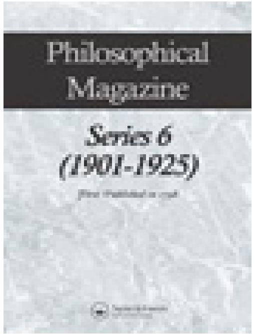

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XI. The potentials required to maintain currents between coaxial cylinders

John S. Townsend

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the case of very large gas-molecules, such as those of ether- and benzine-vapour [*]. For the influence of pressure depends—at least to a great extent—on the circumstance that with the increase of pressure the single absorption-lines gradually broaden, to merge finally into one continuous band [†], and it is likely that this final result is reached the more easily in proportion as the lines lie nearer together. The real cause of this pressure-effect, however, still remains unexplained. If the formula (1) held absolutely, we should have to seek the cause in a change of the moment of inertia brought about by the impacts of the molecules. This could be caused by the momentary effect of the impacts being that the molecules have various rotation-axes, or that they suffer a temporary deformation. But both these assumptions offer considerable theoretical difficulties. Not less difficult to justify, however, seems to me the assumption of Perrin [‡], that immediately after impact the molecule has a given energy of rotation (E), which, however, through some sort of friction or radiation, rapidly decreases, until the remaining rotation-energy satisfies the formula $E = n \cdot h \cdot \nu$.

Uppsala, April 1914.

XI. *The Potentials required to Maintain Currents between Coaxial Cylinders.* By JOHN S. TOWNSEND, *Wykeham Professor of Physics, Oxford* [§].

SEVERAL investigations have been made of the potential required to maintain a current between a wire and a large coaxial cylinder when a glow-discharge is produced by a high electric force. A certain definite potential V_0 is required to start the discharge, and the rise of potential $V - V_0$ required to maintain a given current has been determined experimentally.

When the current i is small the rise of potential $V - V_0$ is proportional to i, and in this case a simple formula [||] connecting i and $V - V_0$ may be obtained on the hypothesis that the ions are produced from molecules of the gas by collisions within a short distance of the wire where the electric force is large.

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[*] E. v. Bahr, *Ann. d. Phys.* xxxiii. p. 585 (1910).

[†] E. v. Bahr, *Verh. d. D. phys. Ges.* xv. p. 710 (1913).

[‡] J. Perrin, 'Die Atome,' 1914, p. 146.

[§] Communicated by the Author.

[||] 'Electrician,' June 6, 1913.

For larger currents dV/di, the rate of increase of the potential with the current, diminishes as i increases, and the relation between V and i may be obtained on the same principles, when the pressure of the gas between the wire and the cylinder is not very low, and ionization by collision takes place only at points whose distances from the wire are small compared with the radius of the cylinder.

The condition that the forces near the wire should be sufficiently large to maintain a current is represented by the equation

$$1=\int_a^c \alpha\, e^{\int_0^n (\beta-\alpha)\,dv}\, dr,$$

where α and β have their usual signification, a being the radius of the wire and c the distance from the axis at which the force becomes so small that α and β may be considered to vanish. This condition is independent of the intensity of the current, so that the forces corresponding to the starting potential V_0, when i is infinitely small, will be sufficient to maintain any current.

The current affects the field of force owing to the electric charge produced by the separation of the ions in the gas, the principal action being due to ions of the same sign as the charge on the wire, which give rise to a volume distribution of electricity ρ in the space between the outer cylinder and the region near the wire where the ions are generated. The distribution ρ' due to ions of the same sign as the charge on the wire that move through a short distance of the order $c-a$ has such a small effect compared with the distribution ρ that it may be neglected.

When a current is flowing the distribution ρ tends to diminish the force near the surface of the wire, and in order to maintain the current it is necessary to counteract this effect by increasing the potential difference between the wire and the cylinder. The relation between V and i may be obtained on the hypothesis that the force at the surface of the wire is equal to X_1 ($=V_0/a \log b/a$), the force corresponding to the starting potential V_0. For simplicity it may be assumed that the velocities of the ions are proportional to the electric force, and although this is by no means true for the negative ions moving in gases at low pressures, the formula obtained on this hypothesis, when compared with the experimental results, shows very clearly many properties of the negative ions.

Let i be the current per unit length of the wire, k the velocity of the ions due to unit force, ρ the charge per cubic centimetre of the gas, and ϕ the potential at the distance r from the axis. At the surface of the wire $r=a$, $\phi=0$; and at the surface of the cylinder $r=b$, $\phi=\mathrm{V}$.

