts⁷ alone. A positive electric current passing, for example, from the air through the Earth's surface, in accordance with Ampere's rule, will deflect the north end of a magnetic needle to the West; a reversed current, on the other hand, would deflect the needle to the East. The distribution and intensity of the vertical currents appeared to be such that for certain regions or parallels the needle, for long stretches, might have a deflection in the same direction amounting to 0°.2 and more—a quantity readily measurable even in ocean work. In mapping out the cruises of the *Galilee* and the *Carnegie* full consideration was given to obtaining closed circuits as frequently as possible, in various regions and under different conditions, in order that sometime adequate tests might be made with regard to the existence of vertical currents having appreciable magnetic effects.

10. On April 23, 1908, I presented at the Washington meeting of the American Physical Society the results of a third investigation on vertical currents, dependent this time upon the 1905 magnetic charts for the United States as constructed by me on the basis of the detailed magnetic survey carried out under my direction by the United States Coast and Geodetic Survey. The line-integrals were computed for circuits around and within the United States. Once more vertical currents were indicated that apparently could not be ascribed wholly to observational error. As their average strength, as seen later, was again of an order 10,000 times that of atmospheric electricity I refrained from publishing the results, although they were based upon the largest land area for which a detailed and carefully-conducted magnetic survey had been made.

11. Others have computed line-integrals, or surface-integrals, of the magnetic force over more or less restricted land regions, but there has been no general consensus of opinion that the question of the existence of vertical currents producing appreciable effects in terrestrial magnetism was definitely settled, one way or the other. It is doubtful whether any reliable results can be

⁷*Idem*, pp. 123-126.

obtained unless the integrals are taken over very extensive areas and unless a method of computation is used which eliminates adequately the effects from local and regional disturbances. My own preference is to deal with the directly-observed magnetic quantities, rather than with adjusted ones. It is quite possible that some of the results obtained by others may be merely the products of adjustment, or as a necessary consequence of the attempt to make the observed quantities fit some particular pattern.

12. Gockel in 1912 published a paper in which he showed interesting correlations between the air-earth currents of atmospheric electricity, the diurnal variations of the Earth's magnetism, and the electric currents in the Earth's crust.⁸ He made no attempt to determine the strength of the vertical current necessary to produce the magnetic variations, but it can readily be shown that it would have to be several thousand times that revealed by the diurnal variations of the atmospheric-electric current. (See paragraph 40.)

13. And now we come to the results of the present paper. Preparatory to the construction of world magnetic charts and the mathematical analysis of the Earth as based upon the accumulated data, a variety of investigations are under way in the Department of Terrestrial Magnetism. One of these researches concerns itself with the computation of the line-integrals, with all possible accuracy and refinement, for a number of circuits in various regions of the globe. To compute these integrals entails considerable labor which, in the end, may prove not to have been wholly necessary. It was, therefore, deemed desirable to make first a reconnaissance and repeat my former computations of line-integrals along latitudinal circuits, using this time magnetic charts, which while not representative of all the available data of the Department of Terrestrial Magnetism are close approximations thereto, namely, the declination chart for 1917 of the British Admiralty and the declination and horizontal-intensity charts for 1920 of the United States Hydrographic Office. These charts are based upon the most recent magnetic data as furnished chiefly by the Department of Terrestrial Magnetism. In brief, *while the present results are not yet as accurate as they ultimately may be, they are based on far superior data than heretofore available.*

14. If W be the total work done by a unit magnetic pole in

⁸*Terr. Mag.*, vol. 17, 1912, pp. 1-19.

moving around a closed curve on the Earth's surface, H , the horizontal component of the magnetic force against which the pole is being moved, ϵ the angle H makes with the tangent to the path traversed, ds the curve-element, I , the intensity in electromagnetic units of any electric currents which may pass vertically through the surface of the enclosed area, then is

$$W = \int_0^\circ H \cos \epsilon \cdot ds = 4\pi I \quad (1)$$

Traversing the circuit in an anticlockwise direction, a positive value of the integral implies upward positive electric currents, i. e., earth-air currents, and a negative value implies downward positive currents, or air-earth currents, such as believed generally indicated by atmospheric-electric observations.

