No. 1,585.—“On the Strength of Flat Plates and Segmental Ends of Boilers and other Cylinders.” By DANIEL KINNEAR CLARK, M. Inst. C.E.

PART I.

THE end of a cylindrical boiler, consisting of a perfectly flat unstayed plate of uniform thickness, springs and bulges outwards when it is subjected to internal pressure. The plate is extended radially under tensile stress, and the resistance to radial extension is supplied, for the most part, by the circumferential compression of the plate at right angles to the radius. The form of the cavity is spheroidal rather than spherical, the shortest radius of curvature being at the circumference. But, for the purpose of calculation, it may be assumed to be spherical. The line of ultimate fracture is not diametrical or in the line of a great circle of the sphere; it is circumferential, that is, at right angles to the radius, at the outer circumference, in the line of the fastening,—where failures in fact do take place.

These two stresses in the end-plate—the radial-tensional, and the circumferential-compressive—are balanced by longitudinal stress in the body of the boiler; which, for each inch lineally at the circumference, is measured by half the product of the internal pressure in lbs. per square inch by the radius of the cylinder in inches. The total pressure, in lbs. per square inch on the end of the boiler, is equal to the product of the sectional area of the cylinder by the pressure, or to $\cdot 7854 d^2 \times p$; in which d = the internal diameter in inches, and p = the internal pressure in lbs. per square inch.

For the radial stress, let $a d c$ (Fig. 1) be a transverse section of the flat end-plate of an iron cylindrical boiler $A B$, and let it be exposed to the steam pressure $p p p$. It will be deflected outwards according to the spherical surface $a d' c$. When the plate is stretched to the elastic limit of length, which for iron of average quality is 1-1000th part more than the normal length at rest, the length of the circular curve $a d' c$ is $1 \frac{1}{1000}$ time, or 1.001 time that of the straight line $a c$.

The semi-curve $a d'$ is sensibly equal to the chord of $a d'$, and the length of the deflection $d d'$ is deducible from the known geometrical equation, $(a d')^2 = (a d)^2 + (d d')^2$. Since $a d' = 1.001 a d$,

then, by substitution and reduction, $(d d')^2 = .002 (a d)^2 = \frac{1}{500}$ (radius of plate)². Taking the square root of both sides of the

FIG. 1.

equation, $(d d') = .04472 (a d) = \frac{(a d)}{22.36}$. Dropping the fraction, the formula becomes—

Ultimate elastic deflection of end-plate.

$$\delta = \frac{\text{radius}}{22} = \frac{r}{22} \quad \dots \dots \dots (1);$$

$$\delta = \frac{\text{diameter}}{44} = \frac{d}{44} \quad \dots \dots \dots (2);$$

in which δ is the deflection, r is the radius, and d is the diameter, all in inches, measured to the circular line of fastening, which is supposed at present to be at the edge of the cylindrical shell. By the properties of the circle—

$$(a d)^2 = (d d') \times (D - (d d')) \quad \dots \dots \dots (a);$$

D being the diameter of the sphere of which the bulged plate forms a segment. The deflection $d d'$ is very small with respect to the diameter D ; and it may be taken that $(a d)^2 = (d d') \times D$; whence—

Diameter of the segmental sphere.

$$D = \frac{(a d)^2}{(d d')} = \frac{r^2}{\delta} \quad \dots \dots \dots (3);$$

where r = the radius $a d$ of the circular junction in inches, or of the cylinder, when not otherwise described;

d = the diameter $a c$ of the circular junction, in inches;

δ = the deflection $d d'$, in inches;

D = the diameter of the sphere, in inches;

r' = the radius of the sphere, in inches.

Substituting for $d d'$ its special value, $\cdot 04472(ad)$, the radial extension became equal to 1-1,000th—

$$D = \frac{(ad)^2}{\cdot 04472(ad)} = \frac{ad}{\cdot 04472} = 22\cdot 36 \times (ad).$$

Dropping the decimal fraction—

Diameter and radius of the segmental sphere at the elastic limit.

$$D = 22(ad) = 22r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4);$$

$$D = 11(ac) = 11d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5);$$

$$r' = 11r = 5\cdot 5d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

At the limit of elastic tension, the tensile stress on iron of average quality amounts to 12 tons per square inch. Now a hollow sphere, like a hollow cylinder strained longitudinally, is capable of resisting twice the bursting pressure per square inch that would rupture a hollow cylinder transversely of the same diameter. Doubling the co-efficient in the usual formula for the bursting pressure of a hollow cylinder, it becomes—

$$p = \frac{4,480 ts \times 2}{D} = \frac{8,960 ts}{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

This formula may be adapted directly for a flat end-plate, strained by internal pressure to the elastic limit, by substituting for D its value when the radial extension of the plate is 1-1,000th, namely $11d$ —

Relative internal pressure and stress in the end-plate strained to the elastic limit.

$$p = \frac{8,960 ts}{11d} = \frac{815 ts}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8);$$

where p = the internal pressure in lbs. per square inch ;

t = the thickness of the plate in inches ;

s = the tensile stress in the plate in tons per square inch at the elastic limit.