If all the electrical quantities are expressed in electrostatic units the following relations hold between ϕ, ρ, and i:—

$$i=-2\pi\rho r k\frac{d\phi}{dr}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right)=-4\pi\rho,$$

when ρ is eliminated the relation between $d\phi/dr$ and r becomes

$$\left(r\frac{d\phi}{dr}\right)^2=\mathrm{C}+\frac{2ir^2}{k}.$$

The constant of integration C is obtained from the condition

$$\frac{d\phi}{dr}=\mathrm{X}_1=\frac{\mathrm{V}_0}{a\log b/a}, \quad \text{when} \quad r=a.$$

Hence
$$\left(r\frac{d\phi}{dr}\right)^2=a^2\mathrm{X}_1{}^2-\frac{2ia^2}{k}+\frac{2ir^2}{k}.$$

When $2i/k$ is negligible compared with $\mathrm{X}_1{}^2$ this equation becomes

$$d\phi=a\mathrm{X}_1\left(1+\frac{2ir^2}{ka^2\mathrm{X}_1{}^2}\right)^{\frac{1}{2}}\frac{dr}{r},$$

which may be integrated by changing the variable r to

$$x=\left(1+\frac{2ir^2}{ka^2\mathrm{X}_1{}^2}\right)^{\frac{1}{2}}.$$

The total potential fall V thus obtained is

$$\mathrm{V}=a\mathrm{X}_1\left\{(1+\theta)^{\frac{1}{2}}-1+\log 2b-\log a\left(1+(1+\theta)^{\frac{1}{2}}\right)\right\},$$

where $\theta=\dfrac{2ib^2}{ka^2\mathrm{X}_1{}^2}$, $2i$ being small compared with $k\mathrm{X}_1{}^2$.

Hence
$$\frac{\mathrm{V}-\mathrm{V}_0}{\mathrm{V}_0}\log\frac{b}{a}=(1+\theta)^{\frac{1}{2}}-1+\log\frac{2}{1+(1+\theta)^{\frac{1}{2}}}.$$

When θ is small this equation reduces to

$$V - V_0 = \frac{ib^2 \log b/a}{2kV_0},$$

which shows that $V - V_0$ is proportional to i for small currents.

The general equation connecting $\frac{V - V_0}{V_0} \log b/a$ and $\frac{2ib^2}{ka^2X_1^2}$ may be solved by means of a curve having these quantities as ordinates, so that when $V - V_0$ is determined experimentally for a current i the value of $\frac{2ib^2}{ka^2X_1^2}$ may be determined, and the value of k may therefore be found.

The theory may be tested by finding the values of k by this method and comparing the results with the values obtained by more direct methods, or if the theory is accepted the experiments provide a simple method of finding the velocities for a large range of pressures of the gas.

The following are the results of some experiments made to investigate the values of k_1 and k_2 for positive and negative ions in air at different pressures. The cylinder was 7·49 cm. radius and the co-axial wire ·0268 cm. radius. The cylinder was provided with two small side tubes, one at each end, and a stream of dry air that had passed through tubes of phosphorus pentoxide was drawn through the cylinder. A series of experiments were made before all the moisture was expelled by heating the cylinder, and the results obtained with positive and negative discharges in air at three different pressures are given in Tables I. and II. The pressures p are given in millimetres of mercury, and the potentials V and V_0 and the currents i in electrostatic units.

Table I.

Velocities k_1 of positive ions under unit electrostatic force.

p.	V_0.	V.	i.	$k_1 \times 10^{-3}$.	$\frac{k_1 p}{760}$.
176	15·6	17·3	321	1·46	339
176	15·6	18·53	660	1·58	365
84	9·85	10·95	306	3·40	376
84	9·85	11·73	642	3·65	404
30	5·75	6·35	305	10·8	420
30	5·75	6·72	560	12·0	470

TABLE II.

Velocities k_2 of negative ions under unit electrostatic force.

p.	V_0.	V.	i.	$k_2 \times 10^{-3}$.	$\frac{k_2 p}{760}$.
176	18·3	19·2	317	2·70	630
176	18·3	19·9	655	2·70	630
84	11·5	12·02	350	7·8	860
84	11·5	12·43	750	8·9	980
30	6·22	6·39	305	40·5	1600
30	6·22	6·50	570	43·2	1700

The cylinder was then heated and filled with dry air and exhausted, the process having been repeated several times. The experiments were again made with the drier air, and the results are given in Tables III. and IV.

TABLE III.—Velocities k_1 of positive ions under unit electrostatic force (after the cylinder had been dried).

p.	V_0.	V.	i.	$k_1 \times 10^{-3}$.	$\frac{k_1 p}{760}$.
176	15·50	16·82	290	1·73	400
176	15·50	18·28	680	1·86	430
84	9·83	10·78	337	4·45	492
84	9·83	11·33	570	4·42	490
30	5·65	6·07	305	16·0	630
30	5·65	6·53	687	15·3	610

TABLE IV.—Velocities k_2 of negative ions under unit electrostatic force (after the cylinder had been dried).

p.	V_0.	V.	i.	$k_2 \times 10^{-3}$.	$\frac{k_2 p}{760}$.
176	18·37	19·03	358	3·95	915
176	18·37	19·43	580	3·87	900
84	11·68	12·00	345	13·1	1450
84	11·68	12·28	660	12·3	1360
30	6·44	6·49	300	140	5600
30	6·44	6·55	540	120	4700

It will be observed that the rise of potential $V-V_0$ for a given current is diminished by drying the gas. The velocities are therefore reduced by the presence of water vapour when the ions move away from the strong field in which they are generated. The mean value of $k_1p/760=415$, deduced from the experiments in dry air at 176 mm. pressure, is in good agreement with the velocities 408 and 427 obtained by Zeleny* and Langevin† by direct methods.

