have been discoverable in the part AA'. However, the chlorine spectrum was easily obtained from AA'; and on comparison with BB' the intensity of the chlorine lines was found to be the same in both cases. The tubes were fused on again and the experiment repeated, making A the anode, when the same result was obtained.

This proves that if any convection of the chlorine does take place, it is to an extremely small extent, and is certainly not sufficient to account for the effect observed.

To Prof. H. Kayser I wish to acknowledge indebtedness.

Physical Institute, Bonn,
February 1900.

XXVI. *On some Novel Thermo-Electric Phenomena.* By
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in the Royal College of Science for Ireland**.

IN the course of a determination of some of the physical properties possessed by various new alloys of iron, which had been prepared by Mr. R. A. Hadfield, of Sheffield, I found the thermo-electric behaviour of a particular nickel steel, to which five per cent. of manganese had been added, to be so remarkable that it seemed worthy of a separate note.

The analysis of this alloy (kindly supplied to me by Mr. Hadfield) was as follows :—

Iron	68·8 per cent.
Nickel.....	25·0 "
Manganese.....	5·0 "
Carbon	1·2 "

The specific electrical resistance of this alloy was found by Mr. Brown and myself to be higher than that of any other alloy we had hitherto examined †. It amounted to no less than 97·52 microhms per cub. centim. (at 15° C.). At the same time its variation of resistance with change of temperature was comparatively small, the temperature-coefficient being 0·08 per cent. per degree C. (between 0° and 250° C.).

The thermo-electric behaviour of the various alloys of iron which Mr. Hadfield had kindly placed in my possession was in course of investigation, and the enormous electrical resistance of this specimen, and of another similar alloy with somewhat less nickel, led me to try earlier than I should otherwise have done some of its other physical properties.

A preliminary experiment was made by coupling a wire of this new alloy successively with different metals and testing

* Communicated by Prof. G. F. FitzGerald, F.R.S. From a separate impression of the *Trans. Roy. Dublin Soc.*, vol. vii. pt. v.

† See *Trans. R.D.S.* vol. vii. pt. 4.

the thermo-electric power of the various junctions up to a red heat. When an iron wire was used as the second metal the thermo-electric force quickly rose till a certain temperature much below redness was reached, and then remained almost stationary, notwithstanding that the temperature of the junction rose from a black up to a bright white heat; the cool junctions being throughout kept in water at the temperature of the room, or in ice. Here, then, we have the change in the thermo-electric force arrested when a certain temperature is reached, and the force remaining nearly the same in spite of the increasing difference of temperature between the cool and hot junctions. We know that so long as the *difference* of temperature between the cool and hot junctions in most thermo-couples remains constant, and the circuit is unchanged, the potential-difference also remains constant; but if the difference of temperature between the cool and hot junctions alters, then the potential-difference, as a rule, also changes; and for *small* changes of temperature the electromotive force thus set up is, in most cases, proportional to the change of temperature. This, of course, is the principle upon which the thermo-electric pile is used as a delicate thermometer; and with certain alloys this proportionality holds good through a wide range of temperature.

In order to measure the exact temperature of the hot junction, a thermo-electric couple, formed of a platinum wire twisted with a wire of an alloy of platinum containing 10 per cent. of rhodium, was employed. The vapour of boiling water (100°C.) and of boiling sulphur (445°C.) are the most convenient and reliable fixed points for plotting the lower part of the scale*. For the higher parts I have used the freezing-point of pure silver (961°C.), and of potassium sulphate (1066°C.)†.

A reflecting galvanometer was employed, a dead beat high-resistance instrument, of the D'Arsonval type, made by Ducretet, of Paris. Owing to its high resistance the alterations in the resistance of the circuit during the heating and cooling of the couple introduced no sensible error, the deflexions being proportional to the E.M.F. and not to the current.

The value of the scale-reading of the galvanometer in

* The thermo-couple must, of course, be protected from the sulphur by being enclosed in a hard-glass tube. To obtain the boiling-point of sulphur it is best to employ a hard-glass flask with a long neck, all except the lower portion of the flask being jacketed with asbestos. A convenient arrangement is supplied by the Cambridge Instrument Company; this I used.