This equation is good for steel plates, as well as for iron plates, since the elastic limit of strain is the same for both metals, namely, 1-1,000th of the length. The values of s , the elastic strengths, are—

For iron, 12 tons per square inch,
 „ steel, 14- „ „

Substitute these values for s in the formula (8), and the following formulas are obtained:—

Elastic strength of circular flat-end plates of iron and of steel, of uniform thickness, fastened at the circumference, exposed to bulging pressure uniformly distributed.

$$\text{For iron, } p = 10,000 \frac{t}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9);$$

$$\text{For steel, } p = 11,500 \frac{t}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10);$$

p = the bulging pressure in lbs. per square inch;

t = the thickness of the plate, in inches;

d = the diameter of the plate, measured to the circular line of junction.

Since the end-plate is thrown by deflection into a state of tension throughout its thickness, the resistance to bursting pressure follows the ratio of the thickness simply.

To find the magnitude of the second stress—the circumferential-compressive stress—let $a d' c$, Figs. 2, be the deflected plate, forming a segment of a sphere, and draw the radii $a b$, $c b$. Through the point c draw the tangent $e' c e$, produce $a c$ to f , and construct the parallelogram $c f e g$. Then $c e$ is the radial tensional force in the spherical segment at c , and it is resolved into $c g$ and $c f$, of which $c g$ is the longitudinal resistance of the body of the boiler to the longitudinal action of the radial force $c e$, and $c f$ is the compressive resistance at c to the centripetal action of the radial force $c e$. These forces are likewise represented by the sides of the similar triangle $c d e'$, of which $c e'$ is the radial force, $c d$ the compressive stress, and $d e'$ the longitudinal stress; or by the similar triangle $c d b$, of which $c b$ measures the radial force at c , $c d$ the longitudinal stress, and $d b$ the compressive stress. Now, the actual value

of the longitudinal stress in the side A, per inch-lineal along the upper edge at c , is, as before stated, equal to $\frac{(cd) \times p}{2}$, in which cd , the radius, is expressed in inches, and p is the internal pressure in

FIGS. 2.

lbs. per square inch; and the compressive stress, correspondingly, is equal to $\frac{(db) \times p}{2}$. Now $db = \sqrt{(cb)^2 - (cd)^2}$; and the compressive

stress per lineal inch at the junction c , is equal to $\frac{p}{2} \sqrt{(cb)^2 - (cd)^2}$.

The total compressive stress may then be calculated for any diametrical section ac , Fig. 2, in plan, where the stress is collected, acting tangentially, at a and c . At each of these points, the total

stress is equal to the stress per lineal inch multiplied by the radius $a d$ or $c d$, or to $\left(\frac{p}{2} \times (c d) \times \sqrt{(c b)^2 - (c d)^2}\right)$

This product, it may be observed, is obtained on the same principle as that on which the reverse, or bursting pressure, was calculated; though, in this case, the diameter was employed as the multiplier. Finally,

The compressive stress at the circumferential junction of a flat end plate.¹

$$c = \frac{p \times r \times \sqrt{r'^2 - r^2}}{2} \quad \dots \quad (11);$$

where c = the total compressive stress at the circular line of junction, in lbs.;

p = the internal pressure in lbs. per square inch;

r = the internal radius of the cylinder, in inches;

r' = the radius of the spherical end, in inches.

Though the compressive stress has been supposed, for the sake of illustration, to be concentrated in the line of junction between the end-plate and the cylinder, it is actually distributed over the whole of the diametrical section $a d c$, Fig. 2, in plan, for both sides of the boiler. But the distribution is not uniform: the stress is a maximum at the circular line of junction at a and c , and it obviously must vanish at the centre d , where the bulging, or deviation from a plane, is equal to nothing.

For the case in which the metal—iron or steel—of the flat end-plate, is strained radially to the limit of its elastic strength, the formula may be simplified by substituting for r' its elastic equivalent $11 r$, in formula (11), and reducing. Thus, the expression $\sqrt{r'^2 - r^2}$ becomes $\sqrt{(11 r)^2 - r^2} = \sqrt{(121 - 1) r^2} = \sqrt{120 r^2} = 10.95 \sqrt{r^2}$, or, say, $11 r$. Then—

Compressive stress at the circumferential junction of a flat end-plate when the radial strain reaches the elastic limit.

$$c = \frac{p \times r \times 11 r}{2}; \text{ or } c = 5.5 p r^2 \quad \dots \quad (12).$$

Example.—The flat end-plate of a cylindrical wrought-iron steam

¹ This formula is the same in construction as that given by Rankine in 'A Manual of Civil Engineering,' 1863, p. 228.

boiler is $\frac{1}{2}$ inch thick, and is 24 inches in diameter, measured to the centre line of the circle of rivets forming the joint.

