risulteranno negli stessi punti discontinuità della linea elastica e per ciascun tratto sarà possibile trovarne l'equazione.

Se i valori di Y_{C_1} e di Y_{R_1} risultano uguali fra di loro, vuol dire che l'asse ottico riesce parallelo all'asse delle ascisse, ma questo fatto si può verificare in modo rigoroso soltanto quando il cannocchiale è in direzione orizzontale o di poco inclinato all'orizzonte ed inoltre è tale per cui $P = P_1$, $Q = Q_1$; $l = l_1$, $E_C = E$ ed ha tutte le sezioni trasversali uguali fra di loro.

L'effetto della flessione è evidentemente massimo nei cannocchiali spezzati. Questi cannocchiali sono muniti di contrappesi all'estremità opposta all'obbiettivo ed il loro effetto è unicamente quello di mantenere in equilibrio il cannocchiale nelle sue varie posizioni d'altezza, senza distruggere per nulla l'effetto della sua flessione. Il reticolo (punto da noi indicato con R) in questi cannocchiali è situato ad una delle estremità dell'asse di rotazione, il che è quanto supporre nelle nostre formole $l_1 = 0$.

Se supponiamo di avere un cannocchiale il cui asse ottico coincida perfettamente col proprio asse meccanico in modo che allo zenit si possa ritenere nullo l'effetto dei pesi

P e Q e trascurabili nelle altre posizioni del cannocchiale i valori di $P \cos z$ e $Q \cos z$ ed inoltre se $P = P_1$, $Q = Q_1$, $l = l_1$, $E_C = E$ ed F è costante, fatte le debite sostituzioni nella (8) si ha:

$$\operatorname{tg} \varepsilon = \frac{8 P + 3 Q}{24 \cdot l \cdot E \cdot I} \cdot \sin z \quad (9)$$

In questa espressione il coefficiente di $\sin z$ è costante per quei cannocchiali i quali soddisfano a tutte le condizioni ora supposte; ne segue che le nostre formole comprendono come caso speciale la legge del seno ritenuta vera da molti astronomi,* e dicono che per eliminare l'effetto della flessione dei cannocchiali coll'aggiunta di contrappesi al suo tubo, bisogna variare opportunamente il braccio di leva di ciascuno di essi per ogni valore di z e ciò a causa della pressione e trazione che per loro effetto si manifesta lungo il tubo del cannocchiale.

È da avvertire che prima di applicare le formole trovate a qualche caso pratico, bisogna modificarle a seconda della forma del cannocchiale che si considera.

Torino 1895 Gennaio.

Ingegnere *Vittorio Baggi*.

*) Vedi i Trattati di Astronomia del Chauvenet e del Gaspari ed il Trattato di Geodesia dello Schiavoni.

Note on the Determination of the Division Errors of a Straight Scale.

By *Harold Jacoby*.

In two valuable papers contained in the *Astr. Nachr.* (Nos. 3134, 3236) Dr. Lorentzen has discussed a method for determining the errors of a straight scale, which must be regarded as a material advance upon the method of Hansen. Dr. Lorentzen's method is not new; for the second and final method which he recommends is identical both in the programme of observations and in the process of reduction with that given by Dr. Gill (Monthly Notices, R. A. S., Vol. XLIX, p. 110). However, Dr. Lorentzen has added considerably to the subject, since he has deduced a formula for calculating the weights with which the division errors of the several lines are determined. Dr. Gill had assumed that these weights are all equal, an assumption which is not quite exact. Dr. Lorentzen finds for the weight of the division error of the line numbered p on a scale of $n + 1$ lines numbered from 0 to n , the value:

$$P_p^n = \frac{n}{2 \left(1 - \frac{1}{n} + \frac{p(n-p)}{n^3} \right)}$$

or, as it may be more conveniently written:

$$\frac{1}{P_p^n} = \frac{2}{n^2} (n-1) + \frac{2p}{n^4} (n-p) \quad (1)$$

Dr. Lorentzen also finds for the weight of a quantity

he calls δ , which equals the difference between the lengths of the mean unit-spaces on the two scales, the value:

$$P^n(\delta) = \frac{n^3}{4}$$

this should be

$$P^n(\delta) = \frac{n^3}{4 - \frac{2}{n}}$$

This error is of no consequence, however, since the essential formula (1) is correct. The object of the present note is to show how the method itself may be greatly strengthened. Let us, in each series of observations, observe the first and last comparisons of the two scales i times instead of once, and use the mean of the i observations in the further computations. In other words, in carrying out Gill's observation programme, let us observe i times every comparison between the two scales in which line 0 or n of either scale appears. The weight formula (1) then becomes:

$$\frac{1}{P_p^n} = \frac{1}{n^2} (n-1) + \left[\frac{1}{n^2} (n-1) + \frac{2p}{n^4} (n-p) \right] \frac{1}{i} \quad (2)$$

