matter when he invented his three cylinder locomotives, which, notwithstanding his high reputation, the necessity for is scarcely as yet acknowledged. And, therefore, I do not expect to make converts to the proposition which I advanced several years ago, and for the doing of which I have only obtained the abuse of those who assume to be the elite of railway engineering

New York, April 2, 1855.

For the Journal of the Franklin Institute.

Investigation of the Angle-Block of Howe's Truss. By J. M. RICHARDSON.

A point of great practical importance in construction is, that all the parts should fit; so that, when put together, they may constitute a perfect whole. Under-pinning, or wedging and keying, to remedy any deficiency of length, breadth, or thickness of the components, is in the highest degree deleterious, and always endangers the stability of the structure. The dimensions, therefore, of each part should be accurately determined beforehand, and no one should be received which does not correspond, in every particular, with the conditions previously ascertained.

With the view of contributing my mite towards perfecting the details of bridge construction, I submit the following general investigation of the angle-block of Howe's Truss, the diagonal brace being perpendicular to the abutting surfaces of blocks.

The quantities given in this investigation are, the distance between the upper and lower strings, the width of bay or pannel, width of brace, height of block, and distance between abutting surfaces of block.

The quantity to be determined is, the degree of inclination of abutting surfaces of blocks.

Let A B C D be a panel of Howe's truss. $AB = a'$; $AC = d''$; $A'c' = c'$; $c'c' =$ half distance between abutting surfaces of block $= e$; $eg = c$; $ef = x$; $fg = y$.

$ac = d''$; $-2c' + y = b + y$; $ae = a' - 2e - x = a - x$.

The right triangles, aec and efg , are similar, and give the proportion, $ef : fg :: ac : ae$, or $x : y :: b + y : a - x$. Hence,

$$ax - x^2 = by + y^2. \quad (1)$$

From efg ,

$$x^2 = c^2 - y^2. \quad (2)$$

From (1) and (2),

$$x = (b y + c^2) a^{-1}. \quad (3)$$

The limits of y are $y = 0$, and $y = c$. When $y = 0$, $c = 0$, or $c = a$, and

$$x = 0, \text{ or } x = a. \quad (4)$$

When

$$y = c, b = -c, \text{ and } x = 0. \quad (5)$$

If $y = m c$ where m is fractional,

$$x = (b m c + c^2) a^{-1}. \quad (6)$$

Solving (1) with regard to x ,

$$x = 2^{-1} (a \pm [a^2 - 4 (y^2 + b y)]^{\frac{1}{2}}). \quad (7)$$

Substituting in (2) and reducing

$$y = -c (a^2 + b^2)^{-1} [b c + a (a^2 + b^2 - c^2)^{\frac{1}{2}}]. \quad (8)$$

Putting this value of y in (3) there results,

$$x = a^{-1} c (c - b (a^2 + b^2)^{\frac{1}{2}} [b c + a (a^2 + b^2 - c^2)^{\frac{1}{2}}]). \quad (9)$$

Having found x and y , the inclination of abutting surface of block becomes known; for, if we denote the angle, $g e f$, by ϕ , we get

$$\tan. \phi = y x^{-1}. \quad (A)$$

Also, if l is the length of the brace,

$$l = [(b + y)^2 + (a - x)^2]^{\frac{1}{2}}. \quad (10)$$

The greatest width of block is,

$$e' h' = 2 e + 2 x = 2 e + 2 a^{-1} c (c - b (a^2 + b^2)^{-1} [b c + a (a^2 + b^2 - c^2)^{\frac{1}{2}}]). \quad (11)$$

If $a' = 0$, the height of the block is the unknown quantity, y , and equation (1) becomes

$$a x - x = a' y - 2 y^2. \quad (12)$$

From (2) and (12),

$$x = (c^2 + d'' y - 2 y^2) a^{-1}. \quad (13)$$

The limits of x and y are the same as before, and if $y = m c$,

$$x = (c^2 + c d'' m - 2 c^2 m^2) a^{-1}. \quad (14)$$

Solving (12) with regard to x ,

$$x = 2^{-1} (a \pm [a^2 + 4 (y^2 - d'' y)]^{\frac{1}{2}}). \quad (15)$$

Substituting in (2) and reducing

$$y^4 - d'' y^3 + 4^{-1} (a^2 + d''^2 - 4 c^2) y^2 + 2^{-1} c^2 d'' y = 4^{-1} c^2 (a^2 - c^2). \quad (16)$$

(16) may be put under the form

$$y^4 + b' y^3 + c' y^2 + d' y = e'. \quad (17)$$

which is the general of the equation of the 4th degree.

