A CONDENSER TRANSMITTER AS A UNIFORMLY SENSITIVE INSTRUMENT FOR THE ABSOLUTE MEASUREMENT OF SOUND INTENSITY.

By E. C. Wente.

THE various methods that have been used with more or less success for measuring the intensity of sound may be divided into five general classes: observation of the variation in index of refraction of the air by an optical interference method; measurement of the static pressure exerted on a reflecting wall; the use of a Rayleigh disc with a resonator; methods in which the motion of a diaphragm is observed by an optical method; the use of some type of telephone transmitter in connection with auxiliary electrical apparatus. The apparatus of either of the first two methods is non-resonant and hence the sensitiveness is fairly uniform over a wide range of frequencies. These methods are not sufficiently sensitive, however, to be of use in general acoustic measurements. On the other hand, instruments of the last three classes possess a natural frequency and are consequently very efficient in the resonance region. However, in the neighborhood of the resonant frequency the efficiency varies greatly with the pitch of the tone. It is possible to use a Rayleigh disc without a resonator, but its sensitiveness in that case is so low that it is of little practical value.

Because of the recent advances in the development of distortionless current amplifiers, the last class, in which use is made of some form of telephone transmitter, seems to offer the greatest possibilities. In the following pages a transmitter is described which has been calibrated in absolute terms for frequencies from 0 up to 10,000 periods per second and which has a nearly uniform sensibility over this range. The apparatus is easily portable, and possesses no delicate parts, so that, when once adjusted, it will remain so for a long period of time.

Except in cases where measurements are made with a single, continuous tone, it is desirable that the instrument for measuring the intensity of sound should have approximately the same sensibility over the entire range of frequencies used. This is especially important if the sound under investigation has a complex wave form. To avoid any great variation with frequency in the sensibility of a phonometer employing a vibrating system, it is necessary that the natural frequency lie outside the range of frequencies of the tones to be measured. Even if the natural frequency be compensated for in other ways, small variations in the constants of the instruments, which are always likely to occur, may change conditions appreciably at this frequency. It is pretty well recognized that for several reasons the natural frequency should lie above rather than below the acoustic range. If the instrument is to be used in studying speech, the natural frequency must indeed be very high. The upper limit of the frequencies occurring in speech is not definitely known, but it probably does not come below 8,000 periods a second. Titchener[1] found that if a Galton whistle was set so as to give a frequency of 8,500, the tone emitted could not be distinguished from an ordinary hiss.

An instrument that is to be used in studying speech should have high damping as well as a high natural frequency in order to reduce distortion due to transients. This is not so important if the natural frequency lies beyond the acoustic range, but nevertheless is desirable even in this case. Aperiodic damping is the best condition, but it is in general hard to obtain when the natural frequency is very high.

It seems best in this paper to give a rather complete treatment of the condenser instrument; for the sake of clearness, however, breaking up the matter into a number of sections as follows:

1. Theory of the Operation of an Electrostatic Transmitter.
2. General Features of the Design of the Instrument.
3. Deflection of the Diaphragm under a Static Force. Measurement of Tension and Airgap.
4. Sensitiveness of the Transmitter at Low Frequencies.
5. Sensitiveness at Higher Frequencies Determined by the Use of a Thermophone.
6. Natural Frequency and Damping of the Diaphragm.
7. Possibilities of Tuning.
8. Characteristic Features of the Instrument.
9. The Electrostatic Instrument used as a Standard Source of Sound.
10. Summary.

Some of these sections deal with theory and some with experimental work as need arises, the general aim being to put in proper order the material necessary for a full account of the condenser instrument.

1. Theory of the Operation of an Electro-static Transmitter.

The device to be described is a condenser transmitter, the capacity of which follows very closely the pressure variations in the sound waves. The use of such a device as a transmitter is not a new idea; in fact it

[1] Proc. Am. Phil. Soc., 53, p. 323.

was suggested almost as early as that of the corresponding electromagnetic instrument.[1] However, before good current amplifiers were available little or no use was made of electrostatic transmitters because of their comparatively low efficiency.

A simple circuit that may be used with such a transmitter is shown in Fig. 1. When the capacity of the transmitter is varied, there will be a corresponding drop of potential across R, which may be measured with an A.C. voltmeter or some other suitable device.

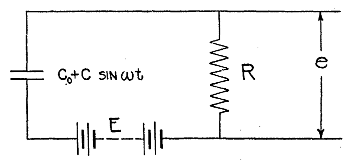

Fig. 1.

In order to get a quantitative expression for the magnitude of this voltage let us assume that the capacity at any instant is given by

$$C = C_0 + C_1 \sin \omega t,$$

in which $\omega = 2\pi \times$ frequency. For the circuit shown in Fig. 1

$$E - Ri = \frac{1}{C}\int idt. \qquad (1)$$

By differentiation and substitution we obtain

$$(C_0 + C_1 \sin \omega t)R\frac{di}{dt} + (1 + RC_1\omega \cos \omega t)i - EC_1\omega \cos \omega t = 0. \qquad (2)$$

In order to evaluate this equation let us assume as a solution

$$i = \Sigma I_n \sin (n\omega t + \phi_n).$$

Substituting this value of i in (2) and determining the coefficients, we have

$$\left.\begin{aligned} i = {} & \frac{EC_1}{C_0\sqrt{\left(\frac{1}{C_0\omega}\right)^2 + R^2}}(\sin \omega t + \varphi_1) \\ & - \frac{EC_1^2R}{C_0^2\sqrt{\left[\left(\frac{1}{C_0\omega}\right)^2 + 4R^2\right]\left[\left(\frac{1}{C_0\omega}\right)^2 + R^2\right]}}\sin(2\omega t + \varphi_1 - \varphi_2) \\ & + \text{terms of higher order in } C_1/C_0, \end{aligned}\right\} \qquad (3)$$

in which

$$\phi_1 = \tan^{-1}\frac{1}{C_0\omega R} \quad \text{and} \quad \varphi_2 = \tan^{-1}\frac{1}{2C_0\omega R}, \quad \text{etc.}$$

[1] La Lumiere Electrique, Vol. 3, p. 286, 1881.

For the best efficiency R should be made large in comparison with $1/C_0\omega$. In this case, the expression for the voltage e becomes

$$e = Ri = \frac{EC_1}{C_0} \sin (\omega t + \varphi_1) - \frac{EC_1^2}{2C_0^2} \sin (2\omega t + \varphi_1 - \varphi_2) + \cdots.$$

From this equation we see that in order to get a voltage of pure sine wave form for a harmonic variation of capacity, C_1 must be small in comparison with $2C_0$. This condition is satisfied as long as the A.C. voltage is small compared with E.

