Astigmatism in lens system

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ASTIGMATISM IN LENS SYSTEM.

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Presidential Address delivered on 9th February, 1911.

The subject I have chosen for this address is the very old and perhaps hackneyed one of astigmatism, particularly in relation to systems of spherical surfaces. You are so familiar with the formation of astigmatic images by cylindri-

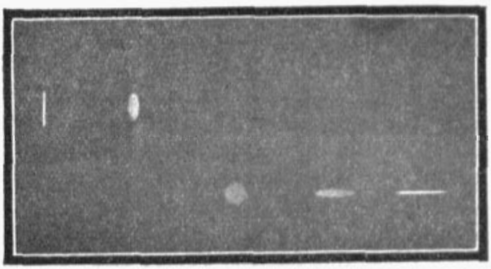

Fig. 1.

cal and sphero-cylindrical lenses that I need only refer to this for the purpose of settling the notation I will use. Fig. 1 shows the images of a point formed by a photographic lens of focus 160 and aperture 6 mm., having a + 5 D cylin-

Fig. 2.

drical lens in front. The plate was focused to show the images formed at various places. You will note that the image may be a vertical or horizontal line, or even a circle. These lines will be referred to as focal lines, and the circle as the circle of least confusion. On the whole this circle gives the best image of a point, while lines parallel to the focal lines are best reproduced at the corresponding focal line.

Fig. 2 shows the corresponding images of a small circular aperture when the value of the cylindrical lens is much smaller, .25 D. In this case one can hardly speak of focal lines.

Fig. 3 shows the images of a test chart formed by the systems (1). It is interesting as show-

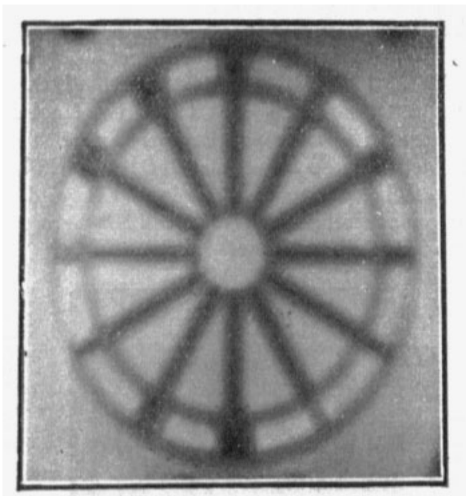

FIG. 3.

ing that there is no intermediate focusing position for which oblique lines are quite as sharp as the horizontal and vertical ones may be made.

Effects similar to the above may be produced by a lens or mirror when the object is not on the axis. Fig. 4 shows the two focal lines formed by a spherical mirror, the object being a small point 15° from the axis. In this case, if the object O_1 be on the line CO_1 at right angles to the axis CB of the mirror, one focal line will fall on CO_2 produced, but the other and the circle of least confusion will lie off the plane of CO_2.

FIG. 4.

The diagram (fig. 5) shows these rays proceeding from a point O and focusing sharply at O_2. Thus O_2 is a focal line image of O. If the point O be moved back to O_1 the focal line will move in a corresponding distance.

In fact, for the object point O_1 we will have two focal lines, one on the plane CO_1 and the other on a circle of curvature 2 F where F is the power of the mirror.

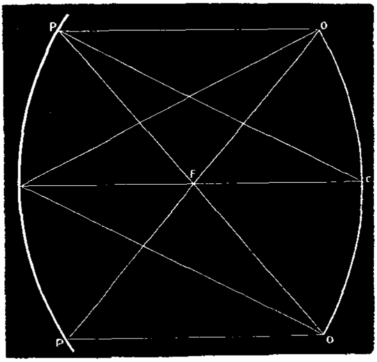

FIG. 5.

The curvatures of the three lines at which the focal lines and the circle of least confusion are formed will be O or F − F, and 2 F or 3 F − F, and F or 2 F − F.

When the object is at infinity an additional complication comes in.

If three rays be drawn parallel to each other, as in fig. 6, they will not meet in a point, and at this place the actual shape of the image will be as shown in fig. 7. Figure 8 shows the images at three other positions of the plate.

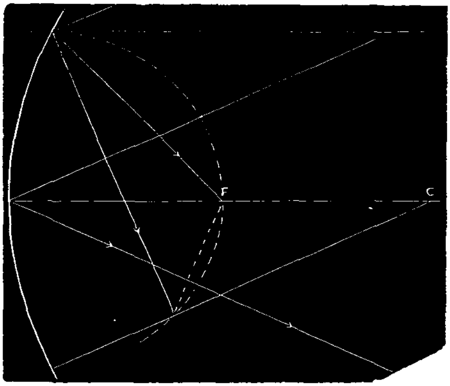

FIG. 6.

