cost complete about twelve cents each. In case a certain reagent is no longer needed the label is readily scraped off and the bottle may be relabeled as desired. If a bottle of a set is broken it can be at once replaced. All of the reagent bottles at the Massachusetts Institute of Technology are labeled in the way described.

The care of brass apparatus. Brass screw clamps, test tube holders, blow pipes, etc., become badly corroded after a year's use, and often are not fit to reissue to students. All such brass ware may be readily cleaned by immersion in concentrated hydrochloric acid for a few minutes. After all corrosion is removed, or so loosened that it readily washes off, wash the article thoroughly in running water and dry on a wire gauze over a flame. In the case of screw clamps or other apparatus having threads and taps, touch them with a piece of paraffin while they are still hot. The melted paraffin spreads quickly over the article and both lubricates it and protects it from further corrosion. Corroded Bunsen burners that have screw adjustments can be very satisfactorily cleaned in this way and in fact the worst looking brass articles can be made as good as new. I am indebted to Professor J. A. Culler of the department of physics for the suggestion that led to the use of this process for cleaning chemical brass ware.

ON THE DEVELOPMENT OF ELEMENTARY GEOMETRY IN THE NINETEENTH CENTURY.

By G. A. Miller,

University of Illinois.

Under the above title Max Simon recently issued a volume which was published in the form of a report to the German Mathematical Society. Probably no one occupying a chair of mathematics in a German university has taken a more active interest than Professor Simon in the recent reform movements relating to the work of teachers of secondary mathematics. He is earnest in his advocacy of reform, zealous in his application of the history of mathematics to the principle of teaching, and full of that good-humored argument that makes a man acceptable as a speaker in an assembly of teachers.* While his writings are not as free from inaccuracies as one might expect, they contain

*D. E. Smith, Bulletin of the American Mathematical Society, Vol. 13 (1906) p. 139.

many instructive partial truths and are characterized by a boldness of statement which is refreshing and very helpful in the acquisition of some great truths. The main object of the present note is to exhibit some of the conclusions of Professor Simon's report, which seem of especial interest to teachers of elementary geometry.

We first note a remark in regard to popular text-books. On page 33 Professor Simon mentions the work on elementary mathematics by L. Kambly, which went through more than one hundred editions, and he observes that this success is without a parallel, although the work had no scientific value. He adds that the great success was perhaps due to the fact that it had no scientific value. This reminds one of the statement, "Any text-book will be extensively adopted if it is only poor enough," and shows that such remarks are not confined to conditions in this country. On page 42 he calls G. A. Wentworth the American Kambly. An important remark in regard to the use of text-books is found on page 26, where it is stated that it is impossible to make a sufficiently sharp distinction between the teacher and the text-book. For pupils below twelve the use of a text-book is regarded by the author as simply a crime and he thinks that a table of logarithms is the only mathematical book that needs to be in the hands of pupils. The teacher should, as far as possible, present the subject in such a genetic manner that the pupil feels that he has discovered the theorems independently, while a text-book, on the contrary, has to be dogmatic.

In summarizing the changes of methods of instruction in elementary geometry during the nineteenth century the following facts are emphasized: There has been a gradual abandonment of the dogmatic viewpoint in favor of the genetic and this change has been attended by greater emphasis on exercises and constructions. More attention has been paid to intuition as an independent and important factor in teaching geometry, and this gave rise to the so-called fusion of plane and solid geometry, which has been especially emphasized in France by the works of Meray, and has found strong exponents in the other countries. Toward the end of the century a strong wave of critical investigations in regard to the foundations of geometry started in Italy and was strengthened and popularized by Hilbert's *Grundlagen der Geometrie*.* While this has affected the advanced work in geometry

*Translated into English by E. J. Townsend.

more than the elementary, yet it has made itself felt in the latter, and in our country has led to such a work as Halsted's *Rational Geometry*.

The gradual abandonment of Euclid and the arithmetization of geometry are described as follows: The French Revolution which destroyed all authority was also turned toward Euclid. Legendre's Elements arose, perhaps through the influence of d'Alembert, but more in opposition to the important work of the Jesuits, and they were rapidly spread over all the Romanic countries, including Belgium and Holland. The arithmetization of geometry began with these elements and acquired momentum especially through the works of Weierstrass and through George Cantor's theory of point sets (Mengenlehre), which aims to give an arithmetic definition of the purely geometric concept continuity.

The Elements of Legendre (the second Euclid) have not only been conducive to the arithmetization of geometry but they have also contributed much toward the critical study of the axioms of geometry—especially of the one relating to parallel lines and known as the eleventh axiom of Euclid. In the first edition of the Elements (1794) Legendre gave what he considered a proof of this axiom and in the later editions he gave what appeared to him simpler proofs of what is really unprovable. The extensive circulation of Legendre's Elements attracted attention to these efforts to prove one axiom by tacitly assuming another and thus intensified the interest in geometries in which one or more of the axioms of Euclid is not assumed—the Non-Euclidian geometries—whose development is one of the greatest scientific triumphs of the nineteenth century.

In his excellent critique of the fundamental concepts of geometry, Wellstein gives expression to the following ideas relating to this development:* The criticisms of Euclid should have begun with the first definition (the definition of a point as that which has no parts) instead of with the eleventh axiom. This concept of point arises from that of a material point through the process of passing to a limit. One thinks of a grain of sand or some other particle of matter and conceives it as becoming smaller and smaller without limit. During this process it appears to become less and less possible to divide the grain of sand into smaller parts and the concept of point is said to arise as a

*Weber and Wellstein Encyklepadie der Elementa Mathematik, Vol. 2, 1905.

limit of this process and hence as something without parts. This conception of a point is untenable, for it is impossible to conceive of the particle as becoming smaller after it has become so small that it cannot be seen. When this point has been reached we are in perfect darkness since we cannot conceive that the process of becoming smaller and smaller has necessarily an end, but we must believe or postulate that there is some limit beyond which the particle cannot be decreased and which it cannot reach.

To get a clearer idea of the given conception of a point we may think of viewing the particle, which becomes smaller and smaller, through a microscope whose power is continually increasing at such a rate that the particle will appear to keep the same size. Hence, no matter how small the particle has become, the possibility of dividing it remains unchanged, and if we want to associate the concept of point with that of a particle which becomes smaller and smaller it should be clearly observed that this is an act of the will and not one of reason. Hence the first definition of Euclid is based upon the postulate that points exist. They have, however, no spatial existence. It is evident that similar remarks apply to the definitions of line, plane, etc., and we are reminded that nothing is a greater hindrance to a deep understanding than that which appears self-evident at the first glance. As the elements with which one operates in elementary geometry have their existence in postulates, it follows that pure geometry relates only to thinking beings. Fortunately, the trend of the developments has been such as to make the applications to other subjects easy.

In an appendix to his report Professor Simon speaks of the efforts that are being made in Germany to secure a better general understanding of the importance and of the nature of a mathematical training. He says that the chief enemies to a proper appreciation of mathematics are the classical philologists who do not want to be convinced, but are anxious to maintain their position in the educational system. The philologists are, however, not the only ones who have hindered proper mathematical training. Even the children of mathematics—those engaged in technical education, and the natural sciences—have threatened to block the way toward proper mathematical training. The former have gone so far as seriously to demand that only such mathematical problems as present themselves in practice should be considered in the training of students. They were even so kind as to offer to prescribe the problems since they themselves were unable to solve them. These unfortunate tendencies have been largely checked by the wise and untiring efforts of Professor Klein, and still more by the power of the facts which is stronger than that of men.