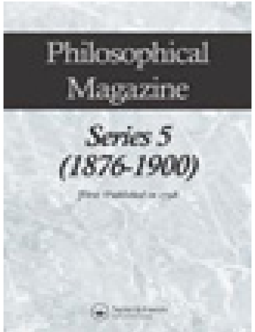

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LIII. Phase-reversal zone-plates, and diffraction-telescopes

R. W. Wood

LIII. *Phase-Reversal Zone-Plates, and Diffraction-Telescopes.* *By* R. W. Wood*.

[Plate XXIV.]

IN a paper published in Poggendorff's *Annalen* (1875) Soret showed that if we describe a number of small concentric circles on a glass plate, with radii proportional to the square roots of the natural numbers, and blacken the spaces between the alternate rings, the plate will have the property of bringing parallel rays of light to a focus, like a condensing-lens. The dark rings check the disturbances on the alternate Huygens's zones on the wave-front, which by destructive interference with the disturbances from the other zones ordinarily produce darkness at points within the geometrical shadow.

Soret's method was to draw circles of suitable size on a sheet of paper, blackening the alternate rings, and make a reduced photographic copy of the whole. He showed that such a plate formed real images of luminous objects, and could be used as the objective of a telescope or as the eyepiece. He also showed that in addition to acting as a condensing-lens, the zone-plate acted as a concave or dispersing-lens. Moreover, he pointed out that the plate has multiple foci at distances $\frac{a^2}{\lambda}, \frac{a^2}{3\lambda}, \frac{a^2}{5\lambda}$, where a is the radius of the central circle.

Since the appearance of Soret's paper very little seems to have been done with the subject, though it is alluded to briefly in the textbooks, and it has been pointed out by Lord Rayleigh that if it were possible to provide that the light stopped by alternate zones were replaced by a phase-reversal, a fourfold effect would be produced. After some experimenting I have succeeded in producing such a zone-plate, perfectly transparent over its entire extent, which yields an image at least six or eight times as bright as those produced with the old form of plate. This increase in intensity is greater than one would expect, which is probably accounted for by the lack of perfection of the black zone-plates, the black rings being partly transparent and the clear ones not wholly so. Using one of the new plates as the objective of a telescope in connexion with a low-power eyepiece, I have distinctly seen the lunar craters, and have constructed telescopes in which both objective and eyepiece were zone-plates.

The largest plate made by Soret contained 98 dark circles, and since the scale on which it was drawn was rather small, the outer zones could not have been very accurately placed.

* Communicated by the Author.

The preliminary drawing which I made contained 115 dark rings, and was drawn on a scale three times as large as that used by Soret. In order to avoid the errors due to shrinkage and expansion of the paper during the progress of the work, the following method was adopted. A sheet of heavy smooth surface drawing-paper, considerably over a metre square, was thoroughly dampened and the edges glued down to a heavy drawing-board. The paper was kept damp until the edges had thoroughly dried to prevent the shrinkage from tearing them loose. On drying, the paper was in close contact with the board and as tight as a drumhead. A copper tack with a small conical dent in its head was driven into the centre of the board to serve as a centre for the beam-compass used in drawing the circles. The pen of the compass was accurately set for each circle by means of a Brown and Sharp steel metre-bar, care being taken to set by the inner and outer edge of the pen alternately, in order that no error due to the width of the line should be present. Two hundred and thirty circles were drawn in this manner, using the greatest care possible to ensure accuracy. The spaces between the alternate rings were then blackened by means of the beam-compass and a broad pen. In spite of every precaution errors crept in which manifest themselves as slight shadings at certain places; their effect, however, is small.

The labour involved in the preparation of such a drawing was so great that it has seemed worth while to publish with this paper a reduced photographic copy, from which other still more reduced copies can be prepared on glass by anyone. I have made photographic negatives of this drawing on lantern-slide plates of various sizes, and find that they act very satisfactorily. These copies were made with a very fine Zeiss lens, and had focal lengths varying from half a metre to ten metres. Smaller plates could not be made with this lens, since the limit in the defining power is about ·02 millim. In order to produce plates of very short focus I made some copies with a Beck microscope-objective of about 2 centim. focus. The focal lengths of these plates varied from 3 to 10 centim., and they were found to act admirably as eyepieces. The central circle of one of these micro-plates had a radius of ·158 millim.; its focus for yellow light is therefore $\frac{(\cdot 158)^2}{\cdot 0006}$, or 4·1 centim. In spite of its small size, the rings were sharp out to the edge, where their width, measured on a small dividing-engine, was found to be only ·005 millim.