† Heycock and Neville, *Trans. Chem. Soc.* 1895.

microvolts was accurately determined by means of a standard cell and potentiometer, one scale-division being found to be equal to 26·5 microvolts. With a similar couple Professor Callendar found that the E.M.F. in microvolts at three points ought to be as follows :—

100° C.	.	.	650 microvolts
445°	.	.	3630 „
1000°	.	.	9550 „

the other junctions being in ice. My own determinations at 100° C. and 445° C. gave almost identically the same values in microvolts ; this may be accidental, but the variations of thermo-electric power in different specimens of platinum is probably very slight. I therefore took Professor Callendar's value in microvolts for 1000° C. in preference to the rather different number I obtained*. The results are plotted in the curve shown in the figure, marked "pyrometer couple."

We will now return to the thermo-electric behaviour of this nickel-manganese-iron alloy. An iron wire, drawn from the purest commercial iron, was coupled with a wire of the alloy. The wires were twisted together at the junction and then brazed. After insulation with asbestos they were lashed alongside of the platinum-rhodium pyrometer couple, and the pair of couples was then inserted in the centre of a thick iron tube held horizontally in, and heated by, a gas furnace, the temperature of which could be raised to a white heat. The cooler junctions of both couples were kept in ice, and pairs of readings were taken as the temperature was raised, and again as it was lowered ; the rise and fall being slow and steady in both cases. As the readings of each couple were taken alternately, to obtain a true comparison three readings were necessary in each case : first, of one couple A ; then, of the other B ; then, of A again ; the mean of the first and last readings of A being comparable with B. The galvanometer being extremely dead-beat, all three readings could be taken closely together. This comparison was repeated two or three times every minute during heating and cooling, thus several series of readings for about every 10° C. rise or fall of temperature were obtained ; the readings which corresponded to similar temperatures in heating and cooling being remarkably concordant, allowance being made for a very small constant difference to be mentioned in the sequel.

* As (within the limits of the scale) the E.M.F. of the thermo-couple is directly proportional to the scale-readings of the galvanometer, it is easy, when the E.M.F. in microvolts at 100° and 445° C. is found, to determine the higher points in the curve by means of the ratio given above : the scale-readings being plotted as ordinates and the temperatures as abscissæ.

Thermo-electric Curves of Hadfield's Nickel-Manganese-Steel Alloy.

The results are plotted in the second curve shown on the figure. It will be seen that, up to a temperature of $320^{\circ}\text{C}.$, the E.M.F. of the nickel-steel alloy and iron couple rose rapidly; it then remained absolutely constant until the temperature rose to 500° , and after this, only a small change occurred up to the highest temperature attainable in the gas furnace employed. The mean E.M.F. between 300° and $1000^{\circ}\text{C}.$ was in round numbers 4000 microvolts, and the extreme variations from the mean E.M.F. throughout this range of $700^{\circ}\text{C}.$ —that is, from a low black heat to a white heat—were less than 170 microvolts, or about $+4$ and -4 per cent. of the E.M.F. at $300^{\circ}\text{C}.$, the cooler junction being kept at $0^{\circ}\text{C}.$ It will be noticed that there is a slight and curious oscillation of the E.M.F. about the mean line between 300° and $1100^{\circ}\text{C}.$, the curve cutting the mean E.M.F. at four points, viz., at 310° , 540° , 810° , and $1030^{\circ}\text{C}.$

The couple was next exposed to a very low temperature obtained from solid CO_2 , but though a temperature of $-80^{\circ}\text{C}.$ was reached (the other junction being still kept in melting ice), no other anomalous action occurred, the reverse E.M.F. increasing rapidly as the temperature fell. On again raising the temperature to a white heat, the phenomena previously observed were exactly reproduced, and continued to be so on repeated heating and cooling.