It would hardly be right to describe these images as focal lines, but there is a sense in which this term may still be used. If the object be a vertical or horizontal line, the position of best definition will correspond with one of the lines, and this will be very nearly at the

FIG. 7.

place of the focal line for a small aperture. The one-sided effect which is seen in these images is described as coma, and our results show that there is no coma in the case of an object placed near the centre of curvature of a spherical mirror.

Lenses.

Similar results may be obtained in the case of lenses, but the presence of the coma will depend on the shape of the lens. We will consider the images of points on a plane at right angles to the axis.

FIG. 8.

Fig. 9 shows the images of a point C_1 on the axis, at 5° and at 10° from the axis respectively, when the plate is not moved. The lens

FIG. 9.

is a plano-convex lens +8 D, with the convex side towards the distant object.

Fig. 10 gives best focus in each case.

Fig. 11 shows the images when the plate is moved towards the lens to give the best circle, the object being in the direction at 15° from

FIG. 10.

the axis. The next figures show similar effects for the same lens reversed.

So long as there is no stop away from the lens the two focal lines lie on curves whose curvature is $3F + F/n$ and $F + F/n$, where F is the power of the lens. If the lens be not thin the expression F/n is replaced by the sum of the powers of the surfaces divided by n, and this expression is known as the Petzval curvature.

Fig. 11.

Fig. 12 shows four ~~curves:~~

(*a*) The curve in which one focal line comes.

(*b*) The curve in which the best average definition comes.

(*c*) The curve in which the second focal line comes.

(*d*) Given by the Petzval curvature.

The curves are equally spaced.

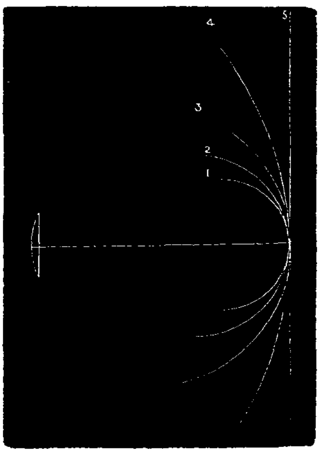

Fig. 12.

The curvatures are:

$$3F + F/n.$$
$$2F + F/n$$
$$F + F/n.$$
$$F/n.$$

In the case of the mirror F/n is replaced by $-F$, and the curves were

$$\begin{array}{r} 2F \\ F \\ O \\ -F \end{array}$$

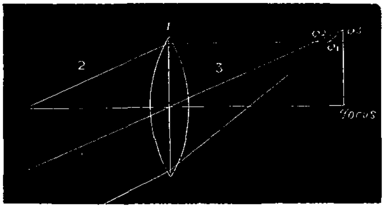

FIG. 13.

When the surfaces are separated, or there is a stop placed in front of the lens, the curves (*a*), (*b*) and (*c*) are generally altered, so that the focal lines are moved towards or away from the lens; but these curves are still equi-spaced. It is easy to see what are the conditions under which the focal lines move.

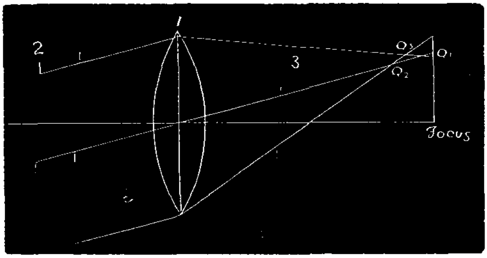

FIG. 14.

In fig. 13 taking the intersection of the two outside rays which pass through the lens as the position of the first focal line, this will be :—

At Q_1 if the stop be at the lens (1).

At Q_2 if the stop be at (2).

At Q_3 if the stop be at (3).

You will notice that this stop effect is only important when the three rays do not pass through one point, *i.e.*, when there is coma in the image, and that, by choosing the stop position, we can bring the focal line forward or move it back towards the focal plane.

By altering the shape of the lens we can alter the coma so that the central ray passes nearer the axis than Q_1 (fig. 14).

In this case a stop in front of the lens has the opposite effect, and moves the focal line further from the lens.

