197. Trilinear Coordinates. The Equation of the Axis of the General Parabola

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(b) Simple constructions, easy to perform with accuracy, are given in Plant's *Geometrical Drawing*.

(c) If the latter are adopted, as seems advisable, what line of argument furnishes the easiest and most natural proofs?

Tentative answer.

(i) Consider a right-angled triangle ABC , right-angled at C , such that $\frac{BC}{CA} = \frac{\sqrt{5}-1}{4}$. Using the '2A' formulæ, prove that $\cos 4A = \sin A$, thus obtaining $\sin 18^\circ$.

(ii) Employ this result in demonstrating the correctness of the constructions referred to. A. F. VAN DER HEYDEN.

195. [X. 1.] Note on the number of feet in a metre.

A high order of accuracy is secured in calculations by taking 1 metre = 39.37079 inches, this being the actual value of a metre to five places of decimals. The object of this note is to show that to this order of accuracy the very simple fraction $\frac{259}{89}$ of a foot may be taken as the equivalent of a metre.

The proof is simple. Expand 39.37079 into a continued fraction. The seventh convergent is $\frac{259}{89}$ and the eighth quotient is 35; thus $\frac{259}{89}$ differs from 39.37079 by less than $\frac{1}{35}(\frac{1}{89})^2$, i.e. by less than .000037. Thus to five places of decimals they are identical, and to this order $\frac{259}{89}$ inches, i.e. $\frac{292}{89}$ feet, represents a metre.

H. L. TRACHTENBERG.

196. [L. 3. a.] Cartesian Coordinates—A new method of finding the equation of the axis of a parabola.

Let the parabola be $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ (1)

Then the equation of its axis is obtained by equating to zero the linear portion of the equation,

$$\frac{(ax + hy + g)^2 - (hx + by + f)^2}{a - b} - \frac{(ax + hy + g)(hx + by + f)}{h} = 0. \dots\dots (2)$$

For had (1) been any central conic, (2) would have been the equation of its axes. But as the centre of a conic goes towards infinity, one of these axes tends to become the line at infinity. Thus in the case of a parabola one of the factors of (2) is a constant, i.e. the equation (2) reduces to a constant multiplied by the factor representing the finite axis. Thus the rule given above for obtaining the equation of the axis of any parabola is true. Since the highest terms of (2) have to vanish in the case of a parabola, we equate them to zero and get the condition that (1) should be a parabola. The vanishing of the three coefficients of x^2 , xy , and y^2 , give the same condition $h^2 = ab$.

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197. [L. 3. a.] Trilinear Coordinates. The equation of the axis of the general parabola.

The equation of the axes of any central conic

$$ua^2 + v\beta^2 + w\gamma^2 + 2u'\beta\gamma + 2v'\gamma\alpha + 2w'\alpha\beta = 0$$

$$\text{is } \left| \begin{array}{l} 2bc(v'\alpha' + v'\beta' + u'\gamma')(v'\alpha + u'\beta + w\gamma) - c^2(v'\alpha + v'\beta + u'\gamma)^2 \\ \quad - b^2(v'\alpha + u'\beta + w\gamma)^2, \quad 2bcu' - c^2v - b^2w, \quad 1 \\ 2ca(v'\alpha + u'\beta + w\gamma)(ua + w'\beta + v'\gamma) - a^2(v'\alpha + u'\beta + w\gamma)^2 \\ \quad - c^2(ua + w'\beta + v'\gamma)^2, \quad 2cav' - a^2w - c^2u, \quad 1 \\ 2ab(ua + w'\beta + v'\gamma)(w'\alpha + v'\beta + u'\gamma) - b^2(ua + w'\beta + v'\gamma)^2 \\ \quad - a^2(w'\alpha + v'\beta + u'\gamma)^2, \quad 2abw' - b^2u - a^2v, \quad 1 \end{array} \right| = 0.$$

