our stand in the organ loft; there we overlook the whole and get the effect that caused the Emperor to exclaim when he stood leaning against the rail, "Beautiful, beautiful!" Indeed, the forms and masses of this late Gothic style are beautiful, and it is unfortunate that they are very seldom used in this way in our church edifices. Slender octagonal pillars divide the nave into three aisles, and the end of the choir is semi-hexagonal. The roof, which is vaulted and deeply ribbed, is ornamented with fresco painting. This is the characteristic feature of the old style. Colors are much used. The coats of arms that are arranged in relief on the railing of the gallery are colored and the capitals of the columns that support the statues of the heroes of the reformation are also colored. The idea of making this a memorial hall for the leading spirits of the reformation is carried still further in the twenty-two bronze medallions in the corners above the arches under the gallery and in the eight windows of the nave. The medallions represent princes, artists, and humanists who lived before and at the time of the reformation: in the windows are 198 coats of arms of the Protestant cities, arranged according to the old provinces. Our view is limited by the three colored glass windows of the choir, the subjects of which were taken from the so-called "Little Passion" of Durer. The dazzling white altar stands out in relief against the rich color tones of these windows, a beautiful piece of work in French limestone, the pinnacles shooting up like delicate ice crystals. When at a distance we see the figure of the Lord through the central opening in the altar, while close to the altar this is not impressive. On the right of Christ stands Peter, on the left Paul. This white altar contrasts finely with the dark oak of the seats arranged on the sides. The chancel, the seats and the organ loft are in the same tone.

The exterior of the church is as imposing as the interior is harmonious. Here we have the old stones of the walls built in the time of Frederick the Wise, and all additions, such, for instance, as the sacristy, have been built in the same style. The burned wooden wings of the Thesis Door were replaced in 1858, by order of Frederick William IV., the new doors bearing the Latin text of Luther's "Theses." This celebrated main entrance is marked by the slender louvre tower vertically above the transept. The principal tower on the northwest corner is very peculiar, and its unusual form shows that it was not always a church tower. In fact, it belonged, in earlier times, to the castle, and later it was several times connected with and disconnected from the church. But this castle tower in connection with this house of God has its special significance, which finds expression in the scroll in the cornice on which the large Gothic letters in mosaic glass work form the words "Ein fester Burg ist unser Gott, ein gute Wehr und Waffen!" (A mighty fortress is our God, a bulwark never failing). This tower, which is 288 ft. high, always attracts the attention of the observer, and when, on October 31, its bells invited to the third consecration, and all the bells of the country round answered in a festal chorus, it was a happy day, not only for Wittenberg, but for all evangelical Christians in Germany and abroad.

In the dedicatory sermon which Luther preached in the church one year before the writing of his "Theses," he reminded his hearers that a consecration of the church should also be a consecration of the heart. Let us hope that the admonition was kept in mind by the congregation during this last ceremony.

For the above and for our larger illustration we are indebted to the *Illustrirte Zeitung*, and for the other illustrations to the *Illustrated London News*.

On Monday, Oct. 31, 1892, says the *Illustrated London News*, the anniversary of Luther's memorable act in 1517, when he affixed to the door of the Schlosskirche at Wittenberg his written "Theses" or theological propositions contradictory of the doctrines of the Roman Catholic Church, a ceremony of great interest to the German Protestants was performed in the presence of the Emperor William II., King of Prussia. This was the reopening and renewed dedication of the Schlosskirche, which has, during seven years past, been undergoing a complete architectural restoration, at the cost of the Prussian government, promoted by the Emperors William I., Frederick III., and William II. In the year 1883, a few weeks previously to the celebration, on Nov. 10, of the four hundredth anniversary of Luther's birth, the Imperial Crown Prince Frederick William, afterward the Emperor Frederick III., visited Wittenberg to open the Luther Hall at the University, with the apartments, including Luther's study, occupied by the great reformer when he was university professor there. The Schlosskirche, containing the tombs of Luther and Melancthon, has been restored from the designs and under the supervision of Professor F. Adler, of Berlin, "Geheime Oberbaurath," by the government architect Herr Paul Groth. The tower has been entirely altered from what it was in 1883, being now raised to the height of 288 ft., instead of 100 ft., and its upper part adorned with a splendid circle or frieze of mosaic, bearing an inscription in yellow letters on blue, and with a cupola and copper spire, richly ornamental. The Emperor and Empress, at the opening ceremony, were accompanied by the Crown Prince of Sweden, the Duke of York, and representatives of all the Protestant German states.

MULTIPLICATION SIMPLIFIED.

By NICHOLAS J. VANDER WEYDE, C.E.

Is the tendency of modern advancement in the direction of simplification or are we striving after more complicated methods of doing? May we not modify the elaborate processes of our forefathers and make life easier for our posterity? Must conservatism block the way to progress in arithmetic as well as in spelling and in weights and measures?

