The Mechanics of Earthquakes*

Recent Views with Regard to the Nature of Seismic Disturbances

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It cannot be said that there is any general mechanical theory at the present time concerning the movements produced by the vibrations of the ground, both upon the surface and in the interior of our planet; although increasingly exact methods of observation have revealed to us both their general properties and their more minute detail.

We say that we possess a mechanical theory of a phenomenon when we are able to construct an ideal model of it comprised within a certain system of finite or differential equations, which determine mathematically the laws of variations of the measurable quantities by which the phenomenon is characterized. In such a case it is possible to make a comparison between the data observed and the theory, thus determining the value of the latter. A typical and very marvellous example of this is the celestial mechanics which comprises within a single law, that of Newton, and in a single system of differential equations deduced therefrom, the laws of movement which govern not only the solar system, but very probably the entire universe.

In the case of seismic movements certain theories of a mechanical character which represent, frequently with great precision, certain features of the phenomenon are generally accepted. But these views do not always agree with each other and, furthermore, they do not possess a common origin about which they can be grouped in such a manner that the problem of the representation of seismic waves can be reduced to a single problem of mathematical analysis, as is the case for many other questions in physics.

As a matter of fact if we consider the subject from a general point of view, it is possible to regard seismic movements, at least in a first approximation, as vibrations of a free sphere, such as our own globe may be represented to be. But the laws of the vibrations of such a sphere have long been studied both with remarkable precision and with regard to their general application, and have been classified and arranged in groups according to definite properties, thanks especially to the researches of Mr. H. Lamb. However, it sometimes occurs in the mathematical treatment of problems in physics that the highly general character of the solution prevents its application to the special case in hand. Furthermore, in the present question there are certain peculiar features which must be taken into account, e. g. the unknown heterogeneities of the interior of the Earth, which limit the value, as regards the seismic problem, of the general solutions, so admirable in themselves, of the problem of the waves in a free homogeneous sphere.

This is why research along these lines has followed an opposite path to a certain extent. Instead of seeking, in the general solutions already known, the special solutions best adapted to the representation of the observed facts, it has sought to make use of the simplest solutions known, attempting to arrive by means of these, at least approximately, at the ideal models, which form, as we have said, the supreme goal of the theory.

We shall attempt to give in these pages a brief summary of these attempts and their results. However, it is proper to first form an idea of the nature of the objects aimed at in theoretic seismology and of the importance with regard to the knowledge of the constitution of our planet which they may possess.

In the majority of physical theories the result aimed at by the investigator is the prediction of the phenomenon. The said prediction constitutes in itself the best confirmation of the theory. But it must be admitted that in the science of seismology absolutely correct prediction lies outside the scope of any theory whatever, no matter how perfect it may be. Seismic movements are phenomena which are produced spasmodically and which depend upon a very special class of circumstances which lie outside the possibility of direct control. These sudden tremors which pass over the surface of the earth are determined by ruptures of the internal equilibrium in localities of limited extent, and these ruptures are due to accidental causes of which we are ignorant. The researches made with the object of establishing in a general manner the frequency of earthquakes in time and in space, have thus far failed to enable us to formulate laws which would permit the making of predictions. The results attained belong to the domain of statistics or of proba-

bilities and have no relation, furthermore, to the mechanical theory of the movement.

The importance of seismic mechanics is of a different nature. It teaches us, above all, to read and to interpret the data yielded by observation which would remain meaningless without the light thrown upon them by theoretic concepts. Seismic diagrams are writings which can be understood only by him who knows the language of mechanics. In the second place seismic mechanics enables us to deduce valuable indications with regard to the constitution of the interior of the Earth, of those vast, mysterious depths which we can have but little hope of penetrating directly by means of research at our disposal. It seems quite certain, in fact, that seismic vibrations penetrate to very great depths in the terrestrial mass, appearing afterwards upon the surface in various forms which are characterized by the special constitution of the strata they have traversed, just as a ray of light is modified in various ways by the transparent mediums through which it passes. The study of these modifications enables us to find our way back to their causes and thus to obtain certain ideas with regard to the peculiar physical properties of deep-lying terrestrial strata. It cannot be claimed, of course, that the analysis of seismic vibrations has led thus far to results at all comparable to those yielded by spectrum analysis with regard to the chemical constitution of the celestial bodies. But these first messages from the interior of the globe to reach us have already furnished valuable contributions to the study of the various hypotheses which have been propounded upon the subject of its internal constitution. It is true that we are able to observe upon the surface of the earth other phenomena which are necessarily connected with its constitution, such as the magnetic field, or the field of gravitation; but the deductions capable of being derived from these are not very extensive and it often happens that they have a purely global character and consequently fail to exhibit localized properties.