Retaining only the first term in (3) we have

$$e = Ri = \frac{EC_1R}{C_0\sqrt{\frac{1}{C_0^2\omega^2} + R^2}} \sin (\omega t + \varphi_1). \tag{4}$$

This equation shows that, so far as its operation in the circuit is concerned, the transmitter may be considered an alternating current generator giving an open circuit voltage $E(C_1/C_0) \sin (\omega t + \varphi_1)$ and having an internal impedance $1/C_0\omega$. It can also be shown that the transmitter can be regarded from this point of view if R is replaced by a leaky condenser or an inductance, so that this result may be said to be true in general.

2. General Features in the Design of the Instrument.

The general construction of the transmitter is shown in Fig. 2, from which the principal features are evident. The diaphragm is made of steel, 0.007 cm. in thickness, and is stretched nearly to its elastic limit. The condenser is formed by the plate B and the diaphragm. Since the diaphragm motion is greatest near the center, the voltage generated, which is proportional to C_1/C_0, will be greatest if the plate is small. On the other hand, since C_0 is proportional to the size of the plate, it cannot be made too small or the internal impedance of the transmitter will be too great. Therefore from the standpoint of efficiency, a compromise has to be made in determining the area of the plate. However, if it is made much smaller than the diaphragm, the natural frequency of the vibrating system will be decreased, as is explained below. On the basis of these factors the size of the plate indicated was judged to be about the best for the transmitter.

After some experiments with various dielectrics between the plate and the diaphragm it was concluded that air was most suitable. The dielectric constant of air is not so high as that of some other materials, but its insulating properties are better. However, the principal advantage of using air is, that it has a high minimum value of sparking potential which lies in the neighborhood of 400 volts, below which there is no appreciable conduction. When E is less than this voltage, the air gap may be decreased without decreasing E, so that the efficiency of the instrument is limited practically only by the fact that when the gap is decreased below a certain value, the electrostatic force between the plate and diaphragm deflects the latter sufficiently to short circuit the condenser. When a potential difference of 320 volts was applied to the transmitter shown in Fig. 4, no appreciable current flowed across the air gap, certainly not more than 10^{-8} amperes. The fact that the air has such a high minimum sparking potential is one of the principal reasons why it is possible to design a successful condenser transmitter of the type shown in Fig. 2.

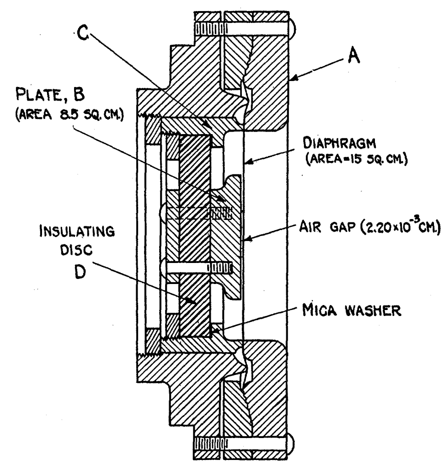

Fig. 2.

Sectional drawing of transmitter.

A word may be said in regard to the method of adjusting the transmitter so as to obtain a small uniform air gap. The surface of part A, next to the diaphragm, was ground plane before assembling. Small irregularities in the surface of the diaphragm facing the plate were removed by grinding with fine carborundum. Parts B, C and D were first assembled without the mica washer. The face of the plate and the ends of part C were then ground to the same level. Finally the mica washer was inserted between C and D and the whole apparatus assembled as shown. The mica may be split into washers of very even thickness, and a uniform air gap so obtained. The diaphragm is clamped between parts A and C, and is thus held in a true plane. In assembling the parts, the greatest care must be taken that no dust is caught between the plate and the diaphragm, for the insulation may be considerably reduced by the presence of any small particles in the gap.

Part C does not fit so perfectly against the diaphragm that the space surrounding the plate is shut off completely from the outside air. Changes in temperature and atmospheric pressure will therefore not affect the equilibrium position of the diaphragm.

The instrument used in these experiments was constructed just as shown in Fig. 2. It is evident from this figure that the diaphragm may be brought into contact with the plate if a mechanical pressure is accidentally exerted on the diaphragm. This will cause a spark to pass, if the transmitter was previously charged. In order to avoid damaging the metal surfaces in this way it may be advisable to glue to the face of the plate, A, a very thin layer of mica of uniform thickness, while still retaining an air-gap sufficient to allow free motion of the diaphragm.

3. Deflection of the diaphragm under a Static Force; Measurement of Tension and Air Gap.

It is not difficult to calculate the sensitiveness of the transmitter for low frequencies from the dimensions of its various parts, provided the magnitude of the deflection of every point of the diaphragm produced by a given static force is known. Since the diaphragm is made of very thin material and the tension is high, we may expect the diaphragm to behave very much as an ideal membrane, at least for frequencies near zero. In order to determine how closely this condition is approximated the following experiment was carried out.

When a static potential is applied between the plate and the diaphragm, the latter is deflected by the electrostatic force. The deflection produced

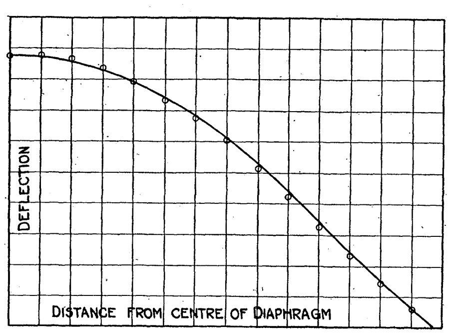

Fig. 3.

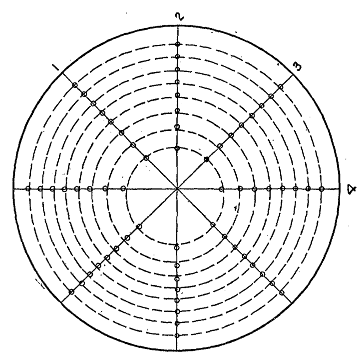

Fig. 4.

in this way by a known potential was measured by a device very similar to that used by Prof. D. C. Miller in his phonodeik.[1] By this arrangement the deflection of the diaphragm was magnified 30,000 times. The mean values of the deflections produced at various points along eight evenly spaced radii when a potential of 320 volts was applied are shown in Fig. 3. Points of equal displacement of the diaphragm are plotted in Fig. 4. The fact that the curves drawn through these points are

[1] D. C. Miller, Science of Musical Sounds, p. 79.

practically circles shows that the tension of the diaphragm was very nearly the same in all directions.