While the author of this article has been endeavoring to make the path of elementary arithmetic easier to the learner, by simplifying the process of multiplication, a puzzle fiend of the periodical order has invented, patented and caused to be published an arithmetical puzzle for cranks of mathematical tendencies to rack their brains over and perhaps wreck their intellects on. If the former can save his fellow creatures as much worry and exasperation as the latter has been the cause of, he will not feel that his labors in perfecting and introducing this new method have been in vain.

Arithmetic is acknowledged to be one of the most

interesting branches of elementary education, and if the tedious processes hitherto necessary for obtaining required results can in any way be modified, a decided gain will have been achieved in the saving of time and patience of those giving as well as receiving instruction; and the mental worry incident to a constant liability to error will be greatly obviated by a less complicated process than that of "old style" multiplication.

Let the brilliant and fascinating "Algebra" come to the rescue of her well meaning but often misunderstood younger sister "Arithmetic" with the easily proved and convincing formula that "the square of the sum of two numbers is equal to the square of the first, plus twice their product, plus the square of the second." Algebraically and arithmetically expressed, with $x = 40$ and $y = 6$, making $x + y = 46$, we have

$$\begin{array}{rcl} x + y & = & 40 + 6 = 46 \\ x + y & = & 40 + 6 = 46 \\ \hline x^2 + xy & = & 1600 + 240 = 1840 \\ + xy + y^2 & & 240 + 36 = 276 \\ \hline x^2 + 2xy + y^2 & = & 1600 + 480 + 36 = 2116 \end{array}$$

but it is more convenient to do all the squaring first and the cross multiplying (doubled) afterward, of which two examples are given below:

$$\begin{array}{r} 46^2 \\ 1636 \\ 48 \\ \hline 2116 \end{array} \quad \begin{array}{r} 3951^2 \\ 09812501 \\ 549010 \\ 3018 \\ 06 \\ \hline 15610401 \end{array}$$

The advantage of squaring by this method can be readily appreciated. No more figures used than by the old method, much less chance of error and not nearly so much strained attention required to keep track of the operation.

Let us do the same thing with ordinary multiplication, with this difference, that while we multiply every figure in the multiplicand by every figure in the multiplier, we do not double any of the products, but put them all down as shown in the following examples:

$$\begin{array}{r} 37 \\ 95 \\ \hline 2735 \\ 63 \\ 15 \\ \hline 3515 \end{array} \quad \begin{array}{r} 856 \\ 914 \\ \hline 720524 \\ 4506 \\ 0820 \\ 54 \\ 32 \\ \hline 782384 \end{array}$$

Where the number of figures in the two factors is not the same it is only necessary to prefix one or more ciphers to the lesser and proceed as before:

$$\begin{array}{r} 4793 \\ 0256 \\ \hline 00144518 \\ 001815 \\ 083554 \\ 0006 \\ 2042 \\ 24 \\ \hline 1227008 \end{array}$$

If a line of products is going to be all ciphers, viz., 0×3 in above example, it may be omitted as in "old style" multiplication, and if decimals are involved, the decimal point finds its place by the old rules.

The process just described and illustrated might be known as "long multiplication" and should be taught first; but where the number of figures in the multiplicand far exceed those in the multiplier, another process, to be known as "short multiplication," will be found more convenient and practicable. In the following example the work is left unfinished in order to show its progress, and the annexed "process" example illustrates how it is done, the products being put down on the first and second lines alternately.

$$\begin{array}{r} 4893185 \\ 7 \\ \hline 2863 \text{ etc.} \\ 5621 \end{array} \quad \begin{array}{l} \text{PROCESS.} \\ 7 \times 4, \times 8, \times 9, \times 3, \text{ etc.} \\ 7 \times 4, 7 \times 9, \text{ etc.} \\ 7 \times 8, 7 \times 3 \end{array}$$

It will be noticed that the products are put down alternately on the first and second lines, so that the units will come in consecutive places.

The part of the process of multiplication most liable to error is that of carrying the tens when the units are put down. In the new way everything is put down and there is no carrying until all the products of multiplication are being added together. In order to check any tendency to carry the tens, the multiplication is begun at the left hand end, but the adding is done from right to left as in the "old style." The following is a finished example worked out according to this idea:

$$\begin{array}{r} 749843109 \\ 39 \\ \hline 2127120327 \\ 12240900 \\ 6381360981 \\ 36722700 \\ \hline 29243881251 \end{array}$$

Below is the same example worked out according to the "old style," with the mental part of the process fully illustrated:

$$\begin{array}{r} 749843109 \\ 39 \\ \hline 6748587981 \\ 48732008* \\ 2249529327 \\ 12210002* \\ \hline 29243881251 \end{array}$$

The small figures in a line with the asterisks are the tens to be carried. As a rule they are never put down, but have to be remembered and considered all the same. Should the attention be distracted during the process, a carried figure may be forgotten and time lost in going back to find it. Arithmeticians of the present age are so familiar with the old style and are so used to its workings that it would be unreasonable to expect them to take up the new; but the new style is for those to whom arithmetic is new, and for them it cannot fail to be simpler, easier and quicker than the other. It may necessitate putting down a few more figures on paper, but it certainly saves putting them down in the brain. Any unnecessary tax on the memory, either for short or long periods, is nothing more nor less than the mental carrying of so much dead weight.