The elastic deflection at the centre, by formula (2), in which $d = 24$, is equal to $\frac{24}{44} = .55$ inch. The elastic bulging pressure by formula (9), for which $t = \frac{1}{2}$, is equal to $p = \frac{10,000 \times \frac{1}{2}}{24} = 208$ lbs. per square inch. The compressive stress at the circumference is by formula (12),

$$c = 5.5 p r^2 = 5.5 \times 208 \text{ lbs.} \times 12^2 = 164,736 \text{ lbs., or } 73.5 \text{ tons.}$$

It has been found that the elastic resistance of wrought iron, when in quantity, to compression, may be taken at 20 tons per square inch. An area of $\left(\frac{73.5}{20} = \right) 3.675$ square inches, would therefore have elastic resistance sufficient to oppose the compressive stress in this example.

To compare the elastic resistance of this flat end-plate with that of the cylindrical portion of the boiler to which it is attached, 24 inches in diameter and of $\frac{1}{2}$ -inch plate. By the usual formula,

$$p = \frac{4,480 t s}{d} = \frac{4,480 \times \frac{1}{2} \times 12}{24} = 1,120 \text{ lbs. per square inch,}$$

reckoned for solid plate. This value is 5.4 times the value for the elastic strength of the flat end-plate, which is 208 lbs. per square inch. Passing over, meantime, the reduction of strength at the joints, which would have the same proportion for both the parts, it would follow that a flat end-plate, to be equally strong elastically with the cylindrical portion, would require to have 5.4 times the thickness of the other, which would amount to $(\frac{1}{2} \times 5.4 =) 2.7$ inches. For $\frac{3}{8}$ -inch body-plates, the required thickness for the end-plate would be $(\frac{3}{8} \times 5.4 =) 2$ inches.

The ratio of resistance for flat unstayed end-plates and cylindrical bodies of equal thicknesses, namely 1 to 5.4, is the same for boilers of all diameters, supposing that the line of junction is at the edge of the cylinder.

In these analyses of stress and strain, the effect of riveted joints in reducing the strength has not been considered. The ratios of strength remain unaffected by the joints, although the absolute values are reduced by them.

A diametrical riveted joint across a flat end-plate does not necessarily reduce the resistance of the plate to bulging stress,

since the tensional stress is radial, and is therefore coincident with the seam.

Mr. S. Nicholls¹ gives the results of four experiments made by him in 1876, partly under the direction of Mr. Robert Wilson, for testing the strength of unstayed flat ends of boilers.

A sample piece was constructed by Mr. Nicholls, 2 feet 6 inches in diameter inside, and 2 feet long. The body (Fig. 3) was made

FIG. 3.

FIG. 4.

of one $\frac{7}{16}$ -inch iron plate, butt-jointed, with a double-riveted single welt. One end *a* was closed with a $\frac{3}{8}$ -inch flat plate, formed with a $2\frac{1}{2}$ -inch flange turned to a radius of 1 inch inside, and riveted to the body. The other end *b* was closed with a $\frac{9}{16}$ -inch plate riveted to an angle-iron ring, 3 inches by $\frac{1}{2}$ inch thick, on the outside of the body. The extreme diameter outside of the ring was 3 feet $\frac{7}{8}$ inch, and the end-plate was joined to the angle-iron by a line of $\frac{7}{8}$ -inch rivets at a pitch of $2\frac{1}{4}$ inches, of which the centre-line formed a circle 2 feet $10\frac{1}{2}$ inches in diameter. The rivet-holes were punched. When tested by hydraulic pressure, the $\frac{9}{16}$ -inch plate failed in the line of the rivet-holes. For the second trial, a $\frac{3}{8}$ -inch plate was substituted for the $\frac{9}{16}$ -inch plate which had failed. When proved, it failed in the same way—across the rivet-holes. For the third trial, an inverted dished plate, $\frac{3}{8}$ inch thick, Fig. 4, was substituted for the $\frac{3}{8}$ -inch flat plate which failed in the second trial. This plate was dished to a radius of 3 feet, and was formed with a 3-inch flange; it was inserted with the convex surface inwards, and riveted direct to the body. The pressure-gauge got out of order, and the trial was abandoned. For the fourth trial, the ends were cut out and a 3-inch angle-iron,

¹ 'The Theoretical and Practical Boiler Maker,' 1876, p. 141.