In place of nearly pure iron, other substances were tried as the second metal in conjunction with this alloy. Ordinary commercial iron wire gave a very similar result, the oscillations from the mean E.M.F. being slightly greater. With an ordinary mild steel wire as the second metal, the nickel manganese alloy now gave a different result; this is shown on the lower, dotted curve in the figure. The E.M.F. is less, remains constant only between 400° and $600^{\circ}\text{C}.$, then begins to fall and continues falling slowly to the highest temperature reached, over $1000^{\circ}\text{C}.$

Platinum, copper, and other metals were also tried in conjunction with this alloy, but in no case was there observed the singular constancy of E.M.F. through a wide range of temperature which occurred when iron was the second metal. With platinum the E.M.F. has an opposite sign (to that occurring when the alloy is coupled with iron) up to a temperature of $210^{\circ}\text{C}.$; inversion takes place at this temperature, after which the E.M.F. rapidly rises with increasing temperature in an approximately parabolic curve.

A small thermo-electric battery was formed of strips of this alloy with strips of iron, the strips being insulated by asbestos, and brazed at their junctions; 25 of these couples

give an E.M.F. of $\frac{1}{10}$ of a volt when heated over any flame, the cooler junctions being kept in ice-cold water. A convenient standard of E.M.F. might thus be made if the means of the readings between 300° and 1000° C. were taken. Whether repeated heating and cooling of the alloy will affect its E.M.F. I cannot say, but I have not yet observed any injury resulting from this cause.

We know so little of the whole subject of thermo-electricity, that the explanation of the remarkable behaviour of this alloy can only be a matter for conjecture. Some light may be thrown on it by the results obtained from the other alloys of iron, when their thermo-electric behaviour is examined. So far I have found that reducing the nickel in the alloy from 25 to 19 per cent., the other constituents remaining the same, does not destroy the sudden arrest of E.M.F. at about 300° C.; but the range of temperature where the E.M.F. is nearly constant is less, extending from about 400° to 750° C.

It has been suggested that the peculiar thermo-electric property of this alloy may be connected with the effect observed by Lord Kelvin, the so-called Thomson effect, whereby a kind of electric convection of heat occurs. It is very possible that it may have some connexion with this, and hence with the *neutral point*, which occurs in the thermo-electric behaviour of certain pairs of metals, such as copper and iron. It is well-known that at a certain critical difference of temperature between the hot and cold junctions of, say, a copper-iron couple, the potential-difference due to heat disappears; as the temperature rises, inversion of the current occurs; and a second inversion may occur at a still higher temperature. In the phenomenon described in this paper, the E.M.F., it is true, does not fall to zero. If, however, the cooler junction were kept at a temperature of 310° C., this would be the case; and we should then have a series of three successive small inversions of E.M.F., occurring at 540° , 810° , and 1030° C. In a copper-iron couple the neutral point is 275° C.

I have noticed that the temperature of the neutral point in a copper-iron, or copper-steel couple, is not the same during the *heating* as during the *cooling* of the couple. Moreover, in a couple formed of copper and mild carbon steel, the neutral point becomes lower in successive heatings. Thus, at the *first* heating the temperature of inversion was approximately 640° C., and in cooling, 500° C.; in the *second* heating, 550° C., and in cooling, 465° C.; in the *third* heating, 520° C., and in cooling, 465° C.: the difference between the temperature of inversion in heating and cooling becoming less in each successive heating and cooling. The cool junc-

tion was here at 16°C . throughout ; the neutral points at the 1st, 2nd, and 3rd heatings and corresponding coolings of the couple, would therefore be approximately as follows :—

Neutral Points of Copper-steel Couple.

	First,	Second,	Third Heating.
When heating ...	328°	283°	268°C .
When cooling ...	258°	241°	241°C .