The distance between the plate and diaphragm was also measured with this apparatus by applying a mechanical force until the diaphragm touched the plate. The value obtained in this way was 2.20×10^{-3} cm. The capacity of the transmitter was measured on a capacity bridge and found to be 335×10^{-12} farads, from which the computed value of the air gap is 2.25×10^{-3} cm. The mean of the values obtained in these two ways is 2.22×10^{-3} cms.

In order to determine how closely the diaphragm approximates an ideal membrane, we may calculate the form that the latter would have assumed under the conditions of the preceding experiment.

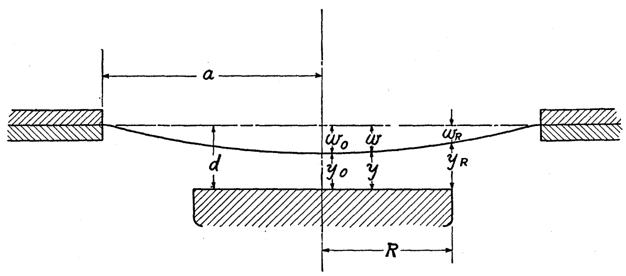

Fig. 5.

Referring to Fig. 5, if V is the potential between the plate and the diaphragm, and T, the tension of the membrane, we have

$$T\left[\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right] + \frac{V^2}{8\pi y^2} = 0.$$[1]

This relation holds from $r = 0$ to $r = R$. Let

$$A = \frac{V^2}{8\pi T}$$

and $x = \log r$, then since $(w - w_R) = (Y_R - y)$

$$\frac{d^2y}{dx^2} = A\frac{\epsilon^{2x}}{y^2},$$

or, since $(w - w_R)/(w_0 - w_R)$ is very nearly equal to $(R^2 - r^2)/r^2$ and $(w_0 - w_R)$ is small compared with y_0,

$$\frac{d^2y}{dx^2} = \frac{A}{y_0^2}\left(1 - 2k\frac{r^2}{R^2}\right)\epsilon^{2x},$$

in which

$$k = (w_0 - w_R)/y_0.$$

[1] Rayleigh, Theory of Sound, II., p. 318.

From this we get

$$w = \frac{A}{4y_0^2}(R^2 - r^2)\left[1 - \frac{k}{2} - \frac{k}{2}\frac{r^2}{R^2}\right] + w_R. \tag{5}$$

The total force on the diaphragm is

$$F = \frac{\pi R^2 V^2}{8\pi d_1^2}$$

where d_1 is defined by the equation

$$\frac{\pi R^2}{d_1^2} = \int_0^R \frac{2\pi r dr}{y^2} = \frac{2\pi}{y_0}\int_0^R \left(1 - 2k\frac{r^2}{R^2}\right) r dr = \frac{\pi R^2}{y_0}(1 - k).$$

so that

$$F = \frac{\pi R^2 V^2}{8\pi y_0^2}(1 - k).$$

In the region extending from $r = R$ to $r = a$,

$$F = -2\pi r T \frac{dw}{dr}.$$

From this

$$w_R = \frac{AR^2(1 - k)}{2y_0}\log\frac{a}{R}. \tag{6}$$

From (5) and (6)

$$w = \frac{A}{4y_0^2}\left\{(R^2 - r^2)\left[1 - \frac{k}{2}\left(1 + \frac{r^2}{R^2}\right)\right] + 2R^2(1 - k)\log\frac{a}{R}\right\}. \tag{7}$$

This equation gives the form into which the diaphragm will be bent if it behaves like an ideal membrane. The curve representing this equation is shown in Fig. 3. The observed points do not lie very far from this curve. We therefore conclude that the diaphragm behaves sufficiently like an ideal membrane, so that no great error will be incurred if this assumption is made in calculating the sensitiveness of the transmitter for low frequencies.

From equation (7)

$$w_0 = \frac{V^2}{32\pi T y_0^2}\left\{R^2\left(1 - \frac{k}{2}\right) + 2R^2(1 - k)\log\frac{a}{R}\right\}$$

or

$$T = \frac{V^2 R^2}{32\pi y_0^2 w_0}\left\{\left(1 - \frac{k}{2}\right) + 2(1 - k)\log\frac{a}{R}\right\}. \tag{8}$$

Hence, if the deflection at the center of the diaphragm produced by a known voltage is measured, the tension may be calculated from (8). Results obtained in this way for the diaphragm used in these experiments are tabulated below.

Volts.	Deflection (w_0) (cm.).	Tension (T) (dynes)/(cm.).
200	6.0×10^{-5}	6.59×10^{7}
240	6.8	6.58
280	12.4	6.55
320	16.9	6.55
Mean.		6.57×10^{7}

4. Sensitiveness of the Transmitter at Low Frequencies.

Having satisfied ourselves that the diaphragm behaves sufficiently like a perfect membrane, and having determined the tension and air gap, we can now proceed to calculate the efficiency of the transmitter for low frequencies. To do this it is necessary to find the change in capacity produced by a given pressure on the diaphragm, since by equation (4) the voltage generated is proportional to C_1/C_0.

Referring to Fig. 5, we see that the capacity is $R^2/4d$ if the diaphragm is not deflected. From the curve of deformation when a potential is applied (Fig. 3), it is evident that w_R is very nearly equal to $0.45\ w_0$. Hence the air gap at any point is given by

$$d - w_0 + \frac{.55}{R^2} w_0 r^2,$$

and since the surface of the diaphragm deviates but little from a plane area, the normal capacity to the first approximation is

$$C_0 = \int_0^R \frac{2\pi r dr}{4\pi\left(d - w_0 + \frac{0.55 w_0}{R^2} r^2\right)} = \frac{R^2}{4d^2}\left[1 + \frac{w_0}{d} - \frac{.55 w_0}{2d}\right] = \frac{R^2}{4d'}, \quad (9)$$

in which d' may be called the effective air gap.