In (17) put

$$y = 2 - 4^{-1} b'. \quad (18)$$

Reducing

$$2^4 + p 2^2 + q 2 = r. \quad (19)$$

in which

$$p = c' - 6 \cdot 4^{-2} b'^2, \quad q = 2 \cdot 4^{-2} b'^3 - 2^{-1} b' c' + d', \quad r = e' + 3 \cdot 4^{-4} b' d' + 4^{-1} b' d' - 4^{-2} b'^2 c'.$$

Assume

$$2 = s + t. \quad (20)$$

(19) then becomes

$$s^4 + s^2 (6 t^2 + p) + t^4 + p t^2 + q t + s (4 t^3 + 2 p t + q) + 4 t s = r. \quad (21)$$

Since but one condition has been placed on s and t , we are at liberty

to impose another; let the sum of the terms involving the odd powers of s , equal zero. Reducing, we get

$$3^2 = -(t^2 + 2^{-1}p + 4^{-1}q t^{-1}). \quad - \quad (22)$$

(21) now becomes

$$s^4 + 3^2 (6 t^2 + p) + t^4 + p t^2 + q t = r. \quad - \quad (23)$$

Eliminating s between (22) and (23),

$$t^6 + 2^{-1}p t^4 + 16^{-1}(p^2 + 4r) t^2 = 16^{-1}q^2. \quad - \quad (24)$$

(24) is of the 6th degree, but may be reduced to the 3d, since it contains none but even powers of t . Assume

$$t^2 = 4^{-1}u. \quad - \quad - \quad (25)$$

(24) now becomes

$$u^3 + 2p u^2 + 4(p^2 + 4r)u = 4q^2. \quad - \quad (26)$$

$$\text{or, } u^3 + m u^2 + n u = v. \quad - \quad (27)$$

$$\text{Let } u = 2^1 - 3^{-1}m. \quad - \quad (28)$$

Reducing

$$2^3 + p' 2' = r'. \quad - \quad (29)$$

Where

$$p' = n - 3^{-1}m^2 r = v + 3^{-1}m n - 2 \cdot 27^{-1}m^3. \quad (30)$$

$$\text{Let } 2' = s' + t'. \quad - \quad (31)$$

After reduction, (29) will become

$$s'^3 + t'^3 + (3s't' + p')(s' + t') = r'. \quad - \quad (32)$$

Since s' and t' have been subjected to but one condition, we are at liberty to impose a second. Let

$$3s't' + p' = 0. \quad - \quad (33)$$

(32) becomes

$$s'^3 + t'^3 = r'. \quad - \quad (34)$$

Eliminating t' between (33) and (34),

$$s'^6 - r's'^3 = 27^{-1}p'^3. \quad - \quad (35)$$

Solving (35) as a quadratic,

$$s' = (2^{-1}[r' + (r'^2 + 4 \cdot 27^{-1}p'^3)^{\frac{1}{2}}])^{\frac{1}{3}}. \quad - \quad (36)$$

Substituting in (34) we get

$$t' = (2^{-1}[r' - (r'^2 + 4 \cdot 27^{-1}p'^3)^{\frac{1}{2}}])^{\frac{1}{3}}. \quad - \quad (37)$$

Now, we can find z' , then u , s , t , and z , and finally x and y . Substituting the value of z' , found from (36) and (37), in (28), we get one root of (27). Denote this root by u' , then from (25),

$$t = \pm 2^{-1}u'^{\frac{1}{2}}. \quad - \quad (38)$$

which, in (22), gives

$$s = \pm (-[u' + 2p + 2u'^{-\frac{1}{2}}q])^{\frac{1}{2}} 2^{-1}. \quad - \quad (39)$$

