LVIII. On the distribution of flow in a strained elastic solid

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The method of using the still is as follows:—The tap E is opened, F is closed ; a water-pump then exhausts the whole system, and the mercury to be acted on rises from the cistern A. The cistern being large and shallow, only a slight change takes place in the height of the mercury in the bulb, when the level of the mercury in the cistern changes. While the pump is exhausting the ring of gas-jets is lit, and in about ten minutes, in the case of the still in our laboratory, the mercury fills the tube D, any metal which comes over being caught in the bulb G. The tap E is then closed and F opened ; the still then continues to work by virtue of the vacuum formed by its own mercury. It has been found necessary to place a gas-regulator on the pipe which supplies the jets, as the change of pressure in the gas-mains is considerable. An automatic arrangement, depending for its action upon the height of the mercury in the cistern, shuts off the gas when the surface of the mercury falls below a certain point. In using stills of this class the mercury before distillation should be carefully freed from moisture, as a minute quantity of water will often cause a fracture in the heated tube or bulb.

LVIII. *On the Distribution of Flow in a Strained Elastic Solid.*
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IF a metal bar of uniform section throughout its length be subjected to uniform longitudinal stress, the elements of the bar will become distorted ; if the strain should exceed the limit of elasticity the distortion will be partly permanent.

In consequence of the straining of the elements, every element will experience a displacement relative to three fixed axes in the bar ; say ox parallel to the length of the bar, oy parallel to one side, and oz parallel to a second side, both oz and oy being at right angles to ox : such displacement is generally spoken of as “flow.”

It is clear that if the metal be homogeneous, the stress uniform, and the section at right angles to the axis everywhere the same, an element will experience a displacement at right angles to the planes xoz , yoz proportional to its distance from these planes ; since such displacement depends on the straining of the elements between it and these planes. Hence a bar with its sides originally parallel to ox will remain so

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after a certain amount of flow has taken place, and lines drawn on the sides respectively at right angles to and parallel to ox will remain so, since the displacement parallel to ox will be proportional to the distance from the plane yoz .

The question I propose to discuss is how to determine the distribution of flow when the section at right angles to ox varies in any way as we pass along ox .

I will for simplicity consider a plate in which the thickness parallel to oy is considerable compared with the thickness parallel to oz , and I will consider the displacements in the plane xoy .

We may take a plate in which there is a gradual reduction of section, in which there is what is commonly called a "shoulder," as in fig. 2.

It is required to determine the displacements in the plane of the paper—the plane xoy : in other words, supposing the plate covered with a series of lines parallel to ox and to oy respectively, it is required to determine the curvature of these lines due to flow.

The distortion of the elements is caused by the shearing-strain induced in the bar when subjected to longitudinal stress. Since this shearing-strain is a maximum at 45° to the axis ox of the bar, it will be convenient to consider the bar divided

Fig. 1.

into a number of small elements with sides making angles of 45° to the axis ox . Every such element as $abcd$ (fig. 1) will

be subject to a shearing-strain of amount $\frac{p}{n}$; *i. e.* the angle $b a d$ will be diminished by an amount $\frac{p}{n}$, where p is the intensity of the shearing-strain on the element, and n is the coefficient of rigidity of the metal. The effect of this shearing-strain on the element may be described as a rotation of $a d$ through an angle s_1 , and of $a b$ through an angle s_2 , where $s_1 + s_2 = \frac{p}{n}$; s_1 and s_2 may be equal, when the displacement of c will be wholly parallel to $o x$; or they may be unequal, when c will have a displacement parallel to $o x$ and a resolved displacement parallel to $o y$, which will be +ve or -ve according as s_1 is less than or greater than s_2 , and which will be a maximum if either s_1 or $s_2 = 0$.

The value of $s_1 + s_2$, and the ratio of s_1 to s_2 for any element, appears to be determined by the following considerations. If a side $a b$ of the element be produced in both directions so as to cut the sides of the bar, there is seen to be a tendency of that part of the bar below this line to slide over that part which is above. Assuming the resulting shearing-strain to be uniform over the length l_1 of this oblique section, the shearing-strain at any element in this direction, *i. e.* s_1 , will clearly be inversely proportional to l_1 ; s_2 will also be inversely proportional to l_2 , where l_2 is the length of an oblique section parallel to $a d$, and $s_1 + s_2$ will be proportional to $\frac{l_1 + l_2}{l_1 l_2}$ —say to $\frac{1}{\rho}$.