But we have seen that the four curves of fig. 12 are equally spaced, and that (*d*) does not alter, if only the shape of the lens is altered, the sum of the powers of the surfaces remaining the same.

Thus, if we wish to make the curves (*a*) and (*c*) coincide, we must choose the stop to move the points on the curve (*a*) back towards the curve (*d*). But with each shape lens there is a stop position which enables us to do this, unless there is no coma.

When the curves (*a*) and (*c*) coincide the focal lines are shortened till the image of a point is practically a point, and there is then no astigmatism, all lines being reproduced at the same place, though there may be a certain amount of coma. In practical cases the stop position being fixed by other considerations, the form of the lens must be properly chosen if it is desired to correct the astigmatism.

The expression which is obtained by dividing the power of each surface by the refractive index of the glass, and adding the results, is called the Petzval expression (P).

This gives the curvature of the curve (*d*), and it is only when the two focal lines coincide with each other and with (*d*) that this represents the best image position.

Still, the curve (*d*) is important, and it is always desirable to reduce the value of the expression (P), particularly in photographic lenses.

If we suppose for the moment that the lenses are all of the same glass, all thin, and all close together, we have $P = F/n$.

i.e., the radius of the curve (d) is $n \times$ the focal length.

If, however, the + and − lenses are separated, we can still retain the same total power, while increasing the actual power of the negative lens, and reducing the Petzval expression. In this way we can make the curve (d) approach the plane.

It is true that by using crown lenses of higher refractive index than the flints we can also slightly reduce the value of P; but in many practical lenses, the average value of the refractive index of the + lenses is nearly equal to the refractive index of the flint lens, and it is the effect noted above that is most important. The same effect may also be produced by separating the + and − surfaces of a thick lens, as in the Goerz lens.

Unless the astigmatism is corrected, it is preferable that the curve (b) should be plane rather than (d).

The astigmatism of the system will be corrected if the two curves (a) and (c) coincide with each other and with (d).

If the lenses are thin and in contact the difference of curvature is 2 F where F is the power of the system; but when they are separated this expression is replaced by twice the sum of the powers of the lenses, and the additional effects due to the stop not coinciding with the various lenses. We may proceed in two ways, either make the sum of the powers of the surfaces zero, and make all the coma effects balance out, or we may leave the positive lenses rather more powerful than the negative ones, and use the coma of the individual lenses to balance out the error.

In the Cooke lenses the first procedure was aimed at, each lens being made free from coma, and the powers of the plus lenses practically equal to the power of the negative lens; but in the final empirical correction of the lens some departures were made, and the plus lenses are rather more powerful than the minus lenses.

In the Unofocal, Steinheil aimed at the working out a single system of two lenses in which the + and − lenses were of equal powers and of the same glass. It was then necessary that the effects of the coma of the two lenses should balance out. Such a system can be constructed, but it has the defect that the half system shows coma, and it is difficult to get rid of this completely in the whole system.

Thus we can in various ways fulfil both the conditions necessary for the correction of the astigmatism and the reduction of the Petzval expression, if the sum of the powers of the plus surfaces is nearly equal and opposite to that of the negative surfaces.

It would therefore seem desirable to separate these surfaces as far as possible, getting as much power in the final system as we can, but there are other difficulties when the surfaces are

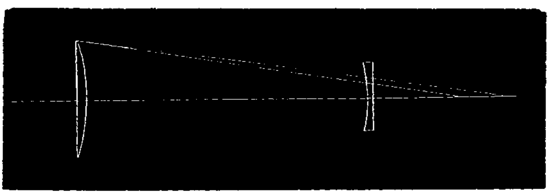

FIG. 15.

too widely separated. It is no longer fair to simply add together the errors of the two surfaces, because a ray which is not proceeding to the theoretical image of the first lens will meet the second lens in a different point, and so the errors of the lens will be modified. Thus, in fig. 15 the real ray passes along the dark line while the theoretical path is the dotted line. The real ray, therefore, meets the lens at a different point, and is differently deviated.

It is in consequence of these secondary defects that the design of photographic lenses of large field aperture is so difficult.

We find that when the astigmatism and curvature of field are corrected for a moderate angle, say, 10° to 15°, serious errors occur for larger angles of field.