It will be seen that formula (2) gives a much greater weight than (1), and at the same time the ratio of the maximum and minimum weights for the same value of n is brought nearer to unity. On the other hand, the total number of observations required is

$$\begin{array}{ll} \text{for Gill's (Lorentzen's) method} & (n+1)^2 - 2 \\ \text{for new method} & (n+1)^2 - 2 + (4n-2)(i-1) \end{array}$$

so that for large values of n the number of observations is not increased materially. Even for small values of n the increase in the time required for the work is not much, since the additional observations of the new method consist simply in additional readings of the micrometer after the scales &c. have been brought into adjustment. It does not seem desirable to make i greater than 4, since it is doubtful whether the mean of a large number of observations of this kind really possesses a weight corresponding to that number. In computing the following table, i has therefore been taken equal to 4. The column »No. of obs.« gives for the new method the number of observations required in addition to that needed in Gill's method. In computing the column »Time used« I have assumed that each original observation in Gill's method requires a unit of time, and each additional one in the new method only one-fourth of

a unit of time. The last two columns bring out clearly the advantage of the new method. It may be objected that these columns depend upon a somewhat arbitrary assumption with regard to the time required for the additional observations. But it should be noted that even without this assumption, these columns would show an increasing advantage for the new method as soon as n becomes larger than 10 or 12. The same would be true even for smaller values of n , if we were to put i equal to 2 or 3 instead of 4.

I may add that it is even possible, by varying the weight with which the several comparisons of the scales are effected, to make the weights of all the division errors come out equal. This possesses perhaps, a theoretical rather than practical interest, and leads to a somewhat lengthy discussion, which cannot be brought within the limits of the present note.

n	Gill Method Weight		New Method Weight		Max. Weight Min. Weight		No. of Obs.		Time used (arbitrary unit)		Min. Wt. $\times 10^2$ Time used	
	Max.	Min.	Max.	Min.	Gill	New	Gill	addit. new	Gill	New	Gill	New
3	2.03	2.02	3.45	3.44	1.00	1.00	14	30	14	21.5	14.3	16.0
4	2.49	2.44	4.13	4.10	1.02	1.01	23	42	23	33.5	10.5	12.2
5	3.00	2.95	4.93	4.88	1.02	1.01	34	54	34	47.5	8.5	10.2
6	3.47	3.40	5.65	5.62	1.02	1.01	47	66	47	63.5	7.1	8.8
7	4.08	4.00	6.62	6.58	1.02	1.01	62	78	62	81.5	6.4	8.1
8	4.39	4.31	7.09	7.04	1.02	1.01	79	90	79	101.5	5.6	6.9
9	5.15	5.05	8.33	8.26	1.02	1.01	98	102	98	123.5	5.1	6.7
10	5.49	5.41	8.93	8.85	1.02	1.01	119	114	119	147.5	4.3	6.0
15	8.06	7.94	12.90	12.82	1.02	1.01	254	174	254	297.5	3.2	4.3
20	10.53	10.42	16.86	16.78	1.01	1.01	439	234	439	497.5	2.4	3.4

Royal Observatory Capetown, 1895 Jan. 9.

Harold Jacoby.

Elements of the orbit of α Pegasi = β 989.

By Dr. T. J. J. See.

This remarkable double star was discovered by Prof. S. W. Burnham on August 12th, 1880, with the 18 inch Refractor of the Dearborn Observatory. Its extreme closeness led to the belief that it would prove to be binary, and accordingly it has been found to be in rapid revolution. Dr. Engelmann of Leipzig succeeded in making one measure of the pair in 1883, and his observation indicated a retrograde motion. Prof. Burnham's measures were continued at the Lick Observatory from 1888 to 1892, and the new data thus obtained enabled him for the first time to get the approximate period of revolution (M. N., March 1891).

At the request of Prof. Burnham and the writer Prof. Barnard has since followed the star, and has obtained additional measures which seem sufficient to give us a reasonably good approximation to the elements of the orbit. In his first examination of the motion of this pair, Prof. Burnham made the orbit nearly circular, but the recent obser-

vations show that the orbit has about the usual eccentricity prevailing among binaries, and that the inclination of the orbit is very high. In the Monthly Notices for November 1894 Mr. Lewis has given a set of measures recently obtained with the Greenwich 28 inch Refractor, and has sketched an apparent orbit which would better satisfy the latest observations.

Having collected all the observations hitherto made, including some unpublished measures kindly sent me by Prof. Barnard last autumn, I have investigated the orbit of α Pegasi by the method of Klinkerfues, and find the following elements:

$$\begin{array}{ll} P = 11.42 \text{ years} & i = 81^\circ 2 \\ T = 1896.03 & \Omega = 116.25 \\ e = 0.49 & \lambda = 89.2 \\ a = 0.4216 & n = -31.5236 \end{array}$$