If a pressure, P, uniform all over the diaphragm produces a deflection, u, the capacity of the condenser will have been changed by the amount

$$C_1 = \int_0^R \frac{u\cdot 2\pi r dr}{4\pi y^2}. \quad (10)$$

The quantity in brackets of equation (9) does not differ greatly from unity in any practical case, so that no great error will be incurred if we set y in (10) equal to the constant value d'. Since

$$u = \frac{P}{4T}(a^2 - r^2),$$ [1]

equation (16) may be written

$$C_1 = \frac{P}{8Td'^2}\int_0^R (a^2 - r^2) r dr = \frac{PR^2}{32Td'^2}[2a^2 - R^2]. \quad (11)$$

[1] Lamb, Dynamical Theory of Sound, p. 150.

C_1 is the change in capacity produced by a static pressure, P; but this differs very little from the maximum value of the alternating capacity resulting from a pressure, $P \sin \omega t$, provided $\omega/2\pi$ is small compared with the natural frequency of the diaphragm.

Having determined C_1 per unit value of P from equation (11), and C_0 from (9), we may calculate C_1/C_0 and hence the sensitiveness, *i. e.*, the volts per unit pressure. In practically all the experiments that have been made with the electrostatic transmitter, the D.C. voltage was 321. Under this condition we obtain 315 E.S.U. for C_0 from (9) and 1.96×10^{-3} E.S.U. per dyne per sq. cm. for C_1/P from (11). Hence we have for the sensitiveness

$$\frac{EC_1}{PC_0} = \frac{1.96 \times 10^{-3} \times 321}{315} = \frac{2.00 \times 10^{-3} \text{ volts}}{\text{dynes per sq. cm.}}.$$

In order to check this value directly by experiment, the apparatus diagrammatically shown in Fig. 6 was constructed. A receptacle was

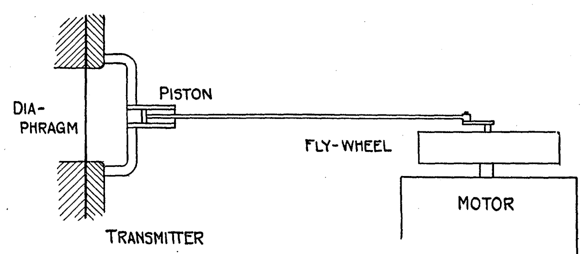

Fig. 6.

placed over the diaphragm as shown in the figure, thus forming an air-tight enclosure. Connected to this was a cylinder containing a piston. The connecting rod was long compared with the stroke of the piston so that with the motor running, the piston was given practically a simple harmonic motion. The fly wheel was fairly heavy and the connecting rod was made of stiff tubing, so that but little vibration was noticeable even when the motor ran at the highest speed.

The pressure variation is given by

$$\delta P = 1.4 P \frac{\delta V}{V},$$

in which δV is one half the total piston displacement and P is the maximum value of the alternating pressure.

$$V = 45.2 \text{ c.c. (volume of chamber)}$$

$$\delta V = \frac{0.68 \times 0.418}{2} = .142 \text{ c.c.}$$

Hence

$$\delta P = 1.4 \times 10^6 \times \frac{1.42}{45.2} = 4{,}400 \text{ dynes per cm.}^2$$

The root mean square value of the pressure is

$$\frac{4{,}400}{\sqrt{2}} = 3{,}120 \text{ dynes per cm.}^2.$$

The circuit used in this test is shown in Fig. 7. The electrostatic voltmeter had a very small capacity, giving it at low frequency an impedance large compared with the 80 megohm resistance in shunt.

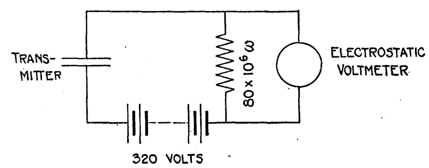

Fig. 7.

We may then calculate the open circuit voltage given by the transmitter from the voltmeter reading and the constants of the circuit, remembering that the transmitter may be regarded as a generator having an internal impedance $1/C_0\omega$. The following values were obtained in this way.

Motor Speed, R.P.M.	Frequency (P.P.S.).	Voltmeter Reading (Volts).	Open Circuit Volts.
1239	20.7	5.31	6.22
1074	17.9	5.31	6.27
950	15.8	5.31	6.33
824	13.75	5.20	6.29
584	9.75	4.92	6.27
Mean..........	....	....	........6.28

We therefore have for the sensitiveness,

$$\frac{6.28}{3120} = 2.02 \times 10^{-3} \frac{\text{Volts}}{\text{dynes per sq. cm.}}.$$

This value is in very close agreement with that given before, so that we may consider 2.00×10^{-3} volts per dyne as a reasonably correct value for the sensitiveness at low frequencies.

5. Sensitiveness at Higher Frequencies as Determined by the Use of a Thermophone.

By the methods just described the values of sensitiveness may be determined for very low frequencies only. In order to measure the sensitiveness at higher frequencies and also to get an idea of the natural frequency and damping of the vibrating system, use was made of the principle involved in the action of the thermophone as described by Arnold and Crandall.[1] A block of lead about 1.5 inches thick was placed against the face of the transmitter so as to form a cylindrical enclosure in front of the diaphragm, 1¾ inches in diameter and ⅜ inch long. The general arrangement is shown in Fig. 8. All crevices were sealed up so that the only openings to the cavity were two capillary tubes several inches long and of about 0.01 cm. bore. Two strips of gold foil were mounted symmetrically inside of this enclosure, the ends being clamped between small brass blocks. The supports were arranged in such a way that a current could be passed through the two strips in series. The connection between them was in electrical connection with the diaphragm.

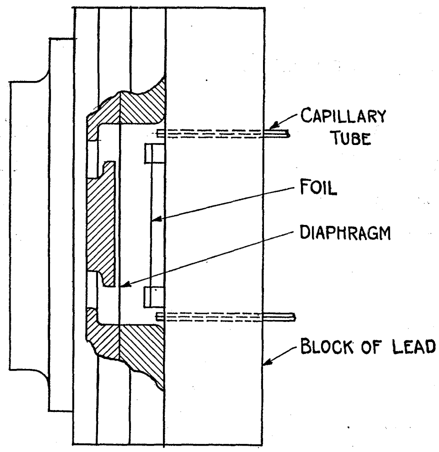

Fig. 8.

In the paper just cited it is shown that within an air-tight enclosure

$$\delta P = \frac{.0106 R i^2 P \sqrt{K_0} \left(\frac{\theta_1}{273}\right)^{1/4}}{\gamma V_0 \theta_2 f^{3/2}}, \tag{12}$$

in which

δP = maximum value of the alternating pressure within the enclosure.
P = normal pressure within the enclosure.
R = resistance of the foil.
i = r.m.s. value of the alternating current passing through the foil.
K_0 = diffusivity at 0° C. of the gas within the enclosure.
θ_1 = mean absolute temperature of the foil.
θ_2 = mean absolute temperature of the gas.
γ = heat capacity per unit area of the foil.
V_0 = volume of the enclosure.
f = frequency of the alternating current.