It is obvious therefore that the curve representing the thermo-electric force of a copper-steel couple at different temperatures is not the same for a rising as for a falling temperature. This I have found to be the case with couples formed of several other metals, provided one element of the couple is iron or steel or other alloy of iron. When the other metal is platinum, the difference in the two curves is well seen, though with a platinum-iron couple the difference of E.M.F. in heating and cooling is less marked than with a platinum-steel couple. In the latter case, a considerable area is enclosed by the curves (representing the relation between thermo-electric force and temperature) during heating and during cooling. Hence, at a given temperature, say 500°C ., of the hot junction, the E.M.F. of a platinum-steel couple is considerably *higher* during heating than during cooling.

The reverse is the case with a couple formed of Hadfield's nickel-manganese-steel and copper, or a couple formed of the same alloy with platinum ; in both these cases at any given temperature, the E.M.F. is *lower* during heating than during cooling. With a couple formed of the same alloy and iron, as described in the earlier part of this paper, there is also a slightly lower E.M.F. at corresponding temperatures during heating than during cooling ; but the difference only exists at certain parts of the scale, and is so small that it could not be shown on the curve as reduced in the text. With the nickel-manganese-steel alloy (containing 19 instead of 25 per cent. of nickel) coupled with iron, the E.M.F. is slightly *higher* at corresponding temperatures in heating than in cooling up to the level part of the curve, *i. e.* about 400°C ., where the E.M.F. becomes almost the same in heating as in cooling—very slightly lower, however,—and remains so until the curve rises, when the E.M.F. again becomes higher at corresponding temperatures in heating than in cooling up to the highest temperature reached : in this case, therefore, the curves showing the E.M.F. during heating and cooling cross each other twice, first at about 400° and next at about 800°C .

I hope, in a subsequent paper, to give the results of further investigation which I am pursuing on this interesting pheno

menon, together with the curves for the E.M.F. of various couples during heating and cooling—*thermo-electric hysteresis curves* as they may be called*. It is very probable that the peculiar thermo-electric deportment of iron, and some of the alloys of iron described in this paper, is intimately associated with the phenomenon of recalescence, or rather of the series of recalescent points which exist in iron and steel.

XXVII. *On Obliquely-crossed Cylindrical Lenses.*

By SILVANUS P. THOMPSON, *D.Sc., F.R.S.*†

CYLINDRICAL lenses have not claimed much attention from writers on geometrical optics. Certain of their properties, which make them invaluable to the ophthalmic surgeon for the correction of astigmatism, are, however, considered in all modern treatises on ophthalmics. Airy applied them for this purpose, but Donders first treated them systematically, in 1862, in his work *Astigmatismus und cylindrische Gläser*. Reusch in 1868 published his *Theorie der Cylinderlinsen*. Javal has written much on the subject from the ophthalmological point of view, as has Mr. S. M. Burnett. Mr. C. F. Prentice has also written of them in two works, a 'Treatise on Ophthalmic Lenses' and 'Dioptric Formulæ for combined Cylindrical Lenses.' Stokes‡ has proposed a cylindrical lens of variable power by combining at a variable angle two equal cylindrical lenses of opposite sign. The problem of the optical properties of crossed cylindrical lenses was touched upon by Reusch and by Donders. Krüss has written upon the aberration of "bicylindrical" lenses, meaning by that term such lenses as have both surfaces cylindrical and of equal curvature, but with their axes mutually at right-angles. Lastly, Prentice has given very elaborate rules for calculating the equivalent sphero-cylindrical lens for any combination of two lenses crossed at any angle. The importance of the solution of this problem arises from a point in modern ophthalmic practice:—In cases of ordinary astigmatism in which the refractive

* As was pointed out by Professor G. F. FitzGerald, F.T.C.D., F.R.S., at the meeting of the Society when this paper was read, the thermo-electric hysteresis here referred to is, no doubt, the cause of the thermocurrent which is produced in an iron wire steadily moved through a flame, a phenomenon first noticed and investigated by Dr. F. T. Trouton, F.R.S. See Proc. Roy. Dublin Society, March 1886. I am also greatly indebted to Professor FitzGerald for other suggestions he has made in reading the proof of this paper.

† Communicated by the Physical Society: read Dec. 8, 1899.

‡ Mathematical and Physical Papers, vol. ii. p. 172.