Suppose, now, we take a series of elements, $a_1 b_1 c_1 d_1$ — $a_2 b_2 c_2 d_2$ touching one another and lying across the bar. If we imagine the points a_1, a_2, a_3 , &c. fixed on a line parallel to $o y$, it is clear that the displacements of c_1, c_2, c_3 , &c. will be proportional to the sum of the shearing-strains in each element. If $s_1 + s_2$ is the same for each element, and $s_1 = s_2$, the points c_1, c_2, c_3 , &c. will remain on a line parallel to $o y$. If, however, $s_1 + s_2$ becomes greater as we approach the middle, and less again towards the further side, the points c_1, c_2, c_3 , &c. will be found on a line concave upwards; if $s_1 + s_2$ diminishes towards the centre these points will lie on a line concave downwards. If the strains should exceed the limit of elasticity the line through c_1, c_2, c_3 , &c. will be permanently curved.

If the points $a_1, a_2, a_3, \dots$, instead of being in one straight line have unequal downward displacements, the form of the curves obtained as above may be modified.

If, now, we consider a series of consecutive elements, $a_1 b_1 c_1 d_1$ — $a_2 b_2 c_2 d_2$, lying in a line parallel to $o x$, if $s_1 = s_2$ for

each element the points c_1, c_2, c_3 , &c. will continue to lie on a line parallel to ox , since the displacement of each of these points, due to the distortion of the corresponding element, is parallel to ox . But if as we descend from the first element we find s_1 becoming greater than s_2 , the points c_2, c_3 , &c. will have a displacement to the left which increases as we descend, so the line passing through c_1, c_2, c_3 , &c. will be bent; and if the shearing-strains become equal again on the lowest element, this line will be concave to the right.

It thus appears that any horizontal line will be curved concave downwards if the sum of $s_1 + s_2$ is greater at the end of the line than in the centre, and *vice versa*, but that the line will remain straight if $s_1 + s_2$ is constant along its length, and $s_1 = s_2$.

Also that a vertical line will remain so if s_1 is everywhere equal to s_2 ; but if s_1 is $> s_2$, the line will be curved and concave towards the right, and *vice versa*.

We can now proceed to determine the curvature of the lines in the case before us.

Commencing at the upper extremity of the bar, fig. 2, a line such as ab will remain straight, because ρ is everywhere constant, and $l_1 = l_2$. The line $cdef$ will be curved concave downwards, since from c to d and from e to f ρ is less than it is from d to e (where it is constant and $l_1 = l_2$); de will therefore remain straight, and cd and ef curved as shown. The line gh will be concave downwards and curved along its entire length, since ρ gets less as we move away from h on both sides.

The line $jklmn$ will remain straight from j to k , since ρ is there constant and $l_1 = l_2$; from k to l ρ becomes

Fig. 2.

less and greater again from l to m , $k l m$ is therefore concave upwards; this line is straight from m to n , since ρ is constant and $l_1 = l_2$.

The line $o p q$ is curved concave upwards throughout the entire length, since ρ diminishes from o to p and increases from p to q .

The line $r s t$ will be concave downwards, since ρ diminishes from r to s and increases from s to t .

The line $u v$ will remain straight, since ρ is constant and $l_1 = l_2$.

To turn now to the curvature of the vertical lines. If we consider one of these as $w x y z$, from w to d $l_1 = l_2$, at x l_2 is $< l_1$, while at x' $l_1 = l_2$ again, so that from d to x' this line will be curved in convex towards $o x$. Again, at y l_2 is $< l_1$, and at z $l_1 = l_2$, therefore $x' y z$ is also curved in convex towards $o x$.

It would appear then that the horizontal lines will have three distinct changes of curvature, while the vertical lines will be pinched in above and below the shoulder.

To test the accuracy of these conclusions, I prepared a copper bar as shown in the following way. The sides and faces were very carefully planed and polished, and on one face I inscribed a series of vertical and horizontal lines in a lathe. I then strained the bar so as to produce a considerable permanent set. An impression was then taken from the bar, which is reproduced in fig. 3*.

By placing the eye very nearly in the plane of the paper and looking along the lines, the curvatures can be seen to follow very closely those sketched on fig. 2. I would specially draw attention to the partial curvature of such lines as $c d e f$ and $j k l m n$.

* The diagonal lines in fig. 3 were ruled on the impression to serve as lines of reference.

Fig. 3.