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As said above, the simplest laws first made use of for the study of seismic waves are those concerning the propagation of plane waves, or of spherical, longitudinal, and transverse waves. If we conceive of a vibrating medium subdivided into a number of parallel planes like the pages of a book, and imagine each of these planes in itself to be subject to small periodical back and forth movements in the direction of their common normal, we shall have an idea of what is meant by a system of longitudinal plane waves. If, on the contrary, we represent each plane as sliding periodically upon itself we shall have the conception of movement by transverse plane waves. It is evident that in the second case the volume of any part whatever, separated in the vibrating medium by a cylindrical surface normal to the planes of the wave, will not perceptibly vary during the movement; just the opposite to this is true in the first case, in which the height of each cylinder is subject to continual variations. Thus the waves of the first class will be waves of *expansion* [dilation] while the others will be called *equivoluminal*. If instead of a series of parallel planes we consider a series of concentric spheres, we may then have two classes of spherical waves, in one of which the spheres undergo periodic changes of radius while vibrating, while the others slide upon themselves. Thus we may say again that spherical waves are either expansional or equivoluminal. It may be proved also that in all cases the movement is propagated along the normal to the surface of the wave; this is why the movement of vibration is parallel to the direction of propagation in one case, while in the other it is perpendicular to it. For this reason the waves of expansion are called *longitudinal*, while equivoluminal waves are called *transverse*. The speed of propagation of the first kind in every uniform elastic medium is always greater than that of the second kind.

But in the seismic diagrams which began to be systematically collected in the last decades of the last century, two phases were at once distinguished: the first consisting of very small and irregular movements, to which the name of *preliminary tremors* has been given, and the much ampler and more regular ones which follow, and to which great catastrophes are due. English seismologists call this second phase the *main shock*.

The first idea which presented itself by way of an

attempt at mechanical explanation of the phenomenon was the comparing of the preliminary tremors to the more rapid longitudinal waves and the ample vibrations which succeed these to the slower transverse waves. But there are various special features of the observed data which fail to agree with this concept.

During this time there appeared in 1885 a remarkable study by Lord Rayleigh announcing the discovery by purely theoretical methods of a new class of waves, with regard to which it is possible to demonstrate the absolute possibility of their propagation in a ground plane of indefinite width and extent. These waves are characterized by the property of becoming very rapidly extinguished beneath the surface of the ground; for this reason they have been called *superficial*. It is a well-known fact in acoustics that the form of a vibrating body is of fundamental importance with respect to the possible vibrations of the said body, and this is why it would be incorrect to assume, as was done at first, the possibility of autonomous longitudinal or transverse plane waves in a body of finite dimensions, such as our globe. The indefinite ground plane in which Rayleigh's waves propagate themselves, is very far from being an exact model of the terrestrial sphere, but in a primary approximation, or, at any rate, by way of obtaining a primary qualitative representation of the facts, it may be accepted as being true within certain limits. It must be admitted also that Rayleigh's waves possess certain special characteristics which has caused them to be of great value in the science of seismology. They result from the superposition of transverse waves and waves of expansion, and possess no actual wave planes; they can also be called *rectilinear* waves, since the vibrating elements are not planes or spheres but straight lines. Furthermore, they possess the fundamental property of propagating themselves with a constant speed along the surface of the ground.

It was R. D. Oldham who in 1900, after a very attentive and systematic study of the numerous data yielded by observation, proposed taking Rayleigh's waves as a mechanical representation of the waves called *long* waves in the final and principal phase of the seismic shock. From this study he concluded that it is necessary to divide the preliminary tremors into two phases which he considers respectively comparable to the two series of longitudinal and transverse waves emanating from the hypocenter, or internal point of origin of the earthquake. These two series of waves must necessarily be distinct from each other because of the different rate of speed which they possess. They are called respectively primary waves and secondary waves. Thus we have altogether three groups of waves: the *primary* waves, which are longitudinal; the *secondary* waves, which are transverse; and the waves of Rayleigh, or *long* waves.