The curves of fig. 14 are not strictly circles, but generally bend back towards the focal plane; this is hardly appreciable in the curves for a single lens, unless very large angles are taken, but when the defects of one lens are almost compensated by those of another, the residual effects are easily seen, even for semi-

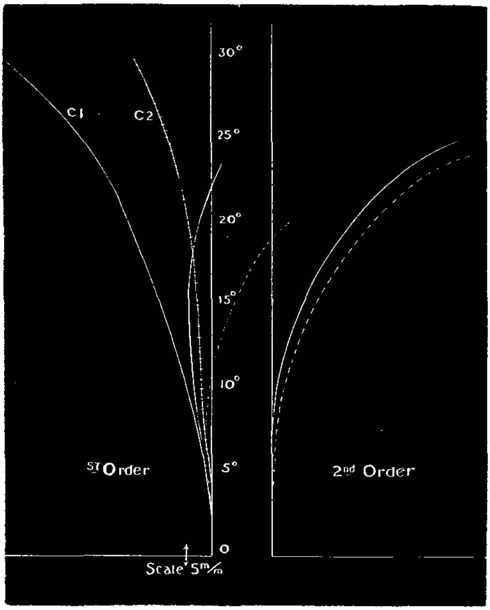

Fig. 16.

angular fields of 15° to 20°. This is evident from the curves in fig. 16, where the curves C_1 and C_2 represent the circles referred to above, and the other curves the real curves. The differences are shown to the right.

If we arrange so that the compensation is complete for objects 20° from the axis, there may be residual errors of appreciable amount at 10° and 30°.

It is necessary to either reduce the angular field for which the lens is to be used, or to select a type in which these secondary effects are small.

We can hardly talk of compensating these defects by others of the same nature, as this would require exceedingly complicated constructions with disadvantages far outweighing their merits. All we can do is to avoid the forms which introduce large errors, and so arrange the final correction of the system that the astigmatism curves give the best average result over the plate.

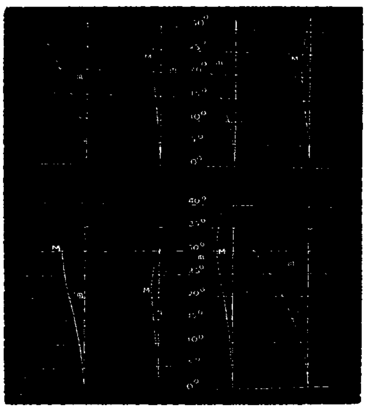

FIG. 17.

Small changes in the powers of the lenses or their separations will generally produce sufficient effect to alter the point of the field for which the astigmatic correction is best, and this enables us to choose the best average result.

Fig. 17 shows some typical astigmatism curves, as calculated by von Rohr.

You will notice that for the smaller angles of field the curves are rather in front of the focal plane; and this is characteristic of nearly all the photographic lenses. This means that the designer should slightly under-correct the error of astigmatism and curvature of field for small angles, and the curves C_1 and C_2 of fig. 16 are generally slightly concave towards the lens.

In conclusion, I would like to express these astigmatism curves as a difference of dioptric power.

For the single thin lens of power F diopters used for an object 20° from the axis, the cylindrical difference is F $\tan^2$ D in diopters, or $\frac{1}{8}$ F approx.

In other systems this expression may be multiplied by a numerical factor depending on the design of the system.

It is interesting to note that the optical system of the eye, taking Tscherning's data for one case, gives a value of this constant of .19. Thus the error, assuming the surfaces spherical, is only about 1/5 of that of a lens of the same power used at the same inclination to its optical axis. The error, if we assume an angle D of 1° is only .16 D.

[This calculation assumes that astigmatism is not affected by the special structure of the lens.]

In the case of the single lens referred to above the constant is rather less than 1, but if the lens were of meniscus form with a stop behind the lens, the constant is steadily reduced as the last surface is made steeper, and may be made zero or even minus. If the stop distance be 20 mm. the constant is 0 for a value of the second lens surface of about -7 D.

From the nature of the astigmatism, it is clear that lenses need not give the same error when used for different object distances, and the correction of the astigmatism in spectacle lenses used for distance by choosing the most suitable form of lens will not generally mean the correction of astigmatism when used for reading; in fact, there is another form of lens more suitable for giving a wide field for reading purposes, in which the negative surface is much shallower.

As regards the curvature of field and astigmatism in photographic lenses, I have endeavoured to give the simpler theory and to show why and how far it does not represent the complete facts. A more complete theory might be given, but it is not yet possible to use the results in such a way as to make the computation of photographic lenses easy. At present we use the general ideas of this theory to enable us to express the results of trigonometrical calculations or measurements in such form that we can more readily determine the effect of any change in the lens. A more complete and simplified theory is much to be desired.