15. Choosing parallels of latitude for our circuits and going around the Earth in an eastward direction, $H \cos \epsilon$ becomes the component Y , directed positive to the East. We have

$$W = \int_0^{2\pi} Y ds = 4\pi I \quad (2)$$

Experience has shown that a sufficiently close approximation to the integral is obtained by summing up the value of Y at longitude intervals, $d\lambda$, of 5° , or even 10° . Setting $ds = 2\pi R \sin u \cdot \frac{d\lambda}{360}$ where u is the co-latitude of the parallel along which the circuit is made and R is the Earth's mean radius $= 6.37 \times 10^8$ cm., and since $\frac{d\lambda}{360} \sum_0^{2\pi} Y = Y_m$, the mean value of Y along the parallel, we have for I expressed in amperes:

$$I = 5R \sin u \cdot Y_m \quad (3)$$

I is the resultant quantity of electricity passing in a second of time perpendicularly through the zone from the north geographical pole to the parallel of latitude around which the complete circuit of the Earth is made. If i is the average strength per unit area of the resultant current passing perpendicularly per second through the surface of the zone bounded by the two parallels whose co-latitudes are u_1 and u_2 , $u_2 > u_1$, and A is the area of the zone,

$$i = \frac{I_2 - I_1}{A} \quad (4)$$

in which $A = 2\pi R^2 (\cos u_1 - \cos u_2)$.

16. Table 1 contains the results regarding vertical electric currents computed with the aid of the preliminary magnetic data as already explained. *The results are subject to some modifications in detail, when the more elaborate computations have been completed;* however, the general conclusions to be drawn from the results may not be materially changed. The present results confirm in general those given in my 1897 and 1904 papers (footnote references 5 and 6).

TABLE 1.—*Residual vertical currents from preliminary magnetic data for 1920.*
(A + sign means upward positive current.)

Zone (N. Hem.)	I (10^4 Amp.)	i (10^{-3} A/km 2)	Zone (S. Hem.)	I (10^4 Amp.)	i (10^{-3} A/km 2)	Zone (N. and S.)	I (10^4 Amp.)	i (10^{-3} A/km 2)
N. P. to 50°N	+127	+21	S. P. to 50°S	+109	+18	Pole to 50°	+118	+20
50°N to 45°N	+67	+45	50°S to 45°S	+51	+34	50° to 45°	+59	+40
45°N to 40°N	—9	—6	45°S to 40°S	—77	—47	45° to 40°	—43	—26
40°N to 35°N	+32	+18	40°S to 35°S	—39	—22	40° to 35°	—3	—1
35°N to 30°N	+15	+8	35°S to 30°S	—58	—31	35° to 30°	—21	—11
30°N to 25°N	—56	—29	30°S to 25°S	+37	+19	30° to 25°	—10	—5
25°N to 20°N	—77	—37	25°S to 20°S	+120	+58	25° to 20°	+21	+10
20°N to 15°N	—41	—19	20°S to 15°S	+86	+41	20° to 15°	+22	+11
15°N to 10°N	—71	—33	15°S to 10°S	—90	—41	15° to 10°	—80	—37
10°N to 5°N	—117	—53	10°S to 5°S	+30	+14	10° to 5°	—44	—20
5°N to Equ.	—39	—18	5°S to Equ.	+20	+9	5° to Equ.	—10	—5

17. To illustrate the figures in Table 1: Through the zone from the North Pole to parallel 50° N, there is a resultant total upward positive current, I , of $+127 \times 10^4$ amperes, giving an average current-density, i , of $+21 \times 10^{-3}$ ampere per sq. km.; for the south polar cap from the South Pole to 50° S, there is a resultant total upward positive current of $+109 \times 10^4$ amperes, the average current density being $+18 \times 10^{-3}$ ampere per sq. km.; on the average, for the two polar caps the resultant upward positive current is $+118 \times 10^4$ amperes, and the average current-density, $+20 \times 10^{-3}$ ampere per sq. km. For the zone bounded by the two parallels, 30° N and 25° N, there is a resultant total downward positive current of -56×10^4 amperes, giving an average current-density of -29×10^{-3} ampere per sq. km.