These hypotheses are corroborated by various facts. We may assume, indeed, that the phenomenon takes place in the following manner: The expansional waves and the equivoluminal waves start from the hypocenter at their respective rates of speed and reach the surface of the earth successively. Neither of these categories of waves can remain separate after the movement has reached the surface; they mingle, therefore, to form a single system, which is the system known as the waves of Rayleigh. The two first groups of waves, expansional and equivoluminal, thus represent in their pure state a brief initial phase of the phenomenon; the waves of the third group, which have been compared to Rayleigh's waves, represent the succeeding phase, which can also be considered, up to a certain point, as the ruling phase [phase de regime]. Observation has proved positively that the traces of these waves can be discovered a number of hours after the beginning of the shock; and they can be perceived at the same observation station, either as coming directly from the epi-center by the shortest surface path, or as coming by way of the arc of the maximum circle which does not contain the said observation station, or else their return path after having traversed the whole extent of the terrestrial maximum circle which passes through the epi-center and the observation station.

This concept has been confirmed in a remarkable manner by figures. Let us take V_1 and V_2 as representing the average value of the speed of propagation of longitudinal and transverse waves; $V_1 = 7.17$ and $V_2 = 4.01$, the values given by Galitzin in his classic

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work, *Treatise on Seismology*, the unit of length being the kilometer, and the unit of time being the second; the proportion they bear to each other may be expressed as follows $V_1 : V_2 = 1.788$.

But the theory of elasticity enables us to deduce from this ratio the value of what is called Poisson's coefficient σ the elastic constant which is characteristic of the matter of which the vibrating medium is composed. Taking as a base the preceding value of the ratio $V_1 : V_2$ we find that $\sigma = 0.27$, a value which is very close to the average value 0.25 which experiment has caused us to assign to the materials of which the superficial crust of the Earth is composed.

Furthermore, the theory of Rayleigh's waves assigns the value V_3 to the speed of propagation of the said waves; and this value V_3 depends upon V_2 , according to the ratio $V_3 = 0.9194V_2$.

Taking the figures mentioned above, 4.01 as the value of V_2 we find that $V_3 = 3.69$, a value which is very close to the values which have been determined by experiment as those of the rate of propagation of the third group of waves, i. e. the long waves.

More recent observations and calculations have slightly modified the average values of the rates of speed quoted above, but without substantially altering the admirable agreement between theory and experiment exhibited in the figures quoted.

The classification of seismic waves into the three groups mentioned above corresponds to a first examination of seismograms. A closer analysis of the latter, and also certain theoretic studies of which we shall speak later, have led us to distinguish in these three fundamental groups of waves sub-groups to which special maximums of amplitude, of periods of vibration, and of rates of propagation correspond. The values deduced from observations made during the great earthquake at Messina in 1908 have given us rates of propagation for the group of long waves comprised between 3.2 and 4.7 kilometers per second, and by means of very close study Prof. Rizzo has been able to distinguish three sub-groups, whose rates of propagation are comprised respectively between 4.2 and 4.7 for the first group, 2.6 and 4.0 for the second group, and 3.2 and 3.5 for the third group.

Various hypotheses may be formulated to explain these facts. Prof. De Marchi has sought the explanation in a more general theory of Rayleigh's waves which propagate themselves in a plane stratum of limited thickness rather than in an infinite depth of ground [plutôt que dans un sol infiniment profond]. This is in accordance with modern views which regard the Earth as being formed of an internal nucleus which is mainly metallic in nature surrounded by a rocky envelope.

But it is possible, even without abandoning the initial concept of Lord Rayleigh, to give to the facts above mentioned, by means of a profounder study of the problem, an explanation which will not depart from the somewhat elementary class of ideas upon which the theories thus far propounded are based.

The determination of the rate of propagation of Rayleigh's waves at the surface of the ground depends upon an algebraic equation of the third degree, which in ordinary conditions has three real roots. The smallest of these roots gives the ratio 0.9194 which enables us, as we have seen, to deduce the rate of speed V_3 of Rayleigh's waves from the rate V_2 of the transverse waves. The two other roots are of no importance with respect to the problem stated by Rayleigh.