Equation (12) may be used for calculating the pressure variation provided the wave-length of sound is large compared with the dimensions of the enclosure. The velocity of sound in hydrogen is about four times

[1] PHYSICAL REVIEW (Preceding Paper).

as great as in air; hence formula (12) holds for frequencies almost four times as high, when the enclosure is filled with hydrogen instead of air. Also, the diffusivity, K_0, is about six times as large for hydrogen as for air, so that greater pressure variation is obtained with the former. For these reasons, hydrogen was passed in a continuous stream through the enclosure by way of the capillary tubes, at a rate sufficiently slow to prevent any appreciable increase of the steady pressure above that of the atmosphere. The hydrogen was obtained from a Kipp generator and then passed through a solution of potassium permanganate and a drying tube containing phosphorus pentoxide.

In order to get the open circuit electromotive force of the transmitter, the circuit was arranged as in Fig. 9. The two resonant circuits were so

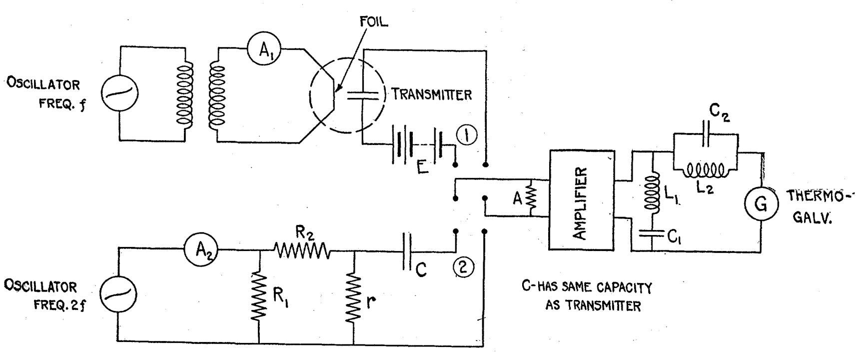

Fig. 9.

adjusted as to prevent current of the same frequency as that given by the oscillator from passing through the galvanometer. If a pure sine wave current passes through the foil, the pressure variation in the enclosure is of pure sine wave form and of double frequency. However, if there is any second harmonic present in the current, there will also be a component of the pressure variation of single frequency.[1] Putting in the resonant circuits eliminates this component from the measurements. The general procedure in making a measurement was as follows. The double-throw switch was first put in position 1, and the foil current and galvanometer current read. The switch was then thrown in position 2; R and r were then adjusted until the galvanometer read approximately the same as before. From the readings of A_2 and the values of R_1, R_2 and r, the voltage drop across r may be calculated. The open circuit voltage of the transmitter is then obtained by multiplying this voltage drop by the ratio of the galvanometer readings. That this gives us the open circuit voltage, follows from the fact that the transmitter

[1] Arnold & Crandall, *loc. cit.*

behaves as a generator having an internal impedance $1/C_0\omega$. Oscillator No. 2, of course, is set at double the frequency of Oscillator No. 1.

The current passed through the gold foil was about 0.5 ampere at all frequencies. Resistance measurements showed that with this current density, the foil was not heated more than 10° C. above the room temperature. The values of the quantities entering into the formula (12) for this experiment were as follows:

$R = 4.18$ ohms.
$P = 10^6$ dynes/cm².
$K_0 = 1.48$ C.G.S. units.
$\theta_2 = 295°$.
$\theta_1 = 305°$.
$\gamma = 4.15 \times 10^{-6}$ calories per sq. cm.
Thickness of gold foil $= 7 \times 10^{-6}$ cm.
Width of each strip $= 1$ cm.
Length of each strip $= 2.6$ cm.

Substituting these values in equation (12), we have for the root mean square value of the alternating pressure

$$2.59 \times 10^6 \frac{i^2}{f^{3/2}} \text{ dynes per sq. cm.}$$

Dividing the measured open circuit voltage by this value should give us the volts per unit pressure for all frequencies within certain limits. Measurements were made in this manner for frequencies from 160 to 18,000 cycles per second. The general shape of the curve obtained by plotting these values is shown in Fig. 10.

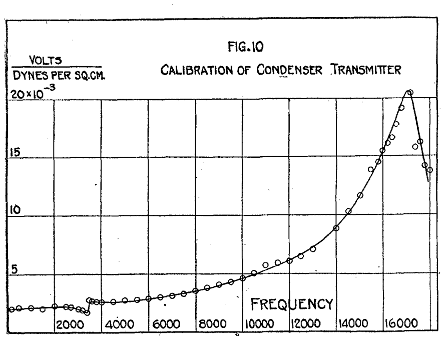

Fig. 10.

The absolute value of the sensitiveness at low frequencies as determined by this method was 0.121×10^{-3} volts per dyne, which is only about one sixteenth of that previously obtained by the piston method and by calculation from the dimensions of the instrument. In order to make further tests within the range of frequencies from 20 to 160 cycles, the gold was replaced by platinum foil, 4.42×10^{-4} cm. thick, and measurements were made as before. However, the size of the enclosure was increased in order to meet the conditions assumed in the derivation of formula (12), and for the same reason air was used instead of hydrogen.

Calculations made in a manner similar to that when gold foil was used gave a value of 1.93×10^{-3} volts per dyne per sq. cm. for the sensitiveness at low frequencies. This is in fair agreement with the value 2.00×10^{-3} obtained theoretically and with the piston apparatus.

Apparently when gold foil is immersed in hydrogen something takes place which is not taken account of in equation (12). The gold foil used was extremely thin (7×10^{-6} cm.) and when placed in hydrogen its specific heat per unit volume was apparently much greater than that of pure gold assumed in the calculations. On account of this discrepancy the gold leaf could not be relied upon for an absolute calibration, but it seemed reasonable to assume that the ratio between the true pressure and that calculated was independent of the frequency, so that a true relative calibration for different frequencies could be obtained. To get the absolute value of the efficiency at all frequencies, the values calculated from the readings on the gold foil were multiplied by the factor $2.0/.121 = 16.6$. The results so obtained are those plotted in Fig. 10.