18. Figure 1 shows the average distribution along the mean geographical meridian of the current-density, i , as given for the various zones in Table 1. The full curve (P) shows the preliminary values of i for 1920; the dotted curve (A) shows the approximate values as obtained from my previous investigation, which was dependent upon the approximate magnetic charts for 1842, 1880, and 1885.⁹ An upward arrow indicates an upward positive cur-

⁹*Terr. Mag.*, vol. 9, 1904, Table 1, p. 121.

rent, or downward negative current; a downward arrow indicates the reverse. It will be seen that the two curves *P* and *A* show considerable similarity, the only marked departure arising from the differences in the line-integrals for parallel 30° S. The parallel 30° S passes through the region of the Indian Ocean where the secular variation is both large in amount and varies rapidly with geographic location; it was in this region that the *Carnegie* observations revealed some of the largest errors in the magnetic charts used at the time.

FIG. 1.—DISTRIBUTION OF VERTICAL CURRENT-DENSITIES
ALONG A MEAN MERIDIAN.

[See Table 1 and Paragraph 18.]

19. It should not be assumed of course that Fig. 1 represents the true meridional distribution for any one meridian. For example, it should not be assumed that everywhere in the zone between parallels 30° N and 25° N, there is a current-density of

-29×10^{-3} ampere per sq. km.; there may be regions in the zone where the resultant positive current is plus, or upward, and in other regions, minus, or downward. The quantities in Table 1, or as shown by Fig. 1, represent only the average excess of upward and downward currents for the zone considered.

20. The average resultant current-density for the mean meridian and for the region 50° N to 50° S, is about one-thirtyfifth (29×10^{-3}) of an ampere per sq. km.; from the previous investigation I had found on the basis of the magnetic charts for 1842-1885 and for the region 45° N to 45° S, an average resultant current-density of about one-thirtieth of an ampere per sq. km.¹⁰ However, the average plus i or average minus i , taken separately and as found from line-integrals around ten-degree quadrilaterals for the region 60° N to 60° S, turns out to be as much as one-eighth of an ampere per sq. km.; as already stated in paragraph 4, Schmidt had found from his analysis of the 1885 magnetic charts for the average current-density, whether plus or minus, one-sixth of an ampere per sq. km.

21. Combining the results for corresponding parallels north and south of the equator in order to eliminate to some extent possible asymmetries caused by the asymmetrical magnetization of the earth, or by the distribution of land and water and other physical agencies, the results for i given in the last column of Table 1 are obtained. It will be seen that in the lower latitudes, on the average, downward currents, or positive air-earth currents, prevail, whereas in the high-latitude zones upward positive currents, or earth-air currents, predominate. It is of interest to note here that the average current-density of the downward positive current for the two hemispheres is apparently largest in the zone 10° to 15° . (Cf. paragraph 33.)

22. Table 2 would seem to show the following general facts:

TABLE 2.—*General distribution of vertical currents according preliminary results for 1920.*

Zone	I (10^4 Amperes)	i (10^{-3} Amp./km. ²)
North Pole to 45° North	+194	+26
45° North to 45° South	-354	-10
45° South to South Pole	+160	+21

¹⁰*Terr. Mag.*, vol. 9, 1904, p. 127.

In the polar caps down to about parallel 45° , the resultant vertical currents are on the average upward for positive currents and downward for negative currents, the average current-density being about 24×10^{-3} ampere per sq. km.; in the region of the Earth between 45° N to 45° S, the resultant vertical currents are on the average downward for positive currents and upward for negative currents, the average current-density being about 10×10^{-3} ampere per sq. km. *If then we speak in terms of negative currents, we apparently have, on the average, negative electricity streaming into the Earth in the polar regions, or regions of pronounced polar lights, and streaming out into the air in lower latitudes. Or, we may say also that we apparently have, on the average, negative electricity streaming into the Earth in polar regions, and positive electricity streaming into the Earth in lower latitudes.*

23. It has already been explained in paragraph 19 that the results in Tables 1 and 2 merely show the resultant currents, or the difference between the total upward and downward positive currents over the region considered. In order to make some tests as to the manner of distribution of the upward and downward currents, the average currents over quadrilaterals bounded by two parallels 10° apart and two meridians, also 10° apart, were derived for the entire region from 60° N to 60° S. As a preliminary result, it was found that, on the average, the positive current was upward over the ocean quadrilaterals and the quadrilaterals of low barometric pressure, and downward over the land quadrilaterals and the quadrilaterals of high pressure. However, to disentangle any effects of distribution of land and water from any effects of low and high barometric pressure, or from the variations in the vertical electric current with latitude, will require further investigation, and, accordingly, no definite conclusion is drawn at present. Acknowledgement is here made of effective computing assistance received from Messrs. C. C. Ennis, C. R. Duvall, and G. H. Keulegan.

