LIV. *On the Dissipation of Energy.* *By* Prof. P. G. Tait.

To Sir W. Thomson, F.R.S.

My dear Thomson,—

I ADDRESS you as one of the Editors of the Philosophical Magazine, but also specially as the first propounder of the doctrine of the Dissipation of Energy. I do so because Prof. Clausius, in the second part of the new edition of his work on Thermodynamics, has challenged your claim to the well-known expression for the amount of heat dissipated in a non-reversible cycle. I think that the time has come for you to speak out on the subject, so as, if possible, to prevent further unnecessary discussions.

I shall endeavour, so far as I can, to keep to matters of scientific importance; but I must introduce the subject by a reference to the comments made by Prof. Clausius upon a somewhat slipshod passage (§ 178) of my little work on 'Thermodynamics.' That passage refers to the integral

$$\int \frac{dq}{t},$$

to which I believe Rankine first called attention, but which is essentially connected with your doctrine.

I cannot altogether complain of Prof. Clausius's comments, because I cannot account for my having called the above integral (in the way in which I have employed it) a *positive* quantity, except by supposing that in the revision of the first proof of my book I had thoughtlessly changed the word "negative" to "positive." This might easily happen from my having used a novel term, "practical value," in a somewhat ambiguous manner, at one place confounding it with "realized value." That the whole section was meant to bear the construction forced on it by Prof. Clausius is, I think, sufficiently disproved by its opening sentence, not to speak of the fact that no one in this country has so interpreted it.

But there is a graver matter involved than any such mere slips of the pen; for Prof. Clausius asserts that the method I employ (and which I certainly obtained from your paper of 1852) is inapplicable to any but reversible cycles. This, I think, is equivalent to denying altogether your claims in the matter. I therefore quote the whole passage, correcting, however, the above-mentioned slip, and slightly extending the latter part to make my meaning perfectly clear.

"§ 178. The real dynamical value of a quantity, dq, of "heat is Jdq, whatever be the temperature of the body which

"contains it. But the extreme practical value is only

$$J\frac{t-t_0}{t}dq$$

"where t is the temperature of the body, and t_0 the lowest "available temperature. This value may be written in the "form

$$Jdq-Jt_0\frac{dq}{t}.$$

"Hence, in any cyclical process whatever, if q_1 be the whole "heat taken in, and q_0 that given out, the practical value is

$$J(q_1-q_0)-Jt_0\int\frac{dq}{t}.*$$

"Now the realized value is

$$J(q_1-q_0)$$

"by the first law; and if the cycle be *reversible*, this must be "equal to the extreme practical value. Hence, in this parti- "cular case,

$$\int\frac{dq}{t}=0.$$

"But in general this integral has a finite negative value, "because in non-reversible cycles the realized value of the heat "is always less than

$$J(q_1-q_0)-Jt_0\int\frac{dq}{t},$$

"which is the extreme practical value.

"Hence the amount of heat lost needlessly, *i. e.* rejected in "excess of what is necessarily rejected to the refrigerator for producing work, is

$$-t_0\int\frac{dq}{t}.$$

"This is Thomson's expression for the amount of heat *dissipated* "during the cycle (Phil. Mag. and Proc. R. S. E. 1852, 'On a "Universal Tendency in Nature to Dissipation of Energy'). "It is, of course, an immediate consequence of his important "formula for the work of a perfect engine.

"[It is very desirable to have a word to express the *availa-* "*bility* for work of the heat in a given magazine; a term for "that possession, the waste of which is called dissipation.]"

As I based the greater part of the last chapter of my work

* On this formula Prof. Clausius remarks, "Die Unrichtigkeit dieses "Resultates lässt sich leicht aus dem blossen Anblicke der Formel "erkennen"!

on your papers, mainly because they appeared to me to be greatly superior to all others on the subject in the three very important qualities of simplicity, conciseness, and freedom from hypothesis, I am anxious to know whether the above passage meets with your approval.

From Prof. Clausius's comments it appears, as I have already said, that he considers the method I have adopted from you to be one which cannot be applied except to *reversible* cycles, and which, therefore, it is absurd to employ in any argument connected with dissipation of energy.

Prof. Clausius also disputes the correctness of my reference to your paper in the Philosophical Magazine, as containing the above expression for the heat dissipated. You ought to be a competent authority on such a question as this.

I do not now reply to the many other remarks of Prof. Clausius, simply because they refer to myself, my motives, and my book, and not to the principles or the history of science. As the matter affects you, however, I may mention that Professor Clausius attributes to me the real authorship of the paper on "Energy" which we jointly wrote for 'Good Words,' and which has been often referred to in the Philosophical Magazine.

But the passage in brackets in the extract above indicates a want of proper nomenclature, which would, I think, be well met by the publication of the paper on Thermodynamic Motivity, read by you some years ago to the Royal Society of Edinburgh.

Yours truly,

38 George Square, Edinburgh,
March 17th, 1879.

P. G. Tait.

Note by Sir W. Thomson on the preceding Letter.