$\frac{1}{2}$ inch thick, was inserted at *a*, inside the body, to which a $\frac{3}{4}$ -inch flat plate was riveted, with a line of $\frac{3}{4}$ -inch rivets at a pitch of 2 inches, in a circle 2 feet $2\frac{1}{4}$ inches in diameter. The plate failed by fracture through the rivet-holes. For the fifth trial, the $\frac{3}{8}$ -inch flanged plate *a*, Fig. 3, originally applied, and then removed, was again inserted and riveted through the flange to the body of the boiler. The flanged plate was drawn inwards at the bend, inasmuch that the angle of the bend—a right angle when the plate was inserted—was widened by the stress, and measured 107° . The angle-irons employed for making the joints were not damaged.

The principal results of the trials are here summarised :—

STRENGTH OF UNSTAYED FLAT ENDS OF CYLINDRICAL BOILERS.

(Mr. S. Nicholls' Experiments.)

No. 1, $\frac{9}{16}$ -inch Plate, Riveted.			No. 2, $\frac{3}{8}$ -inch Plate, Riveted.	No. 4, $\frac{3}{8}$ -inch Plate, Riveted.			No. 5, $\frac{3}{8}$ -inch Plate, Flanged.		
Pressure per Square Inch.	Deflection.	Set.	Pressure per Square Inch.	Pressure per Square Inch.	Deflection.	Set.	Pressure per Square Inch.	Deflection.	Set.
lbs.	Inches.	Inches.	lbs.	lbs.	Inches.	Inches.	lbs.	Inches.	Inches.
30	$\frac{1}{8}$	..	200	10	$\frac{1}{32}$	..	10	$\frac{1}{16}$	Nil.
35	$\frac{3}{16}$	..	Fractured at rivet-holes.	20	$\frac{1}{16}$	..	25	$\frac{1}{16}$	Nil.
40	$\frac{1}{4}$	..		40	$\frac{1}{16}$	..	40	$\frac{1}{16}$	Nil.
45	$\frac{5}{16}$	..		60	$\frac{1}{16}$	..	50	$\frac{1}{16}$	Nil.
120	$\frac{1}{4}$	..		80	$\frac{1}{16}$	..	65	$\frac{1}{16}$	$\frac{1}{32}$
150	$\frac{1}{16}$	..		100	$\frac{3}{16}$	..	80	$\frac{1}{16}$	..
200	$\frac{1}{4}$	..		120	$\frac{1}{16}$	..	100	$\frac{1}{16}$	$\frac{1}{16}$
250	$\frac{1}{8}$	$\frac{1}{16}$		140	$\frac{1}{16}$	..	120	$\frac{1}{16}$	..
270	$\frac{1}{8}$	$\frac{1}{8}$		155	$\frac{1}{16}$	..	150	$\frac{1}{16}$	..
280	$\frac{1}{8}$	$\frac{1}{16}$		170	$\frac{1}{16}$	..	175	$\frac{1}{16}$	..
Fractured at rivet-holes.				185	$\frac{1}{16}$	..	200	$\frac{1}{16}$	..
				200	$\frac{1}{16}$	..	250	$\frac{1}{16}$	..
				Repeated 36 times in five days, no difference.			280	$\frac{1}{16}$	..
				250	$\frac{1}{16}$	..	300	$\frac{1}{16}$	..
				300	$\frac{1}{16}$	..	Fractured at bend.		
				350	$\frac{1}{16}$	..	380	$\frac{1}{16}$	$\frac{1}{16}$
				370	$\frac{1}{16}$	$\frac{1}{16}$	Completely fractured.		
				Fractured at the rivet-holes.					

It appears that deflection by bulging commenced with the application of pressure, and the increase of deflection was practically uniform until about one-half the bursting pressure was laid on. For higher pressures, the deflection increased more slowly; the decreased rate of increase was clearly due to the increasing con-

vexity of the plate and the decreasing diameter of the sphere of convexity. The plates took a set early in the course of proof, and it is likely that the set arose from a yielding at the joints rather than from a yielding in the body of the plate. Calculating the deflection and the pressure within the elastic limits, by formulas (2) and (9)—

No. Elastic Deflection.		Elastic Pressure.	
1	$\frac{34.5}{44} = .78$ inch.	$10,000 \times \frac{.5625}{34.5}$	= 163 lbs. per square inch.
2	$\frac{34.5}{44} = .78$ „	$10,000 \times \frac{.375}{34.5}$	= 109 „ „
4	$\frac{26.25}{44} = .60$ „	$10,000 \times \frac{.375}{26.25}$	= 143 „ „
5	$\frac{28.5}{44} = .65$ „	$10,000 \times \frac{.375}{28.5}$	= 132 „ „