6. Natural Frequency and Damping of the Diaphragm.

It is thought that this curve (Fig. 10) may be relied upon to give the sensitiveness in absolute value for frequencies up to 10,000 cycles. Above this frequency the wave-length of sound approaches the diameter of the cylindrical enclosure. The wave-length in hydrogen at 10,000 cycles is 13 cm. whereas the greatest distance from boundary to boundary of the enclosure is 4.4 cm. Although the absolute values of the sensitiveness above 10,000 cycles are probably not given by the points plotted in Fig. 10, nevertheless, this curve indicates in a general way the behavior of the transmitter at high frequencies. The principal peak in this curve comes at 17,000 cycles, which undoubtedly corresponds to the natural frequency of the diaphragm. The damping cannot be determined with any great assurance of accuracy, although the curve as drawn would indicate a damping factor of the vibrating system of about six or seven thousand.[1]

These high values of natural frequency and damping are in a large measure due to the cushion effect of the air between the plate and the diaphragm. Free lateral motion of the air is prevented by its viscosity. This increases the rate of dissipation of energy when the diaphragm is vibrating and also adds to its elasticity.

To see whether 17,000 cycles is a reasonable value for the natural frequency we may make an approximate theoretical calculation. When

[1] The term damping factor as here used may be defined as the reciprocal of the time required for the amplitude to fall to 1/2.718 of its initial value.

the frequency is as high as 17,000 cycles it seems reasonable to assume that there is practically no lateral motion of the film of air. Let us further assume that the film of air is compressed and rarified adiabatically by the motion of the diaphragm and also that the plate is of the same size as the diaphragm. This latter condition is not quite satisfied in the case of the electro-static transmitter but no great error is introduced by this assumption, since the motion near the edge of the diaphragm is small. Under these conditions if d is the length of the air gap, P, the atmospheric pressure, ρ, the mass per unit area of the diaphragm, and T, the tension, the equation of motion of the diaphragm becomes:

$$\rho \frac{d^2w}{dt^2} = T\left(\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right) - \frac{1.4Pw}{d},[1]$$

or since w varies as $\epsilon^{j\omega t}$,

$$\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr} + \left(\frac{\rho\omega}{T} - \frac{1.4P}{Td}\right) w = 0. \qquad (13)$$

The solution of (13), consistent with the boundary conditions, is

$$w = J_0(lr),$$

in which

$$l = \sqrt{\frac{\rho\omega^2}{T} - \frac{1.4P}{Td}}.$$

The boundary conditions require that $J_0(la) = 0$. The lowest root of this equation is 2.4 so that

$$\left[\rho\omega^2 - \frac{1.4P}{d}\right]\frac{a^2}{T} = (2.4)^2$$

or

$$f_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{T}{\rho}}\left(\frac{2.4}{a}\right)\sqrt{1 + \frac{1.4Pa^2}{Td(2.4)^2}}.$$

This equation gives the natural frequency of the diaphragm when vibrating in its fundamental mode.

For the transmitter used in the preceding tests —

$$T = 6.57 \times 10^7 \text{ dynes per cm.}$$

$$\rho = .05 \text{ gm. per sq. cm.}$$

$$a = 2.18 \text{ cm.}$$

$$P = 10^6 \text{ dynes per sq. cm.}$$

$$d = 2.22 \times 10^{-3} \text{ cm.}$$

Hence

$$f_0 = 6{,}350\sqrt{8.9} = 19{,}000 \text{ P.P.S.}$$

[1] Rayleigh, Theory of Sound, L., 318.

which is slightly higher than the observed value. With the plate removed, the diaphragm would have a natural frequency of 6,350. This shows that the film of air between the plate and the diaphragm increases the elastic factor many times. It is due entirely to this fact that it has been possible to obtain natural frequencies above 10,000 without making the diaphragm exceptionally small.

We may satisfy ourselves that the maximum point in the efficiency curve is not due to resonance in the cylindrical enclosure by calculating its resonant frequencies. These frequencies are determined by the equation

$$J_n'(\sqrt{K^2 - p^2\pi^2 l^{-2}}\, R) = 0 \text{[1]}$$

in which

$$K = \frac{2}{a}\pi f_0.$$

a = velocity of sound.

l = length of cylinder.

R = radius of cylinder.

p = an integer.

Since the foil was placed symmetrically in the enclosure, only the symmetrical modes of vibration need be considered, in which case $n = 0$.

The first root of the equation $J_0'(Z) = 0$ is 3.83. For the lowest resonant frequency, $p = 0$, so that we have

$$f_0 = \frac{a}{2} \cdot \frac{3.83}{R}.$$

In this problem

a = 127,000 cm./sec. (velocity of sound in hydrogen).

R = 2.18 cm.

hence

$$f_0 = 35{,}500 \text{ cycles per second.}$$

which is very much above the frequencies covered in the calibration.

If the enclosure is filled with air instead of hydrogen, the first resonant frequency comes at about one fourth of 35,500 or 9,000 p.p.s. A series of measurements were made with the circuit arranged as in Fig. 9, and air instead of hydrogen surrounding the gold foil. Points were calculated and plotted; the curve so obtained showed a sharp resonant point at 9,600 but none below. This may be taken as further evidence that the maximum point in Fig. 10 is not due to any resonance in the enclosure and so corresponds to the natural frequency of the diaphragm.

[1] Rayleigh, Theory of Sound, II, 300.

There is an irregularity in the calibration at about 3,500 periods per second. This is undoubtedly due to the natural frequency of the back-piece. At any rate, vibration of the plate would have an effect of this general character, *i. e.*, the efficiency would be decreased below, and increased above, resonance. In a later design, the plate and support have been made more rigid so as to form practically one solid piece. It is believed that with the newer model, the irregularity in the curve will have been eliminated.

This completes the account of the experimental work done in calibrating the instrument.

In order to obtain some idea of the sensitiveness of the electrostatic transmitter just described as compared with an electromagnetic instrument, the sensitiveness of the former was compared directly with an ordinary telephone receiver used as a transmitter, over a considerable range of frequencies. Except within a hundred cycles of the resonant frequency of the diaphragm of the receiver the electrostatic transmitter was found to generate a greater voltage for a given sound intensity.

7. Possibilities of Tuning.

Since an electrostatic transmitter is equivalent to an alternating current generator having an internal impedance $1/C_0\omega$, it is evident that, if in the circuit shown in Fig. 1, the resistance R is replaced by an inductance L, the voltage e will be a maximum for a frequency of

$$f = \frac{1}{2\pi\sqrt{LC_0}}.$$

The sharpness of tuning will of course depend upon the possibility of getting an inductance with a small resistance. In many problems in acoustics it is desirable to have a tuned system and in that case it is also better to have a diaphragm of low natural frequency and damping.