The mean values 491 and 620 obtained at the pressures of 84 and 30 mm. respectively, show that the velocity of positive ions increases more rapidly than the inverse of the pressure. Langevin, whose determinations were made with smaller forces, found that k_1p was practically constant for pressures between 75 millimetres and 1420 millimetres.

The numbers given in Table IV. show that even at the pressure 176 mm. the negative ions move much faster than the positive ions, the velocity being eight times greater at the pressure 30 millimetres. This is a well known effect, and is due to the tendency of the negative ions to assume the electronic state when the value of X/p increases. The phenomenon only occurs at the higher pressures when the intensity X is large, and has not been observed at the higher pressure by the direct methods of finding the velocities.

The theory also gives satisfactory results when applied to the experiments made by Watson‡ and Schaffers§ on the discharges at higher pressures. Watson determined the potential required to maintain currents from a wire ·35 millimetres radius and a co-axial cylinder 10·2 centimetres radius. The values of k_1 and k_2 for positive and negative ions deduced from these experiments are given in Tables V. and VI., the current i per unit length of the wire and the potential V being in electrostatic units.

There is a good agreement between the values of the velocities deduced from experiments at the same pressure with different currents. The mean values are rather low for dry air. This may be due to the oxides of nitrogen which are formed by the discharge as they would condense on the ions and diminish the velocities, an effect which may be appreciable with the larger currents when the air is imperfectly dried.

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* J. Zeleny, Phil. Trans. A. cxcv. p. 193 (1900).

† P. Langevin, *Annales de Chemie et de Physique,* [7] xxviii. p. 289 (1903).

‡ E. Watson, 'Electrician,' February 11, 1910.

§ V. Schaffers, *Phys. Zeits.* xv. (1914).

TABLE V.

Values of k_1 deduced from Watson's experiments with air at 748 mm. pressure.

V.	i.	k_1.	$\frac{k_1 p}{760}$.
51·3	0		
58·0	600	380	374
63·5	1200	425	418
72·5	2400	360	354
79·8	3600	340	334

TABLE VI.

Values of k_2 deduced from Watson's experiments with air at various pressures p.

p.	V.	i.	k_2.	$\frac{k_2 p}{760}$.
748	52·8	0		
	58·8	600	446	440
	63·5	1200	446	440
	71·0	2400	422	415
	76·0	3600	450	443
552	43·0	0		
	47·0	600	745	542
	50·2	1200	790	575
	55·3	2400	785	570
	60·2	3600	740	539
356	30·4	0		
	34·3	600	1580	740
	36·4	1200	1660	780
	40·5	2400	1520	712
	43·7	3600	1490	700

In Schaffer's experiments the air was at atmospheric pressure, and the variation of the potential with the current was determined for wires in cylinders of different diameters. The results show that the rate of change of the potential with the current, dV/di, is small with the smaller cylinders, but increases rapidly with the radius of the cylinder. It is difficult to form an accurate estimate of the values of $V-V_0$ for the smaller cylinders, but with the larger cylinders of radii 3·85 and 5·85 centimetres, the rise of potential with the current may be found to a sufficient degree of accuracy, to calculate the values of k.

Table VII. gives the results of some of these experiments. The values of k_2 are in good agreement with the numbers obtained from Watson's experiments, but k_1 is somewhat larger.

TABLE VII.

Values of k_1 and k_2 for air at atmospheric pressures deduced from Schaffer's experiments, b being the radius of the cylinder and a the radius of the wire in centimetres.

V_0.	V.	b.	a.	i.	k_2.	k_1.
8·85	14·2	3·85	·00127	935	410	
15·6	22·6	3·85	·00385	1870	411	
24·5	34·2	5·85	·0099	1870	...	442
57·5	66·8	5·85	·052	3740	...	425

XII. *Atomic Structure and the Spectrum of Helium.* *By* J. W. NICHOLSON, *M.A., D.Sc., Professor of Mathematics in the University of London*[*].

AN earlier paper[†] has indicated that the crucial test of Bohr's theory of spectra is to be found in its application to the ordinary spectrum of helium. For the theory gives a precise specification of the constituents of a helium atom,—a specification which is in accord with the results obtained by a combination of the atomic number hypothesis of Van den Broek and the experiments of Moseley, and with those to which Rutherford and others have been led by some other lines of study. The necessity for this accordance, in fact, dictated the particular model which Bohr has used. His corresponding models for lithium and the heavier elements have been shown to be at fault, in that they involve distributions of electrons which cannot exist by virtue of the assumptions on which the theory is founded, in spite of the fact that these assumptions are of a very general character. The success of the theory therefore rests on its application to the spectra of hydrogen and helium,—leaving out of account, for the moment, the controversial question of its application to X-ray spectra, which was partially dealt with in the earlier paper,—in so far as that application has been made. The theory must therefore stand or fall according to

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[*] Communicated by the Author.

[†] Phil. Mag. April 1914.