As the conic tends to become a parabola, one of the axis tends to become the line at infinity. Thus if the first equation be that of a parabola, the determinant $\equiv (a\alpha + b\beta + c\gamma)(la + m\beta + n\gamma)$, where $la + m\beta + n\gamma = 0$ is the equation of the axis. Thus la = the coefficient of α^2 in the determinant, and l is found. Similarly m and n are found. Thus the equation of the axis is

$$\Sigma \left\{ (2bcw'v' - c^2w' - b^2v') \{ (2cav' - a^2w - c^2u) - (2abw' - b^2u - a^2v) \} \right. \\ \left. + (2cav'u - a^2v^2 - c^2u^2) \{ (2abw' - b^2u - a^2v) - (2bcu' - c^2v - b^2w) \} \right. \\ \left. + (2abuw' - b^2u^2 - a^2w^2) \{ (2bcu' - c^2v - b^2w) - (2cav' - a^2w - c^2u) \} \right\} \frac{a}{a} = 0,$$

$$\text{i.e. } \Sigma [(cw' - bv')^2 \{ (2cav' - a^2w - c^2u) - (2abw' - b^2u - a^2v) \} \\ + (av' - cu)^2 \{ (2abw' - b^2u - a^2v) - (2bcu' - c^2v - b^2w) \} \\ + (bu - aw')^2 \{ (2bcu' - c^2v - b^2w) - (2cav' - a^2w - c^2u) \}] \frac{a}{a} = 0.$$

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198. [L. 3. a.] *A new method of finding the equation of the axes of the general conic.*

$$\lambda(ax^2 + 2hxy + by^2 + 2gx + 2fy + c) \\ = [(ax_0 + hy_0 + g)x + (hx_0 + by_0 + f)y + (gx_0 + fy_0 + c)]^2$$

represents the system of conics having double contact with

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

at the points where it is cut by the polar of x_0y_0 . Now if x_0y_0 is on an axis, and in that case alone, one of the conics of this system will be a circle, for the tangents from such a point to the conic are equal. Let λ_1 be the value of λ for this circle. Then the conditions for a circle give

$$\lambda_1 a - (ax_0 + hy_0 + g)^2 = \lambda_1 b - (hx_0 + by_0 + f)^2, \\ \lambda_1 h - (ax_0 + hy_0 + g)(hx_0 + by_0 + f) = 0.$$

Thus, eliminating λ_1 the equation of the axes is

$$\frac{(ax + hy + g)^2 - (hx + by + f)^2}{a - b} = \frac{(ax + hy + g)(hx + by + f)}{h}.$$

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199. [L. 3. a.] *Trilinear Coordinates.*—The equation of the axes of the general conic.

$$\lambda(ua^2 + v\beta^2 + w\gamma^2 + 2u'\beta\gamma + 2v'\gamma\alpha + 2w'\alpha\beta) \\ = [\alpha(u\alpha' + w'\beta' + v'\gamma') + \beta(w'\alpha' + v'\beta' + u'\gamma') + \gamma(v'\alpha' + u'\beta' + w'\gamma')]^2$$

is the system of conics touching the conic

$$ua^2 + v\beta^2 + w\gamma^2 + 2u'\beta\gamma + 2v'\gamma\alpha + 2w'\alpha\beta = 0$$

at the points where it is met by the polar of $\alpha'\beta'\gamma'$. If $\alpha'\beta'\gamma'$ is on an axis, and in that case alone, one of the conics of this system will be a circle. Let λ_1 be the corresponding value of λ . Then writing down the conditions for a circle, a, b, c being the sides of triangle of reference

$$2bc[\lambda_1 u' - (w'\alpha' + v'\beta' + u'\gamma')(v'\alpha' + u'\beta' + w'\gamma')] \\ - c^2[\lambda_1 v - (w'\alpha' + v'\beta' + u'\gamma')^2] - b^2[\lambda_1 w - (v'\alpha' + u'\beta' + w'\gamma')^2] \\ = 2ca[\lambda_1 v' - (v'\alpha' + u'\beta' + w'\gamma')(u\alpha' + w'\beta' + v'\gamma')] \\ - a^2[\lambda_1 w - (v'\alpha' + u'\beta' + w'\gamma')^2] - c^2[\lambda_1 u - (u\alpha' + w'\beta' + v'\gamma')^2] \\ = 2ab[\lambda_1 w' - (u\alpha' + w'\beta' + v'\gamma')(w'\alpha' + v'\beta' + u'\gamma')] \\ - b^2[\lambda_1 u - (u\alpha' + w'\beta' + v'\gamma')^2] - a^2[\lambda_1 v - (w'\alpha' + v'\beta' + u'\gamma')^2].$$