The calculated elastic pressures range well within the proved bursting pressures, and the elastic deflections for the 4th and 5th experiments are in harmony with the experimental observations of deflection. But the elastic pressure for No. 1 is relatively high for the bursting pressure, whilst the elastic deflection is much lower than the experimental deflection for that pressure. These two comparisons for No. 1 point to the existence of a partial crippling of No. 1 plate, which, for its greater thickness, was no doubt affected to a greater degree by the bending stress at the joint, due to lap-riveting, than the $\frac{3}{8}$ -inch plates. The flanged plate gave evidence of circumferential compressive action in the contracting of the circle, and the widening of the angle at the bend; whilst, by the formation of the square angle for the flange, an element of contrary flexure near the circumference was introduced, which aided in reducing the diameter of the spherical surface in tension.

Cast Iron.—Although there is no definite elastic limit for cast iron, short of the ultimate strength, it is convenient to adopt an extension of $\frac{1}{1000}$ part of the length, the same as for iron and steel, as the working elastic limit, corresponding to a tensile stress of 5 tons per square inch for ordinary castings. The formula for elastic deflection will be the same as that for iron or steel. For the elastic bulging pressure, the co-efficient in formula (9) for iron, is to be reduced in the ratio of 12 tons to 5 tons; then $(10,000 \times \frac{5}{12} =)$ 4,166 is the co-efficient for cast iron, say 4,000.

surface of a screwed bolt in a plate may be taken as only 50 per cent. of the tensile resistance of the bolt per square inch of section. In order that the amount of shearing resistance of a screwed stay-bolt should be equal to that of its tensile resistance, therefore, the area of shearing resistance should be twice the sectional area of the bolt, as is expressed by the equation ($d' \times t \times 3 \cdot 14$) = ($d'^2 \times \cdot 7854 \times 2$), in which d' is the diameter of the bolt at the base of the thread, and t the thickness of the plate. Reducing for d' ,

$$d' = 2 t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

showing that the equality of shearing resistance and tensile resistance of an iron stay-bolt would be attained when the net diameter of the bolt is twice the thickness of the plate, on the condition that the stress is fairly applied, without injurious distortion of the plate. But this is a condition which is not usually fulfilled in practice; and, even if the conditions of simple stress were fulfilled, it may be seen that the two formulæ for the value of the diameter d' of the stay-bolt are not commensurable; for whilst the first value by formula (27) varies as the square root of the pressure, the second, by equation (b), varies as the thickness of the plate, and therefore also as the pressure simply. It is the better course to select the first of these formulæ, (27), for determining the size of the bolt proper for resisting tensile stress, and to make up, by way of compensation, if necessary, the needful power of resistance to rupture between the bolt and the plate.

There are three means of maintaining, wholly or in part, the resisting capacity of the stay-bolt. First, by limiting the extent of the pitch of the bolts, so that the loosening effect of bulging strain may be minimised; second, by riveting over the ends of the stay-bolts upon the plates; third, by extending the ends of the stay-bolts to receive nuts which are to be screwed down with washers on the plates. Riveting, which must be done when the bolt is cold, and therefore done only to a limited extent, is satisfactorily effective only when the stay-bolts are placed at short pitches of about 4 inches. But the addition of a nut at each end supplies an efficient means of ensuring that the resistance of the stay-bolt in its place in a boiler shall be brought up to the full measure of its tensile strength.

Example.—A flat iron plate, $\frac{1}{2}$ inch thick, is stayed by rows of $1\frac{1}{8}$ -inch bolts placed rectangularly at a pitch of 12 inches both ways. The clear distance apart of the stay-bolts is $(12 - 1\frac{1}{8} =) 10\frac{7}{8}$ inches. The deflection at the elastic limit is by formula (22),

equal to $\frac{d}{44} = \frac{10.875}{44} = \frac{1}{4}$ inch; and the elastic limit of pressure is by formula (25), equal to $5,000 \times \frac{0.5}{10.875} = 230$ lbs. per square inch. The proper diameter of the stay-bolts, at the base of the thread, by formula is equal to $.0069 \times 12 \times \sqrt{230} = 1.26$ inch, corresponding to the external diameter, $1\frac{1}{2}$ inch.

RESULTS OF EXPERIMENTAL TESTS OF THE RESISTANCE OF STAY-BOLTS AND FLAT STAYED PLATES.

Screwed Stay-bolts.—Sir William Fairbairn,¹ in 1853, tested the strength of $\frac{3}{4}$ -inch stay-bolts with enlarged ends screwed into $\frac{3}{8}$ -inch plates of copper and of iron, some of them being riveted or headed in addition.