But if we examine from a general point of view the problem of the propagation of plane waves in an unlimited depth of ground, we find that it permits of three solutions, one of which is that of Rayleigh; this is a somewhat singular solution by reason of the fact that the wave planes are reduced to simple straight lines. The two others are composed of couples of ordinary wave planes, one longitudinal and the other transverse, whose directions of propagation in space are so inclined with respect to each other that their rate of propagation at the surface of the ground becomes identical. The rate of propagation of each of these couples is exactly determined by the two larger roots of the equation of the third degree which the first theory neglects. Thus the hypothesis appears plausible which explains the three sub-groups of long waves by assuming that they represent the three solutions of the problem of plane waves in the ground, in whose discovery the foregoing analysis was of assistance, and which comprise the result obtained by Lord Rayleigh. This view is supported by figures. The theoretical value of the ratio of the rates of propagation of the first and the second sub-groups is found to be 1:12; the calculations made by Prof. Rizzo in the Messina earthquake indicate that this ratio is comprised between 1:12 and 1:20, these figures being the result of 22 determinations. This explains the plausibility

of the hypothesis that the three systems of waves indicated by Rayleigh's problem actually exist in the group of long waves if we take into consideration the sub-groups discovered by observation.

Another hypothesis of wider scope may be set up to interpret the results of the preceding analysis, it consists in the comparison of the three fundamental systems of primary, secondary, and long waves to the three systems of associated wave-couples which are obtained as solutions of the problem of plane waves in the ground. In order to make this theory agree with experimental results we must assume that Poisson's ratio has a different value for the planet Earth from that which it possesses for the superficial materials which form the subject of our direct examination—an assumption which is entirely permissible. But if we calculate this value we find that $\sigma = -0.25$, i. e., a negative value; but while the laws of mechanics do not exclude the possibility of this, since it is contained within the limits assigned by the said laws to this coefficient, it has so far not been found by experiment to belong to any known body.

This second hypothesis, which would have the advantage of uniting under a single concept and in a single analytical problem the general problem of seismic waves, and which encounters no difficulties of a mechanical nature, thus confronts numerical values which conform but poorly to known physical data. This leads us to prefer the first hypothesis.

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The mechanical solutions of the seismic problem thus far examined are based upon mechanical concepts of the simplest sort and relate rather to an external representation than to an intrinsic explanation of the problem.

As a matter of fact the theories thus far examined consider a few special cases of vibration discovered by the authorities on mechanics and compare the said cases, in various fashion, with the different kinds of waves distinguished by observation. But from the physical point of view the phenomenon is a single one, and this separation into independent problems appears somewhat arbitrary.

Furthermore a complete theory ought to explain to us how the various classes of waves observed in a seismogram can be derived from a single cause of disturbance.

To Prof. Horace Lamb belongs the credit of having first attacked this problem, in 1904, with breadth in his mechanical concepts and genius in his methods of analysis. He made a study of the propagation, in an unlimited ground plane of the disturbance produced by a superficial impulse which he supposed to be located along a straight line or at a point. This disturbance is naturally propagated first as a rectilinear disturbance, and secondly in the form of concentric circles. The analytical basis of this solution is still Rayleigh's solution, but very ingeniously extended and amplified. The most remarkable result obtained by Lamb is to the effect that the movement which is propagated on the surface is characterized by three *salients*, [saillants] which travel respectively with the speed of longitudinal waves, transverse waves, and Rayleigh's waves. We thus obtain an exact reproduction by a synthetic method of the fundamental characteristics of the observed facts.

However, certain details of the representation are insufficient, e. g. these waves of Lamb are not periodic in character, and, like some of those considered above, they do not comprise vibratory displacements in a direction normal to that of propagation; yet such displacements do actually exist. This does not lessen their importance, however, which is all the more notable from the mechanical point of view, since they afford an explanation of the fundamental fact of the formation of the three waves, while other theories introduce this fact by way of a hypothesis (even though this may be justified by general considerations).