But from (20) $z = s + t$. Hence, the four roots of z are,

$$z = 2^{-1}(u'^{\frac{1}{2}} + [-(u' + 2p + 2qu'^{-\frac{1}{2}})]^{\frac{1}{2}}). \quad - \quad (40)$$

$$z = 2^{-1}(u'^{\frac{1}{2}} - [-(u' + 2p + 2qu'^{-\frac{1}{2}})]^{\frac{1}{2}}). \quad - \quad (41)$$

$$z = -2^{-1}(u'^{\frac{1}{2}} - [-(u' + 2p - 2qu'^{-\frac{1}{2}})]^{\frac{1}{2}}). \quad - \quad (42)$$

$$z = -2^{-1}(u'^{\frac{1}{2}} + [-(u' + 2p - 2qu'^{-\frac{1}{2}})]^{\frac{1}{2}}). \quad - \quad (43)$$

Subtracting $4^{-1}b$ from each of these results, the four roots of (17) are obtained.

The formulas last obtained are of so complicated a character, at least

in appearance, that the merely practical engineer may be deterred from using them. How may the process for finding x and y be simplified? It is evident that the available roots of (17) must be real and positive. Find, then, the limits of the real and positive roots of (17), by trial or otherwise, and, by some method of approximation, determine the roots to at least four places of decimals, if they are fractional.

Or proceed thus, in (13) put $y = mc$, where m is fractional. There results,

$$x = a^{-1}c(c + bm - 2cm^2) = n'c. \quad - \quad (44)$$

Substituting this value of x in (2),

$$y = +c(1 - n^2)^{\frac{1}{2}} = m'c. \quad - \quad (45)$$

$$x = a^{-1}c(c + bm' - 2cm'^2) = n''c. \quad - \quad (46)$$

$$y = +c(1 - n'^2)^{\frac{1}{2}} = m''c. \quad - \quad (47)$$

$$x = a^{-1}c(c + bm'' - 2cm''^2) = n'''c. \quad - \quad (48)$$

&c., &c., &c., until x and y are obtained to four places of decimals.

The solution of the problem in this form is more difficult than in the first. Having found x and y , the length of the brace becomes known, for

$$l = [(b - y)^2 + (a - x)^2]^{\frac{1}{2}}. \quad - \quad (49)$$

The simplest case is when the axis of the brace passes through the points A and D. The triangles ACD, and efg , are then similar, and from them we get the proportion,

$$ef : fg :: AC : CD, \text{ or, } x : y :: d'' : a', \text{ and} \\ x = a'^{-1}d''y. \quad - \quad (50)$$

(50) combined with (2) gives

$$y = +c(a'^{-2}d''^2 + 1)^{-\frac{1}{2}}. \quad - \quad (51)$$

(51) in (50) gives

$$p = +a'^{-1}d''c(a'^{-2}d''^2 + 1)^{-\frac{1}{2}}. \quad - \quad (52)$$

l is found by (10) or (49), according as $a'7$ or $= 0$.

From this discussion, it is seen that for simplicity, a' should be 70, and that the axis of the brace should pass through A and D. Some difficulty may be occasioned by making both of these assumptions at the same time, but this may generally be avoided by giving suitable values to e and c' . When $a'70$, it will be well to make $c' = c$ or $7c$. The value of e is dependent, in a measure, upon the diameter of the tie-rods connecting the upper and lower strings, and which pass through the blocks.

If $a' = 0$, and x and y be assumed, the width of the brace will have to be found.

I owe an apology, perhaps, for rejecting, in this discussion, the ordinary form of fractional expressions, except in the case of exponents, and the radical sign. My reason for so doing is, that I find certain advantages to accompany this method of notation. I cannot, with propriety, point out these advantages now, as it would be foreign to my purpose. The use of fractional exponents to express the extraction of roots, is very frequent now, and I see no reason why this notation should not be adopted to the entire exclusion of the radical sign.