24. One of the most interesting circuits around which to compute the line-integral of the magnetic force is that of the *Carnegie's* sub-antarctic cruise of 17,084 nautical miles. Leaving Lyttleton, New Zealand, December 6, 1915, the *Carnegie* returned there on April 1, 1916, after a continuous voyage of 118 days, except for a stop of two days at King Edward Cove, South Georgia Island. The circumnavigation of the Earth was made between the parallels about 40° S and 60° S, the average parallel being about 52° S; the

average day's run was 145 nautical miles. Declinations were generally observed twice a day and horizontal intensities and inclinations, once a day. A preliminary computation of the line-integral was made by Messrs. W. J. Peters and C. R. Duvall, of the Department of Terrestrial Magnetism. They found apparently a total resultant negative current, I , passing from the air perpendicularly through the surface of the zone, from the South Pole down to the circuit, of 80×10^4 amperes, which corresponds to an average current-density of 0.016 ampere per sq. km. Table 1 shows an average current-density for the zone from the South Pole to parallel 50° S, of 0.018 ampere per sq. km., for upward positive currents, or for downward negative currents. Results from other line-integrals will soon become available, computed with all possible accuracy.

RESULTS FOR VERTICAL CURRENTS IN THE UNITED STATES.

(Abstract.)

25. The 1908 investigation, already referred to in paragraph 10, depended upon the 1905 magnetic charts for the United States. The average results of the line-integrals around and within the area between the meridians 77° W and 117° W and the parallels 33° N to 49° N was a downward positive current of 26×10^{-3} ampere per sq. km.

26. The computations have since been repeated with the aid of the 1915 magnetic charts for the United States which are dependent not only upon the extensive data available for the 1905 charts, but also upon the additional observations made by the United States Coast and Geodetic Survey since 1905. It may be remarked further that, while the 1905 charts were constructed under the writer's direction, those for 1915 were constructed independently under the direction of Mr. D. L. Hazard. The average result of the computations by the Department of Terrestrial Magnetism of the line-integrals for 1915 around and within the area enclosed by the meridians 75° W and 117.5° W and the parallels 30° N and 50° N was a downward positive current of 33×10^{-3} ampere per sq. km. The average result from the line-integrals about and within the area in the middle part of the United States bounded by parallels 35° N to 45° N and the meridians 80° W to 120° W, was a downward positive current of 32×10^{-3} ampere per sq. km.

27. It may be remarked that, before the magnetic charts for

1905 and 1915 were constructed, all observations were referred to mean of day and to the mean date, using for this purpose the data from the magnetic observatories and from the numerous repeat stations.

Giving the 1915 result double weight, it is apparently found that the 1905 and 1915 magnetic charts for the United States show, on the average, downward positive currents, or upward negative currents, the average current-density over the greater part of the United States being about 31×10^{-3} ampere per sq. km.

How the strength of the vertical current varies over the United States with latitude and longitude, is the subject of a separate investigation. There would seem to be a tendency in certain regions towards a zero value of the current or even reversal of current. Turning to Tables 1 and 2, and Fig. 1, it will be seen that the United States in part may be in the transition region where the upward positive currents of northerly latitudes pass over to downward positive currents of lower latitudes.

ON THE RELATION BETWEEN TERRESTRIAL MAGNETISM AND ATMOSPHERIC ELECTRICITY.

28. Let i_m be the strength in amperes per sq. km. of the vertical currents as found from the line-integrals of the magnetic force, and let i_a be the value resulting from atmospheric-electric measurements. We have the following values of these quantities:

$i_m = 2.9 \times 10^{-2}$ (average value along a mean meridian, from 50°N to 50°S , regardless of sign)

$i_a = 3 \times 10^{-6}$ (approximate average value from land and ocean observations).

Hence, approximately

$$i_m = 10^4 \times i_a = f \cdot i_a \quad (5)$$

It is thus seen that the average result from the magnetic observations is about 10,000 times that from the atmospheric-electric observations; since according to paragraph 20 the average $+i_m$, or $-i_m$, taken separately, may be as much as 13×10^{-2} , the factor f may at places be as high as 40,000. But the large value of f presents not the only difficulty in attempting to harmonize the results from the two different sets of measurements.