	Breaking Weight.
	Tons.
1. Copper bolts into copper plates, screwed and riveted . . .	7.2
2. Iron " " " " " " . . .	10.7
3. Iron " " " " " " screwed only . . .	8.1
4. Iron " " iron " " " " screwed and riveted . . .	12.5

In the first test, the bolt broke through the shank; the sectional area was .44 square inch, and the ultimate resistance was at the rate of $(7.2 \times \frac{1}{.44}) = 16.4$ tons per square inch. In the second test, the rivet-head was broken off, and the bolt was drawn out of the plate, stripping the thread of the plate. In the third test, the thread of the plate was stripped by the bolt. In the fourth test, the bolt broke through the shank, the screw and the plate remaining uninjured. The resistance was at the rate of $12.5 \times \frac{1}{.44} = 28.4$ tons per square inch.

Flat Stayed-plates.—The foregoing tests of holding power are confirmed by Sir William Fairbairn's experiments with flat stayed-plates tested by hydraulic pressure. He constructed two flat boxes, 22 inches square and 3 inches thick, having top and bottom plates of $\frac{1}{2}$ -inch copper and $\frac{3}{8}$ -inch iron respectively, inclosing a $2\frac{1}{2}$ -inch water space; stayed with $1\frac{1}{8}$ -inch iron stays, having enlarged ends screwed and riveted into the plates, to represent the conditions of the fire-box of a locomotive. The stays were placed at intervals of 5 inches in the first box, and of 4 inches in the second. They were tested, by hydraulic pressure, for bursting resistance.

The first box bore a pressure which was gradually increased from 245 lbs. to 425 lbs. per square inch. The bulging or deflection

¹ See "Useful Information for Engineers" (1st series). London, 1856. Appendix, p. lxxiii.

was not noted until a pressure of 455 lbs. per square inch was attained, when the deflection amounted to $\cdot 03$ inch, or $\frac{1}{32}$ inch. At a pressure of 785 lbs. the deflection was $\cdot 08$ inch, or $\frac{1}{12}$ inch, and the box gave way at 815 lbs. by the pushing of the copper plate off the end of the central stay-bolt, and the stripping of the copper thread. The ultimate stress amounted to 9 tons on each stay-bolt.

The second box, under a pressure of 515 lbs., showed a deflection of $\cdot 04$ inch, or $\frac{1}{25}$ inch; at 965 lbs. and up to 1,295 lbs. the deflection, as observed, stood without variation at $\cdot 09$ inch, or $\frac{1}{11}$ inch. At 595 lbs. the deflection amounted to $\cdot 34$ inch, or $\frac{1}{3}$ inch; and at 1,625 lbs. the box failed, by the pushing of the iron plate off the end of one of the middle stay-bolts, when the screw-thread in the plate was stripped. The ultimate stress on the stay-bolt was 11.6 tons.

It is deducible from the results of Sir William Fairbairn's experiments, that the fastening of a $\frac{3}{4}$ -inch stay-bolt screwed into a $\frac{3}{8}$ -inch plate of the same metal as the bolt, and riveted, is stronger, when fairly tested, than the shank of the bolt; and that, generally, such a fastening for a bolt, of which the diameter is twice the thickness of the plate, of the same metal, is stronger than the shank. This deduction is corroborative of the argument, that the fastening and the shank would be equally resistant if the bolt be only screwed into the plate. This argument is, moreover, directly confirmed by the result of Sir William Fairbairn's experiment, in which a $\frac{3}{4}$ -inch iron bolt stripped the thread of the screw from a $\frac{3}{8}$ -inch copper plate. Taking the diameter over the thread of the bolt at $\frac{7}{8}$ inch, the area of the shorn surface was ($\frac{7}{8} \times 3\frac{1}{2} \times \frac{3}{8} =$) 1 square inch; the shearing resistance was 8.1 tons, or 50 per cent. of the tensile strength of the bolt, which was proved, by the result of the first experiment, to be equal to 16.4 tons per square inch.

To compare the resistances of the stay-bolts when tested by themselves, and when tested in the chamber, it may be assumed that the hold of the bolt in the $\frac{1}{2}$ -inch copper plate was as much as in the $\frac{3}{8}$ -inch iron plate, since both plates failed, one in each box. The relative behaviours were, then, as follows:—

Three-quarters inch iron stay-bolt screwed into a $\frac{3}{8}$ -inch iron plate and riveted.

	Failure.	Final Stress per Square Inch.
		Tons.
Tested separately,	bolt broke in shank . . .	12.8
In chambers, at 4 inches pitch,	plate forced off . . .	11.6
" at 5 " "	plate forced off . . .	9.0

Here it is seen that the stay-bolts proved nearly as resistant, —within 10 per cent.,—in the chamber, at the 4-inch pitch, as they did when tested separately; but that, at the 5-inch pitch, their resistance was less than three-fourths of that of the separate bolt: clear evidence of the advantage of a shorter pitch over a longer pitch, in bringing into action the shearing resistance of the bolt.