The passage quoted, with amendments, by Professor Tait from his 'Thermodynamics,' seems to me perfectly clear and accurate. Taken in connexion with the sections which preceded it in the original, its meaning was unmistakable; and a careful reader could have found little or no difficulty in making for himself the necessary corrections with which Professor Tait now presents it. It is certainly not confined to reversible cycles; but, on the contrary, it gives an explicit expression for the amount of energy dissipated, or, as I put it, "absolutely and irrecoverably wasted" in operations of an irreversible character. My original article "On a Universal Tendency in Nature to the Dissipation of Mechanical Energy,"

communicated to the Royal Society of Edinburgh in April 1852, and published in the 'Proceedings' of the Society for that date, and republished in the Philosophical Magazine for 1852, second half year, is a sufficient answer to the challenge referred to in the opening sentence of Professor Tait's letter.

I think Professor Tait quite right in referring also to that paper for the formula $t_0\int\frac{dq}{t}$. The whole matter is contained in the formula $we^{-\frac{1}{J}\int_T^S \mu dt}$, which is given explicitly in that paper. At the top of the next page in the Philosophical Magazine reprint the following passage occurs:—"If the system of thermometry adopted be such that $\mu=\frac{J}{t+\alpha}$, that is, if we agree to call $\frac{J}{\mu}-\alpha$ the *temperature* of a body, for which μ is the *value* of Carnot's function (α and J being constants), &c.;" and on the word "adopted" the following footnote is given: "According to Mayer's 'hypothesis' this system coincides "with that in which equal differences of temperature are defined "as those with which the same mass of air under constant "pressure has equal differences of volume, provided J be the "mechanical equivalent of the thermal unit, and $\frac{1}{\alpha}$ the coeffi-"cient of expansion of air." Here the true foundation of the absolute thermodynamic scale now universally adopted was, I believe, for the first time given. I had previously, in Part III. of my "Dynamical Theory of Heat," published in the Transactions of the Royal Society of Edinburgh, and in the Philosophical Magazine for 1852, second half-year, taking advantage of a suggestion made to me by Joule, in a letter of date December 9, 1848, shown that the assumption $\mu=\frac{J}{\alpha+t}$ reduces the formula $we^{-\frac{1}{J}\int_T^S \mu dt}$ to $w\frac{\alpha+T}{\alpha+S}$: and I used this transformation in the concluding formulas of the article referred to by Professor Tait (corrected in the *errata* of Phil. Mag. 1853, first half-year). It was not, however, until the experiments by Joule and myself, made in the course of the years 1852, 1853, and the early part of 1854, on the thermal effects of forcing air and other gases through porous plugs, had proved that my proposed thermodynamic scale agreed as nearly with the scale of an air-thermometer as different air-thermometers agreed with one another, that I definitively adopted it in fundamental formulas of thermodynamics. Thus, for example, in Part VI. ("Thermo-electric Currents") of my "Dynamical

Theory of Heat," published in the Transactions of the Royal Society of Edinburgh for May 1854, and in the Philosophical Magazine for 1855, first half-year, the formula

$$\frac{H_t}{t}+\frac{H_{t'}}{t'}+\ldots+\frac{H_t^{(n-1)}}{t^{(n-1)}}+\frac{H_t^{(n)}}{t^{(n)}}=0$$

is given as an equivalent for

$$\Sigma\alpha_t=\Sigma\alpha_t(1-\epsilon^{-\frac{1}{J}\int_T^t \mu dt}),$$

which was first published in the Proceedings of the Royal Society of Edinburgh for 1851, and Phil. Mag. 1852, first half-year. Tait had actually quoted the formula from my 1854 paper in § 176 of his book, and so left absolutely no foundation for Professor Clausius' objection to his saying "This is Thomson's expression &c.," quoted in his letter above.

As to the 'Good Words' article on Energy which appeared under our joint names, Professor Tait and I are equally responsible for its contents. I claim my full share of the "scientific patriotism" commended in that article, and cannot assent to Professor Clausius' giving all the credit of it to Professor Tait.

In compliance with the concluding sentence of Professor Tait's letter, I hope in the course of a few days to write out, and send to the Philosophical Magazine for publication, a short statement of the communication on Thermodynamic Motivity which I made *vivâ voce* to the Royal Society of Edinburgh on April 3rd, 1876.

LV. *On Thermodynamic Motivity.* *By* Sir W. Thomson, *F.R.S.**

AFTER having for some years felt with Professor Tait the want of a word "to express the Availability for work "of the heat in a given magazine, a term for that possession "the waste of which is called Dissipation"†, I suggested three years ago the word *Motivity* to supply this want, and made a verbal communication to the Royal Society of Edinburgh defining and illustrating the application of the word; but as the communication was not given in writing, only the title of the paper, "Thermodynamic Motivity," was published. In consequence of Professor Tait's letter to me, published in the present Number of the Philosophical Magazine, I now offer,

* Communicated by the Author.

† Tait's 'Thermodynamics,' first edition (1868), § 178: quoted also in Professor Tait's letter in the present Number of the Philosophical Magazine.