29. It is generally concluded from the atmospheric-electric results that all over the Earth there is a downward positive current, called the "air-earth current of atmospheric electricity." How-

ever, the vertical currents from the magnetic work, as shown in Tables 1 and 2, and Fig. 1, while, on the average, downward positive in lower latitudes, appear to be, on the average, upward positive in higher latitudes. The integral of the vertical currents from terrestrial magnetism, if taken over the entire Earth, may be zero, the downward currents being balanced by the upward ones. On the other hand, the conclusion usually drawn from the atmospheric-electric observations is that there is a uniform resultant negative charge over the whole Earth, which, because of the conductivity of the air, requires for its maintenance, a continuous replenishing current of about 1,000 amperes. Hence, the discord between the magnetic and electric results arises both from respective magnitudes of the vertical currents and from the variation in direction over the Earth in the case of the former, and the supposed constancy in direction, during normal conditions, in the case of the latter (the atmospheric-electric conduction current).

30. Unfortunately we are not able at present to make an exhaustive examination into the question whether there are sufficient data to indicate that the atmospheric-electric current, i_a , is, indeed, *everywhere* downward positive during fine weather. Lately observers often fail to state, when publishing potential-gradient observations, whether they have actually made a test of the sign or direction. In view of the very large and rapid fluctuations in the atmospheric-electric current, which take place during the day, and the well-known reversals of current often occurring during cloudy or rainy weather, we must not, of course, expect always complete accord between the vertical currents from the magnetic observations and those from the atmospheric-electric observations.

31. It may be pointed out that the value of the surface density, σ , of the Earth's assumed negative charge, as deduced from potential-gradient measurements, is subject to a geographical variation of at least 50 per cent; the average value is about -3×10^{-4} E. S. U. At Kew Observatory, England, σ is about three times the average value, and according to the observations made at Sobral, Brazil, by Mr. Andrew Thomson of the Department of Terrestrial Magnetism, May-June, 1919, it is about one-sixth of the average value. When σ is subject to such large geographic variations, the question may be raised whether the surface charge may not at some places practically vanish and at other places have reversed sign, even under normal conditions. It may further be recalled that the

results of measurements of the horizontal electric currents in the Earth's crust, have not yet been brought into harmony, quantitatively, with such horizontal currents as might be indicated by variations in the values of σ .

32. The average current, i_m , from the line-integrals in the United States was found to be downward positive and 3.1×10^{-2} ampere per sq. km.; the results of the atmospheric-electric observations being made at Washington by the Department of Terrestrial Magnetism, give a value of i_a of the downward positive current about 3×10^{-6} ampere per sq. km. The average result from Rücker's own line-integrals in Great Britain from the 1886 and 1891 magnetic surveys indicated a downward positive current value of 8×10^{-3} ampere per sq. km.;¹¹ the atmospheric-electric observations at Kew give on an average a positive air-earth current of about 1×10^{-6} ampere per sq. km., or about one-ten thousandth that from Rücker's integrals. For two regions, the United States and Great Britain, for which we may make *preliminary* comparisons the direction of the vertical currents from the two classes of work agree at least as far as direction of current is concerned, but the outstanding numerical factor f is about 10^4 .

33. It is also of interest to record here that, according to Dr. Mauchly's reductions, the atmospheric-electric observations, made on cruises IV and V of the *Carnegie* in the Pacific Ocean, show that the current-density, reduced to mean of day, has a maximum value of about 3.3×10^{-6} ampere per sq. km. in the zones 20° N to 20° S; the current-density then in general decreases until about parallel 47° , when the value is but 1.9×10^{-6} ampere per sq. km. The law of variation with latitude in the values of i_a for the Pacific Ocean runs, in general, parallel with that shown by the values of i_m , given in the last column of Table 1. (See also paragraph 21.)