Following a very different class of ideas, Prof. A. E. H. Love of Oxford has come much nearer to the real nature of the observed facts in his penetrating study of numerous special problems connected with vibrations propagated in a homogeneous sphere or in its superficial strata, either in an indefinite ground plane or in one limited by another deep plane. The vibrations examined are harmonic, of arbitrary frequency, and correspond, therefore, to the vibratory character of seismograms.

Let us recall briefly some of the results obtained. In a homogeneous planet the action of the mutual gravitation of the masses which compose it causes no change in the laws of propagation of equivoluminal waves, laws which are obtained without taking gravity into consideration; expansional waves, on the contrary, are modified. These waves, in a similar case, cannot exist in the pure state, but are accompanied by

a slight rotation of the vibrating particles. Moreover their rate of propagation is not constant, but varies in space, and depends furthermore upon the length of the wave; the shorter waves travel faster than the longer ones, and because of this there are dispersion phenomena. These results accord with the fact, observed in preliminary tremors, that the intervals between the successive maxima tend to constantly increase.

In conformity with the generally accepted hypothesis of the existence of a superficial terrestrial crust having physical properties notably different from those of the subjacent masses, Prof. Love studied the possibility of the propagation, in a plane stratum, of waves for which the horizontal component of the displacement would be either normal or parallel to the direction of propagation. And he demonstrated the possibility of these two types of waves when certain conditions are realized with regard to the physical constants of the stratum and of the subjacent mass. This enabled him to deduce certain interesting conclusions concerning the constitution of the seismic waves of the third phase, the long waves. According to this theory the latter are composed of two types of waves, each constituted by an aggregate of simple harmonics, combined in such a manner as to form a progressive wave, with periods dependent upon the length of the wave. The horizontal displacement of those of the first type is transverse, and they penetrate very slightly into the interior of the terrestrial mass. The others are analogous to Rayleigh's waves, and differ therefrom only through the modifications produced by gravity; consequently they have a horizontal displacement parallel to the direction of propagation. The first type travels faster than the second; hence it follows that the transverse movements must predominate in the first instants of the long waves, while the longitudinal movements will predominate later, on the contrary. Facts of this nature are fully confirmed by experiment.

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While these researches, based upon the broadest and most comprehensive statement of the problem, and upon the certain foundation of rational mechanics, open the way to a profounder study of the propagation of seismic waves, the most recent and striking results of theoretic seismology are to be found elsewhere.

It is the theory of what have been called *seismic radiations* [rayons] which best affords us the hope of penetrating the less accessible sides of the phenomenon and deriving therefrom those ideas concerning the constitution of the interior of the Earth which form perhaps the most attractive object of such researches.

The ideas which underlie the theory of seismic radiations are less solid and certain than those made use of in seismic mechanics. It is no longer differential equations of movement which form the point of departure, but certain principles of optical physics, which are extended, *by analogy*, to seismic waves. These are: Fermat's principle of the minimum time, the concept of the radiation considered geometrically as the orthogonal trajectory of the successive surfaces of propagation of the disturbance, the principle of Huygens of the formation of the surface of the wave as an envelop of spherical waves, and in a general manner the hypothesis of a variation of the physical properties in the interior of the Earth according to concentric spheres. These principles, and the methods of research derived from them, are chiefly applied to the waves of the preliminary tremors, and the legitimacy of such an application may be regarded as being based upon the fact that waves of short duration are concerned, which constitute, as we are in the habit of saying, isolated series [trains] of waves which rapidly disappear. For if this phenomenon were more permanent in character the applicability of these principles would be dubious. We must remember, in fact, that the principle of Huygens presupposes that the wave length is very small, while the length of seismic waves may be reckoned in tens of kilometers. Furthermore, if we desired to give to these radiations [rayons] a mechanical, instead of a purely geometrical, definition, it would be proper to apply the ideas of Poynting and of Kirchhoff, and consider them as lines of the propagation of energy, and in this case the validity of the principles referred to above is far from being proven.

However, many results well worth attention have been obtained by the said method, which is probably justified only by the instantaneous character of the phenomenon; while, on the other hand, the methods of mechanical propagation which are applicable to waves of the final phase are perhaps too heavy and complicated for those of the first two phases.

Thanks to the principles of optical geometry, carried over into the seismic domain, it has been possible to obtain the equation of the curve of the seismic radiation

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