To apply the formula (25), for the strength of these two chambers: In the first chamber, with a 5-inch pitch, the bolts were, say, $1\frac{1}{8}$ inch in effective diameter over the thread, and they were $(5 - 1\frac{1}{8} =)$ 4.2 inches clear. The elastic resistance of the stayed-iron plates was equivalent to a pressure of $(5,000 \times \frac{3}{8} \div 4.2 =)$ 446 lbs. per square inch. For the 4-inch pitch, the clear distance between the bolts was 3.2 inches, and the elastic resistance was equivalent to $(5,000 \times \frac{3}{8} \div 3.2 =)$ 586 lbs. per square inch. Comparing the calculated strengths with the recorded data:—

	Bulging first noted at	Calculated elastic Strength.	Stay-bolt failed at
	lbs.	lbs.	lbs. per sq. in.
5-inch pitch	455	446	815
4-inch pitch	515	586	1,625

The calculated elastic strengths are, certainly, safely within the limits of ultimate strength. For the 4-inch pitch, the elastic strength is little more than one-third of the limiting strength; whilst, if the stay-bolt at the 5-inch pitch had been as effective as the others, the bursting pressure would have been greater in the ratio of 9 tons to 11.6 tons, already noted, or equal to $(815 \text{ lbs.} \times 11.6 \div 9 =)$ 1,050 lbs. per square inch. The elastic deflections were by formula (22), for the 5-inch pitch $(\frac{1}{4} =)$.10 inch; and for the 4-inch pitch $(\frac{3}{4} =)$.073 inch, or $\frac{1}{14}$ inch. These values can scarcely be compared with the deflections recorded by Sir William Fairbairn, the observation of which was evidently wanting in precision.

Mr. Lavington E. Fletcher¹ records the results of tests made in August 1876, at Portsmouth, on an experimental flat stayed-chamber, 8 feet 7 inches long, 4 feet 8 inches high, and $6\frac{3}{4}$ inches thick, as in Figs. 5 (see next page). The plates at the back were of Low-moor iron, $\frac{7}{16}$ inch thick; in the front, they were of Staffordshire quality, partly $\frac{1}{2}$ inch thick, and partly $\frac{7}{16}$ inch thick. The stay-bolts were $1\frac{1}{4}$ inch in diameter in the body, and fully $1\frac{3}{8}$ inch over the

¹ Chief Engineer's Monthly Report to the Manchester Steam User's Association, July-October 1876, p. 31.

thread, with eleven threads to the inch. They were pitched at $15\frac{1}{2}$ inches horizontally and $15\frac{3}{4}$ inches vertically, and had a head or a nut at the front of the chamber, and were riveted over at the back. As the chamber was peculiarly constructed for a particular

FIGS. 5.

purpose, the action at those stay-bolts only which were placed under normal conditions will be noticed here.

At the first test, on arriving at a pressure of 95 lbs., the back plate was forced over the end of No. 2 stay, stripping off the metal that had been riveted over, but leaving the threads on the bolt

and in the plate uninjured. The bulgings noted when the pressure reached 60 lbs. and 75 lbs. successively were as follows :—

BULGING OF FLAT-STATED CHAMBER under HYDRAULIC PRESSURE.

Back of Chamber. Stay bolts 1, 2, 3, 4, 7, 8, 9, 10, Screwed and Riveted.			Front of Chamber. Stay-bolts 1, 2, 3, 4, 6, 7, 8, 9, 10, Screwed and Nuts.		
Gauged at	Bulging at 60 lbs.	Bulging at 75 lbs.	Bulging at 60 lbs.	Bulging at 75 lbs.	Gauged at
	Inch.	Inch.	Inch.	Inch.	
A	$\frac{3}{16}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{3}{16}$	M
B	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{16}$	$\frac{5}{16}$	L
C	$\frac{3}{16}$	$\frac{1}{2}$	$\frac{3}{16}$	$\frac{1}{4}$	K
F	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	R
G	$\frac{1}{8}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	Q
H	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{5}{16}$	P
			$\frac{3}{16}$	$\frac{1}{4}$	W
			$\frac{1}{8}$	$\frac{3}{16}$	V
			$\frac{1}{8}$	$\frac{5}{16}$	U

For the second test, the stays 1, 2, 7, and 8 were replaced by bolts $1\frac{3}{8}$ inch in diameter at the base of the thread, at both ends. At a pressure of 105 lbs. the back plate was forced over the end of No. 3 bolt; the threads were uninjured, but the metal that was riveted over was stripped off. For the third and fourth tests Nos. 3 and 9 stay-bolts were replaced by $1\frac{3}{8}$ -inch bolts, as before, with nuts at both ends. The back plate was forced over the end of No. 4 bolt, as in the second test, under a pressure of 155 lbs. For the fifth test, the two remaining stays Nos. 4 and 10, were replaced by $1\frac{3}{8}$ -inch stays, as before, screwed and nuts. The front plate, cracked at No. 3 hole, above and below, about $1\frac{1}{2}$ inch each way, under a pressure of 150 lbs. For the sixth test, the leakage was stopped with india-rubber washers, and under 165 lbs. pressure, the front plate burst at the lower line of rivets, through thirteen consecutive rivet-holes, which were countersunk.

