The work on d'Arsonval galvanometers is being continued with the intention of getting out an instrument of this type which shall have an extremely high sensibility, rivalling that of the Thomson square pattern galvanometer. The results thus far are very encouraging, and within a few months you may hope to see this instrument in tangible form.

ERRORS IN THE DETERMINATION OF AREAS FROM MEASURED DIAMETERS.

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In scientific determinations made at the present day, the accuracy of the determination is the paramount question. The problem is not to measure a length to '01 millimetre or a resistance to '0001 of an ohm, but to determine a length or a resistance with an accuracy of '01 or '001 of one per cent. Little improvements often increase accuracy to a marked extent. To one of these little wrinkles, I would call attention to-night.

In some recent work in the measurement of conductivity at Queen's Laboratory at Ardmore, I had occasion to study the methods of determining the cross section of the specimens in question. The area could be determined in two ways: by calculation from the volume obtained by the determination of the mass and specific gravity and by calculation from a directly measured diameter. The latter method is of course the simpler and where a good chemical balance is absent the only one to be employed. To call attention to a few points to be observed in this latter method is the object of this paper.

Let δ equal the true diameter of any geometrical cross section, the area of which is to be determined.

Then $k\delta^2$ can be taken as the area of that cross section, k being a constant varying from

$$1 \text{ to } \frac{\pi}{4}$$

for all figures from the square to the circle.

In measuring a diameter by micrometer calipers two things may occur: the diameter may be measured too large by slightly twisting the calipers or by the presence of bends, or made too small by crushing the surfaces measured. Let the error thus made be called a . Then the measured diameter is

$$(\delta \pm a) \quad (1)$$

and the area becomes

$$k (\delta \pm a)^2 \quad (2)$$

the true area being

$$k \delta^2 \quad (3)$$

and, therefore, the error is

$$k \delta^2 - k (\delta \pm a)^2 \quad (4)$$

and the percentage of error is

$$\frac{k \delta^2 - k (\delta \pm a)^2}{k \delta^2} \quad (5)$$

Simplifying expression (5) we obtain

$$\frac{\delta^2 - (\delta \pm a)^2}{\delta^2}$$

and

$$\left. \begin{aligned} \therefore \quad & \frac{\delta^2 - \delta^2 - 2 a \delta - a^2}{\delta^2} \\ & \frac{\delta^2 - \delta^2 + 2 a \delta - a^2}{\delta^2} \end{aligned} \right\} \quad (6)$$

or

$$\frac{2 a \delta - a^2}{\delta^2} \quad (7)$$

$$\frac{2 a \delta + a^2}{\delta^2} \quad (8)$$

From expression (7) will be seen that when the diameter is measured too large, then the percentage of error in measuring the area is

$$\frac{2 a \delta + a^2}{\delta^2} \quad (7)$$

and the percentage of error when the diameter is measured too small

$$\frac{2 a \delta - a^2}{\delta^2} \quad (8)$$

A comparison and discussion of expressions (7) and (8) bring out several interesting points.

The percentage of error in measuring the diameter is

$$\frac{a}{\delta}$$

in measuring the area the maximum error is

$$\frac{2 a \delta + a^2}{\delta^2}$$

the ratio of the two errors is, therefore,

$$\frac{2 a \delta + a^2}{\delta^2} \times \frac{\delta}{a} = \frac{2 \delta + a}{\delta} \quad (10)$$

Expression (10) may be given the form

$$2 + \frac{a}{\delta} \quad (11)$$

Expression (11) shows us two interesting facts. First, when the diameter is measured greater than it really is then the minimum ratio of the error of area to diameter is 2, and the maximum ratio is 3. a is generally small compared with δ . This brings out the fact that the common opinion that the error in the area is equal to the square of the error in the diameter is erroneous. Expression (11) shows that the actual error is never far from double the error of the diameter.

Hitherto we have been considering the case when the measured diameter is greater than the actual. Let us consider expression (8) in which the measured diameter (due say to crushing) is less than the actual diameter. In this case we have the percentage of error as

$$\frac{2 a \delta - a^2}{\delta^2}$$

If we divide this by

$$\frac{a}{\delta}$$

we obtain as before the ratio of the error of area to the error of the diameter.

$$\frac{2 a \delta - a^2}{\delta^2} \times \frac{\delta}{a} = \frac{2 \delta - a}{\delta} = 2 - \frac{a}{\delta} \quad (12)$$

From expression (12) we see, as before, that the ratio is not far from 2, but is always less than 2.

A comparison of expressions (7) and (8) will bring out an interesting fact. From (7) we have

$$\frac{2 a \delta + a^2}{\delta^2}$$

from (8) we have

$$\frac{2 a \delta - a^2}{\delta^2}$$

the difference is therefore

$$\frac{2 a^2}{\delta^2} \quad (13)$$

In other words, admitting the same error, we gain

$$\frac{2 a^2}{d^2}$$

by crushing the wire by an amount a , to having the calipers too loose by the same amount a .

The same thing can be shown more clearly graphically. Let us take the special case of a square, as that permits of simpler demonstration. Three squares are shown, $A B$

C ; B is the correct area we desire to measure; C is the measured area when the calipers are too loose; A when the calipers crush the surface to be measured. The diameter of C is $B + a$. Of A is $B - a$. The errors in measuring the areas are the bands between A and B and between B and C . An inspection of the diagram shows that the band $ghmn$ is balanced by the band $hn oi$; the corner $fjhg$ by $jkhi$; but the part $elfk$ is unbalanced. Similarly for each corner we have a part equal to $elfk$; therefore, the total area between B and C exceeds the total area between A and B by $4elfk$. But $elfk$ has the dimensions $\frac{1}{2}a \times a$ therefore

$$4elfk = 4\left(\frac{1}{2}a \times a\right) = 2a^2$$

or the error in measuring the area as C is $2a^2$ greater than in measuring it as A ; the error percentage gained being

$$\frac{2a^2}{d^2}$$

as shown above. Summing up the foregoing it follows that it is always an advantage to screw up the calipers quite tightly as even if the surface is slightly crushed something

$$\left(\frac{2a^2}{d^2}\right)$$

is gained over having the calipers too loose by the same amount a . This rule I have found especially advantageous to use when measuring the conductivity or specific resistance of wire. The surfaces to be measured are in this case generally of sufficiently hard material to prevent any serious crushing by screwing up the calipers tightly. On the other hand, there is a strong probability that if this is not done the kinks and bends which will exist, no matter how carefully the wire be straightened, will increase the apparent diameter. The usual precautions should of course be observed; *e. g.*, the determination of the mean diameter by reading along the whole length of the specimen under examination, test of the zero, etc. By observing these precautions the probable error can be reduced so as not to exceed the errors made in the measurement of the resistance.