In the new style each product is put down complete, the tens and units falling naturally into their proper places, representing, units, tens, hundreds, thousands, etc., according to location. The separating dots used in the examples given are only intended to keep each product by itself, and may be dispensed with in the regular and practical use of the process. The readiness with which examples can be corrected or their correctness verified by either pupil or teacher is in itself a vast economy of both time and labor in education, as errors are detected in any part of the operation without going back for missing or mistaken tens. The author of these pages is confident that there can be no further use for the "old style" multiplication, either in school, in college or in business, which the new style cannot fill to greater advantage.

In conclusion it may be well to remark that if this easier method of multiplication is not new, a most unjust and iniquitous tax has been levied on the energies of the children of the present century, in not introducing it before into the primary schools of the civilized world. The new method is easy enough to teach, simple enough to understand and convenient enough to use in all the operations of practical mathematics for it to entirely supersede the "old style" in the schools of this country by the beginning of the twentieth century.

OSMOTIC PRESSURE.

By J. W. RODGER.

OF the various properties which have found a common explanation in the new theory of solutions, there are none perhaps to which more interest attaches than to osmotic pressure; and although, on account of the experimental difficulties, the observations as yet accumulated on this subject are but scanty, they have so largely contributed to the novel ideas involved in the new theory that they merit special attention.

Since accounts of osmotic pressure are finding their way into few English text books, it may be worth while glancing at the main features which have led up to the present state of the question.

It has long been known that if an aqueous solution—say of sugar—be separated from pure water by a piece of animal membrane, movements of the water and of the sugar take place through the membrane. If the solution be contained in an open vessel, the base of which is composed of membrane, on partially immersing the vessel in water it is easy to see that more water enters the vessel than solution leaves it. The level of liquid within rises above that without the vessel, different pressures being thus set up on opposite sides of the membrane.

To this process wherein currents pass through a membranous septum, the terms "osmosis," "osmose," and "diomose" have been applied. The last of these is perhaps to be preferred, as it serves to indicate that two currents are involved in the phenomena. Investigations carried out as indicated above were concerned with the measurement of what was termed the "endosmotic equivalent." That is the ratio of the amount of water passing into the solution to the amount of dissolved substance passing in the opposite direction. Consistent measurements of this quantity could not be obtained, however, for it was found that the nature of the membrane exercised a marked influence upon its magnitude. The kind of membrane employed, or, with the same membrane, its thickness or freshness, or even the direction in which water passed through it, was of importance. Thus, in illustration of the last point, water passes more readily outward through eel's skin, more readily inward through frog's skin.

To obtain quantitative relations in this field it thus became essential to eliminate the influence of the membrane, and more recently this end seems to have been attained by the use of membranes artificially prepared.

These artificial membranes differ from those of animal origin in the remarkable particular that although they allow water to pass through, they present a barrier to the passage of certain dissolved substances. On this account they have been termed semi-permeable membranes, and by their use measurements of osmotic pressure have been made possible.

To carry out such measurements the first point to be solved was to obtain a membrane of sufficient strength. The substance which has been found to be most satisfactory as a membrane former is copper ferrocyanide. When aqueous solutions of potassium ferrocyanide and copper sulphate are carefully brought into contact, a pellicle of copper ferrocyanide is formed where the two solutions meet. In this condition the pellicle is much too fragile to sustain even slight differences of pressure; but by the following simple device, employed first of all by W. Pfeffer, satisfactory results have been obtained.

If a cell similar to the ordinary porous pot of a voltaic battery be lowered into a solution of copper sulphate while at the same time a solution of potassium ferrocyanide be poured into its interior, the two solutions meet somewhere within the walls of the cell and deposit a film of copper ferrocyanide. Little diaphragms of membrane are thus produced stretching across the pores of the cell wall, which furnishes the necessary support, and by taking suitable precautions a membrane may thus be obtained capable of withstanding a pressure of several atmospheres.

The behavior of a solution when separated from pure solvent by such a semi-permeable membrane differs markedly from what takes place when an animal membrane is employed. In the latter case, at the outset, water adds itself to the solution; the level of liquid