34. If there is any hope of harmonizing in all parts of the Earth the results from terrestrial magnetism and atmospheric electricity, at least in so far as direction of current is concerned, then a new aspect would be given the problem of the maintenance of the Earth's negative charge. Though this problem has received the attention of most eminent investigators, there is still no generally accepted theory. The difficulty consists in finding an adequate cause for the continual replenishing of a negative charge of the Earth of the required amount; because of the conductivity

¹¹*Terr. Mag.*, vol. 1, 1896, p. 84.

of the air, the Earth's charge would be practically dissipated in about 10 minutes, unless the supply were renewed. If, on the other hand, the sign of the *A.E.* current, i_a , undergoes changes over the Earth such as does the i_m current, then the problem of the maintenance of an Earth-charge may possibly be simplified, as the total charge over the Earth may turn out to be practically zero.

35. It is not to be understood that any definite assertions are being made; the desire is merely to call renewed attention to the need of determining unquestionably the direction of the *A.E.* current during normal weather in all parts of the Earth. It should also be stated that it may easily turn out that the positive upward current (or negative downward) in the polar regions, as indicated by the magnetic line-integrals, may be concentrated over rather restricted areas near the magnetic poles. The density of the inward, or downward, negative currents near these poles may be sufficient so as to make their total equal the total outward, or upward, negative currents prevailing over the very much larger portion of the Earth.

36. The outstanding discrepancy, however, between the magnetic and electric results is the large numerical factor, *f*. Though we are investigating various hypotheses, we have no explanation at present to offer, other than that the usual instrumental appliances for atmospheric-electric observations are designed to detect but comparatively slowly-moving charged particles, such as ions. A corpuscular current, of sufficiently high penetrability, as to get through the atmosphere and down to the Earth, or *vice versa*, might not be measured, or at least not completely, by ordinary electric methods. Another outstanding puzzle of atmospheric-electricity, the so-called "penetrating radiation," seems to indicate that something of a highly penetrating character does actually get through the atmosphere and down to the surface of the Earth. In brief, it may happen that the current density as deduced from magnetic measurements would differ from that deduced from electrostatic measurements. If the two sets of currents show at least a qualitative relationship, as they apparently do, then either both are to be referred to the same cause, or one, the i_a current, may, perhaps, be a secondary effect brought about as the i_m current passes through the atmosphere into the Earth.

CONCLUDING REMARKS.

37. To account for the strength of the vertical currents apparently revealed by magnetic observations, it would seem that we must either assume operating agencies of exceedingly high penetrability, or must look for other causes, which, doubtless will again necessitate postulating agencies appearing improbable on the basis of present knowledge. The question naturally arises as to what reliance may be put upon the results from the magnetic line-integrals, other than the evidence we have already given.

38. Variations in the vertical currents, which apparently took place during the solar eclipse of May 29, 1919, will be described in a future paper. In addition the interesting question has been seriously raised in recent years whether the agencies causing the well-known diurnal variations of the Earth's magnetism have in their entirety a potential. Dr. Annie van Vleuten, who has made an extensive investigation of the question, reached a negative conclusion; it would seem necessary to assume that vertical currents are responsible, partly at least, for the diurnal variation¹⁹ of the Earth's magnetism.

39. On the basis of Dr. van Vleuten's derived coefficients for the diurnal variation of terrestrial magnetism, I have computed the current-densities, Δi_m , for a station situated on the parallel of latitude $38^\circ 44' N$ and for the mean of the summer months (April-September), 1906-1908. As a close approximation, it is generally assumed that the diurnal variation of the Earth's magnetism is the same for all stations along the same parallel of latitude; the same is assumed with regard to the vertical currents disclosed by magnetic diurnal-variation observations. Hence, while the latitude chosen is that of the Cheltenham Magnetic Observatory in Maryland, we may with fair approximation for the reasons stated, apply our computed quantities to any point, let us call it "Magnetism", on the designated parallel. Curve 3 of Fig. 2 shows the diurnal variation, Δi_m , of the vertical currents of terrestrial magnetism. For comparison there have been added the summer diurnal-variation curves, Δi_a , of the atmospheric-electric currents, first

¹⁹Do the forces causing the diurnal variation of terrestrial magnetism possess a potential? *K. Ak. van Wet. Amsterdam*, 26, pp. 297-299, 1917. [For fuller publication see *Mededeelingen en Verhandelungen No. 23*, published as Bulletin No. 102 of the K. Nederlandsch Meteorologisch Instituut, Utrecht, 1917. There are unfortunately a number of typographical errors in this valuable publication. The interested reader should use the coefficients as given on page 89 in preference to those on page 111. As the formulae, for which Dr. van Vleuten's coefficients apply, are to be considered in the nature of more or less empirical ones, it is not admissible to make use of them much beyond the region of the data used in their establishment (about $60^\circ N$ to $15^\circ S$). Accordingly for this reason as well as others, Adolf Schmidt's concluding arguments (*Physik. Zeitschr.*, XIX, 1918, 349-355) against the correctness of Dr. van Vleuten's conclusion do not appear legitimate.—L.A.B.]