In order to get an expression for the sensitiveness as a function of the frequency, let us assume that we have a parallel plate condenser, one of the plates of which is fixed and the other moved perpendicularly to its own plane by a simple harmonic force. Practically this condition is approximated by a diaphragm, the center of which is separated a short distance from a plane plate as is shown in Fig. 2.

Let x = displacement of the diaphragm from its equilibrium position.
d = air gap, assumed large compared with x.

Then

$$\frac{1}{C} = \frac{1}{C_0}\left(1 + \frac{x}{d}\right).$$

The mechanical impedance of the diaphragm is

$$Z_1 = r + j\left(m\omega - \frac{s}{\omega}\right),$$

where r = resistance factor,

m = mass factor,

s = elasticity factor,

$\omega = 2\pi \times$ frequency.

If T = kinetic energy of the entire system,

W = potential energy of the entire system,

$2F$ = rate of dissipation of energy,

then

$$2T = m\dot{x}^2 + L\dot{y}^2,$$

$$2W = sx^2 + \frac{1}{C_0}\left(1 + \frac{x}{d}\right)(Y + y)^2,$$

$$2F = R\dot{y}^2 + r\dot{x}^2,$$

where Y is the permanent, and y the variable electric charge on the condenser. The equations of motion for the system are

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} + \frac{\partial F}{\partial \dot{x}} + \frac{\partial W}{dx} = P\epsilon^{j\omega t},$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right) - \frac{\partial T}{\partial y} + \frac{\partial F}{\partial \dot{y}} + \frac{\partial W}{\partial y} = 0.$$

If second order quantities are neglected, and also the constant terms, which affect only the equilibrium position, these equations become

$$\left.\begin{aligned} pm\dot{x} + r\dot{x} + \frac{s}{p}\dot{x} + \frac{Y\dot{y}}{pC_0 d} &= P \\ pL\dot{y} + R\dot{y} + \frac{1}{pC_0}\dot{y} + \frac{Y}{pC_0}\frac{\dot{x}}{d} &= 0 \end{aligned}\right\} \tag{14}$$

in which p is written for

$$j\omega = \frac{d}{dt},$$

solving equations (14) for $\dot{y}$ and substituting the values,

$$E = Y/C_0, \quad Z_1 = (r + s/p + pm), \quad Z = (R + 1/pC_0 + RL),$$

we have

$$\dot{y} = \frac{PE}{pd[(E/pd)^2 - Z_1 Z]},$$

or

$$e = \dot{y}(R + pL) = \frac{PE(R + pL)}{pd[(E/pd)^2 - Z_1Z]}.$$

In any practical case $(E/pd)^2$ is small compared with Z_1Z so that we may write without much error

$$e = \frac{EP(R + pL)}{pdZ_1Z}.$$

In order to obtain a large value of e/P, which is a measure of sensitivness, Z_1Z should be made small, *i. e.*, the diaphragm should have a natural frequency equal to the frequency, $\omega/2\pi$, and the electrical circuit should be in resonance at the same point.

No extensive measurements have been made with the circuit arranged in this way, although enough has been done to show that it is feasible in some cases. The chief difficulty lies in the fact that the transmitter capacity is so small that the inductance has to be very large to get resonance for ordinary sound frequencies. This difficulty may be overcome by shunting the transmitter with a condenser, which of course reduces the generated voltage.

8. Characteristic Features of the Instrument.

Because of the high internal impedance of the electrostatic transmitter, it is possible to use the instrument efficiently only with high impedance apparatus, such as an electrostatic voltmeter or a vacuum tube amplifier. However, this is no special disadvantage if an amplifier is to be used, because it is not desirable to use a transformer in connection with an instrument for measuring sound intensities, since the ratio of transformation of a transformer is not independent of either frequency or load.

The method for calibration of this instrument as explained in the preceding pages is rather elaborate and requires considerable care. But since the efficiency depends primarily on the air gap and tension, it should not be difficult to make duplicate transmitters to which the same calibration applies, since the desired values of air gap and tension may be obtained without great difficulty, the former being tested by measuring the capacity, and the latter by determining the deflection produced by a known potential between the plate and diaphragm.

The fact that the sensitiveness of this instrument is independent of any properties of material, such as magnetization or electrical resistance, is of considerable advantage. For this not only allows us to make instruments which are almost exact duplicates, and so let the calibration for one instrument serve for all the rest, but, the calibration is also constant with the time. The metal parts are of machine steel throughout; from the construction as shown in Fig. 2, it is therefore evident that temperature can affect the sensitiveness but little. The tension of the diaphragm is, of course, not absolutely independent of temperature, nor is the action of the cushion of air between the plate and the diaphragm independent of the barometric pressure: but these effects are hardly worth considering. Being made of heavy material, the transmitter satisfies the requirement in the way of ruggedness; having once been adjusted, it should remain so, even if subjected to considerable rough usage.

The sensitiveness of the transmitter is not absolutely uniform, but varies only about a hundred per cent. between zero and 10,000 cycles, as the curve in Fig. 10 shows. This variation is much less than would be the case with an electromagnetic instrument with a diaphragm having the same natural frequency and damping. Except for eddy current and iron losses, the voltage generated by an electromagnetic transmitter is proportional to the *velocity* of the diaphragm, whereas that given by the electrostatic transmitter is proportional to the *amplitude*. Below the natural frequency, the variation of velocity with frequency is much greater than the variation of amplitude since the velocity is proportional to the product of the frequency and amplitude.

In most problems the transmitter would be used with an amplifier. Now, the sensitiveness of the transmitter increases, whereas the efficiency of an amplifier sometimes decreases with the frequency; at any rate, it is possible to design a circuit for the amplifier, so that the combination of the two has a constant sensitiveness over a wide range of frequencies.

Since the natural frequency of the transmitter is very high, instantaneous records of sound waves obtained in combination with a distortionless oscillograph would not only give the relative amplitudes of the different frequencies into which the sound may be analyzed, but also the phase relations should be practically unchanged for frequencies up to 10,000 p.p.s.

As yet no instrument is available which will record without distortion currents of frequencies as high as 10,000 cycles. Only after such an instrument has been developed will it be possible to get a true record of consonant sounds. The same is true in regard to the quantitative study of the quality of musical instruments. However, by using an ordinary high frequency oscillograph in connection with a condenser transmitter and amplifier, it should be possible to get curves equal to or better than any obtained heretofore.