It appears from these results that the chamber burst, by slipping over stay-bolts, riveted only, at pressures of from 95 lbs. to 155 lbs. per square inch. When all the stays had nuts, the front plate cracked at No. 3 hole under a pressure of 150 lbs.; and ultimately it bore 165 lbs. without any failure at the stay-bolts. In testing the original marine boiler on which the chamber was modelled, the maximum amount of bulging in the plates of the boiler under pressures of from 60 lbs. to 65 lbs. per square inch, was $\frac{3}{16}$ inch,

in the plates corresponding to the front of the model chamber; and $\frac{1}{16}$ inch full at the shell, represented by the back of the chamber. Of seven boilers tested in this way, only one showed any degree of set: $\frac{1}{16}$ inch on the side of the uptake. These results harmonise with those deduced from the action of the model chamber. The working pressure of the boiler was 30 lbs. per square inch.

The area supported by each stay amounted to $(15\frac{1}{2} \times 15\frac{3}{4} =)$ 244.5 square inches; and the total pressures resisted by the stays were:—

	lbs.	Tons.	
In the first test	$(244.5 \times 95 =)$	10.4	per stay-bolt.
" 2nd "	$(244.5 \times 105 =)$	11.5	" "
" 4th "	$(244.5 \times 155 =)$	16.9	" "
" 6th "	$(244.5 \times 165 =)$	18.0	" "

In the sixth trial, there was no failure at the stay-bolts. All the riveted ends of bolts, $1\frac{1}{4}$ inches in diameter at the base of the thread, supported a pressure of 17 tons, and no doubt they were capable of supporting 18 tons. The riveted bolts failed with from 10 to 17 tons.

A second chamber of the same dimensions as the one just described, with $\frac{7}{16}$ -inch front plates, and $\frac{1}{2}$ -inch back plates, was constructed and tested by hydraulic pressure at Portsmouth, in August 1876. A screwed and riveted bolt was drawn out of the front plate, and the thread was stripped by a pressure of 120 lbs. per square inch. Two others were drawn through the back plate with 125 lbs. and 130 lbs. respectively. The fourth that failed was a riveted bolt, under 140 lbs. Finally the chamber cracked at a rivet-joint, under a pressure of 140 lbs.

The elastic deflection of the Portsmouth chambers is by formula (22) $(15.75 - 1.25 \div 44) = 0.35$ inch, an amount consistent with the average of the recorded observations for deflection, which are very various. The elastic pressure is, by formula (25), $(5,000 \times \frac{7}{16} \div 15.5 =)$ 141 lbs. per square inch. The chambers were burst under pressures of 195 lbs., and 140 lbs. respectively; and, in each instance, the failure took place at a rivet-joint at the edge of one of the plates.

The only instances in which the calculated elastic strength lies well within the range of the actual bursting resistance were those of the experimental chambers tested by Sir William Fairbairn. In the other cases, there is an approximation to the actual bursting strength. It is obvious that the failures arose, not from the weakness of the plates, but from that of the rivet-joints, or of the stay-fastenings, which were loosened and weakened by the bulg-

ings of the plate round the stay-bolts. The greater degree of resistance of the stay-bolts of Sir William Fairbairn's boxes was clearly derived from the comparative nearness of the stay-bolts in those boxes, and the more limited range of the distortion to which the stay-fastenings were subject. Even between these two boxes, the advantage of the closer 4-inch pitch over the wider 5-inch pitch, was perceptible; for the more closely-set stay-bolts made the greater resistance to bulging stress: whilst the 5-inch stay-bolts resisted rupture with a force of 9 tons, the 4-inch stay-bolts resisted with a force of $11\frac{1}{2}$ tons.

Upon the whole, the values for elastic deflection and elastic strength of flat-stayed surfaces given by means of the formulæ may be accepted as approximately correct values, on the assumption that the stay-bolt fastenings are sufficiently strengthened to resist the weakening influence of distortion other than that which arises from direct stress fairly applied. The nuts of the stay-bolts employed in the Portsmouth boxes proved to be effectual for securing that object.

The Paper is illustrated by several diagrams, from which the woodcuts, Figs. 1 to 5, have been engraved.