(Curve 1) as shown by Dr. Dorno's observations¹³ at the Alpine station, Davos (latitude $46^{\circ} 48' N$; longitude, $9^{\circ} 49' E$; altitude

FIG. 2. SUMMER DIURNAL VARIATION OF AIR-EARTH CURRENT-DENSITY OF ATMOSPHERIC-ELECTRICITY (CURVES 1 AND 2) COMPARED WITH DIURNAL VARIATION OF VERTICAL CURRENT-DENSITY OF TERRESTRIAL MAGNETISM (CURVE 3).

[Note the remarkable similarity of Curve 3 with Curves 1 and 2. Plus value of ordinate means a value of the current-density higher than the mean value for the day (24 hours). Scale of ordinates: For Curves 1 and 2, $d = d' = 10^{-8}$ Amp/km²; for Curve 3, $d'' = 10^{-8}$ Amp/km². See paragraph 39.]

above sea-level, 1,560 meters), and secondly (Curve 2) at the Potsdam Observatory ($52^{\circ} 23' N$; $13^{\circ} 04' E$; altitude above sea-level, 86 meters). The three curves are plotted according to local mean time; it will be seen that they show a good general agreement. As the curves are plotted, a minus ordinate indicates a value of the positive air-earth current-density less than its mean daily value. Accordingly during the daytime (6 A. M. to 6 P. M.) both sets of currents, Δi_a and Δi_m , unite in showing, on the average, a decreased positive downward current. During the night hours, on the average, the positive downward current is increased.

At some other occasion the matters in this paragraph will be discussed more fully. From various preliminary investigations of magnetic storm-effects which I have made, it also seemed as though some of the observed effects would have to be ascribed to vertical currents.

¹³Studie ueber Licht und Luft des Hochgebirges, table 78, p. 138. Braunschweig, 1911.

40. Table 3 contains the chief facts shown in Fig. 2. We see that:

TABLE 3.—*Diurnal variation of vertical currents for summer months (April-September).*

[Unit 10^{-6} Amp./km.²; i =current-density; Δi =diurnal variation. A + sign means a greater value than daily mean.]

Station	Curve	Observations	Current-Density, i		Average Δi		
			Daily Mean	Daily Range	Day Hours	Night Hrs.	24 Hours
Davos	1	At. Elect.	1.7	0.8	-0.24	+0.22	± 0.3
Potsdam	2	At. Elect.	2.2	1.0	-0.15	+0.20	± 0.3
Magnetism	3	Magnetic	3×10^4	2.8×10^3	-0.40×10^3	$+0.43 \times 10^3$	$\pm 0.7 \times 10^3$

Average Δi_a = about $0.15 i_a$ (Davos and Potsdam).

Average Δi_m = about $0.02 i_m$ (Magnetism).

Average Δi_m (Magnetism) = about $2 \times 10^3 \Delta i_a$ (Average, Davos and Potsdam).

41. The only approach in strength of the atmospheric-electric current to that of the vertical currents, shown apparently by magnetic observations, is in the case of shower-clouds. From C. T. R. Wilson's observations¹⁴ it would appear that below a thunder-cloud, during lightning flashes, a current of electricity amounting to some amperes may pass from the atmosphere through a few square kilometers of the surface of the Earth. Under shower-clouds, the vertical current-density will, of course, be less than under thunder-clouds, still it would appear that it may be of the order of the vertical current-density from the magnetic line-integrals.

42. The questions raised in this paper, both with regards to terrestrial magnetism and atmospheric electricity, will receive further rigid investigation. It is believed that the present paper shows at least a *qualitative* relationship between certain phenomena of terrestrial magnetism and of atmospheric electricity.

¹⁴Investigations on lightning discharges and on the electric field of thunderstorms: *Phil. Trans. Roy. Soc. of London, Ser. A*, vol. 221, pp. 112-113.