9. The Electrostatic Instrument Used as a Standard Source of Sound.

There is of course no theoretical reason why the instrument described in the preceding pages cannot be used in a reversible manner: that is, as a source of sound when an alternating voltage is applied between the plate and the diaphragm. If the instrument is to be used in this way, it is better to have the plate the same size as the diaphragm, in order to get the maximum electrostatic force for a given voltage and air gap. The resulting increase in capacity is in general no disadvantage in this case. Also for convenience in using the instrument it may be desirable to have the face of the plate covered with a thin layer of mica.

Because of the simplicity of this type of instrument it is not difficult to calculate the output of sound energy for a given voltage *after its efficiency as a transmitter has been determined.* It is evident that the instrument can be excited in two different ways; (*a*) the alternating voltage can be applied alone, and (*b*) it may be superimposed on a static potential maintained by a battery in exactly the same way as when the instrument is used as a transmitter. The main principles underlying the two kinds of excitation in this case are quite similar to those discussed by Arnold and Crandall in connection with the excitation of the thermophone by pure A.C. and by A.C. with D.C. superimposed. For this reason neither type of excitation need be discussed at length; but a brief treatment of the condenser instrument excited by pure alternating current will be given.

When a pure alternating voltage is applied, the mean deflection of the diaphragm will depend on the magnitude of this voltage and the efficiency may vary somewhat because of the change in mean air gap, and the consequent change in the cushion effect of the air sheet on the motion of the diaphragm. It is therefore necessary to have curves corresponding to the curve in Fig. 10 but for a series of applied static potentials. These are most easily obtained by determining for a number of frequencies the generated voltage as a function of the static potential when sound of a fixed intensity falls on the transmitter. It will be found that the alternating voltage generated is so nearly proportional to the static potential that for most acoustic work this may be assumed to be the case.

When an alternating potential $\sqrt{2}v \sin \omega t$ is applied to the plates, the electrostatic force per unit area acting on the diaphragm is

$$\frac{v^2}{8\pi d^2}(1 - \cos 2\omega t). \qquad (15)$$

Now refer to Fig. 10; assuming that the curve there shown gives the efficiency of the instrument, *used as a transmitter*, for an applied static potential v. If we multiply the ordinate (*i. e.*, the voltage per unit pressure) at frequency $\omega/\pi = f$ by the quantity

$$\frac{C_0}{v} \equiv \frac{C_0}{E}$$

we can obtain (cf. (4)) $\overline{C}_f$, the change in capacity per unit pressure. The total change in capacity due to the electrostatic force is then, (if $\overline{C}$ is the change per unit pressure at zero frequency)

$$C_1 = \frac{v^2}{8\pi d^2}(\overline{C} - \overline{C}_f \cos 2\omega t) \qquad (16)$$

from which we can proceed to calculate the amplitude of motion of the diaphragm.

It is necessary of course to have a mean value of d, the air gap, but it is sufficiently accurate to take an arithmetic mean of the values at the center and at the edge of the diaphragm. The motion at the center is greater, but the motion near the edges extends over a greater area.

In computing the mean amplitude of the diaphragm we shall introduce very little error if we take the form of the diaphragm as that of a paraboloid. u, the amplitude at any radius, r, is given by the relation already quoted

$$u = \frac{P}{4T}(a^2 - r^2), \qquad (17)$$

in which a = the radius of the diaphragm and plate. Equation (11) gives the total change in capacity in terms of P and T, that is (since $a = R$)

$$C_1 = \frac{a^4}{32d^2}\frac{P}{T}, \qquad (11')$$

or, eliminating P/T between (11′) and (17) we have, for displacement at any radial distance r, in terms of maximum capacity change

$$u = \frac{8d^2}{a^4}(a^2 - r^2)C_1.$$

Substituting for C_1 the value given in (16) we have

$$u = \frac{v^2}{\pi a^4}(a^2 - r^2)(\overline{C} - \overline{C}_f \cos 2\omega t), \qquad (18)$$

in which v is the r.m.s. value of the applied alternating voltage, and $\overline{C}_f$

is the change in capacity per unit pressure, determined in the manner described from the calibration curve of the instrument used as a transmitter. Equation (18) is rigorously true for all frequencies within the range of calibration, because the quantity $\overline{C}_f$ is taken from the calibration curve.

If, however, T is known, we can obtain an approximate value of u good at low frequencies, without any knowledge of C_1. (This is merely "equilibrium theory" and makes use only of the elastic factor, leaving the inertia and mechanical resistance of the moving system out of account). Substituting the value of electrostatic force (15) for P in (17) we have

$$u = \frac{v^2(a^2 - r^2)}{32\pi d^2 T}(1 - \cos 2\omega t). \tag{19}$$

The actual acoustic effect may be determined by the usual methods. If the diaphragm forms a wall of a small enclosure, the intensity is determined by the ratio of the volume displaced by the diaphragm as it vibrates to the volume of the enclosure. In other cases the intensity at a given point is calculated by determining the velocity potential due to the motion of the diaphragm.

It has been tacitly assumed that the amplitude of motion of the diaphragm is small compared with the air gap. This is necessary in order to get a pure tone when a sine wave E.M.F. is applied. While the instrument will not take care of a very large amount of energy, sound of the same order of intensity may be obtained as from an ordinary telephone receiver without appreciable distortion.

Summary.

1. A description is given of a transmitter of the electrostatic type which is especially adapted for measurement of sound intensities over a wide range of frequencies. The instrument is portable and is sufficiently rugged to retain its calibration.

2. A discussion is given of the necessary auxiliary apparatus and the precautions necessary for proper use.

3. A theory of the transmitter has been developed by which its operation can be predicted from a few simple measurements.

4. A description is given of the calibration of such an instrument in absolute terms over a wide range of frequencies. It is found that its efficiency may be made practically uniform for frequencies up to 10,000 cycles per second, and the results of the calibration are in agreement with the theory.

5. The apparatus when once adjusted may be used for the measurement of the intensities of sound at any frequencies throughout this wide range without further special adjustment.

6. Due to the uniform response through this wide frequency range it will be possible to secure correct indications of complex wave forms and to determine not only the relative intensities of the components but also their phase differences.

7. When properly calibrated this apparatus can be used as a precision source of sound.

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Telephone & Telegraph Co. and Western Electric Co., Inc.