A number of experiments were tried with these plates; but as they can be shown to a much better advantage with the transparent plates, I will postpone the discussion of them until a little later in the paper. Before taking up the subject of the phase-reversal plate I may as well call attention to a very pretty method of preparing a zone-plate to illustrate its action.

If a profile-face or other design is cut out of black paper and pinned on to the original drawing somewhere between the centre and the edge, and a photograph of the whole about 3 centim. in diameter be made, on looking at a point of light through the plate, the now perfectly transparent profile will appear jet-black on a brightly illuminated back-ground —that is to say, the space within the design being devoid of black lines does not fill up with light.

To prepare a zone-plate in which the light, instead of being cut off by the alternate rings, shall suffer a phase-reversal, the best method appeared to be to prepare a plate with transparent zones of a thickness sufficient to retard the light one-half wave-length. Sheets of thin plate-glass with the surface carefully cleaned were flowed with a warm solution of gelatine (strong enough to just set into a jelly on cooling), and set on edge to dry. The films were then sensitized by immersing the plates for about five seconds in a weak solution of bichromate of potassium, and dried in the dark. Properly prepared films should exhibit no trace of crystallization, and should have a barely perceptible yellow tint. These plates were placed in contact with the photographic reduction, and printed in direct sunlight; the exposure varying from 20 seconds to 1 minute according to the density of the negative. They were then removed from the frame, washed for a moment in cold water, and then immersed in rather warm water, which dissolved and washed away the gelatine which had been protected by the dark zones, leaving the alternate rings attached to the glass. On drying these plates they were perfectly transparent, but by holding them in certain lights the rings could be seen. The effect produced by them far surpassed my expectations. On viewing a brightly luminous point through them, they fill up with a blaze of light which compares very favourably with a lens. They are not uniformly good, however, everything depending on the thickness of the film, and, moreover, the exposure must have been exactly right. I find that if one of these plates 3 centim. in diameter be set up at a distance of $2\frac{1}{2}$ metres from an arc-light and a paper screen be placed at a suitable distance (3 or 4 metres) behind the plate, an exceedingly bright and sharp, somewhat enlarged image of the arc is projected; the surface irregularities of the heated carbons can be seen, and the blue flame between them, the small incandescent particles that are thrown off being also clearly visible.

If the screen is near the plate, the image of the carbons is orange-red; and as it is moved back the colour changes to yellow, green, and blue, the foci for different colours being far apart. The best definition appears to be in the yellow and green. On setting the plates up at a greater distance from the arc and placing a sheet of paper in the focus, a large part of the light incident on the plate is brought to a very small sharp focus surrounded by a dark area, corresponding to the geometrical projection of that part of the plate which is effective. With some plates this dark area is very small, and the central bright spot correspondingly feeble, showing that what little action there is comes from a few rings in the centre. This forms a very good method of testing the plates; and in general I find that about one out of four can be considered first class. With more care in the preparation of the films greater uniformity in the results could doubtless be secured.

In making these plates it is necessary to use mirror-glass with a ground surface if good definition is to be obtained. I find the German mirror-glass which comes with a deposit of metallic silver backed by varnish gives very good results. The varnish can be removed by alcohol, the silver with nitric acid, and the surface cleaned. It is best to prepare a large number of plates and select the best, either by throwing an image of an arc-light with them on a screen, or by the following method which is perhaps better.

A small hole a millimetre or so in diameter is punched in a black card or a metal plate and set up in front of a bright sodium flame; the zone-plate is mounted at a distance of three metres from this, and the eye brought into the focus. If the plate is good, it lights up brightly and uniformly. Most plates, however, show irregularity in the illumination, and may even have spots that appear quite devoid of light. These dark spots may be caused by the film being so thin as to give no appreciable retardation, or so thick as to give a retardation of a whole wave or a number of whole waves. Plates that do not come up to this test often give very good images, however, and need not be rejected. Those which light up feebly and but a short distance from the centre should be thrown away.

Although a large percentage of the light is wasted by a zone-plate, the focal images produced by these plates were so sharp and brilliant, that I determined to see just what results could be obtained with a telescope constructed with one.

Removing the objective from my 5 in. Clark, I put a 3 centim. gelatine zone-plate in its place. A low-power eyepiece was used, and on viewing terrestrial objects, such as distant chimney-tops and trees against the sky, with this combination, the result seemed to be very unsatisfactory; the amplification was about 60 diameters and the outlines were sharp, but there was very little difference in light value between the sky and dark chimneys. In other words the field was bright. The most suitable objects for a telescope of this description are small, brightly illuminated ones on a black background; under these conditions the general illumination of the field is reduced to a minimum. I tried the telescope on the moon, and although the image was somewhat dim and hazy, as seen through thin clouds, I could distinctly see a number of the larger craters.

The zone-plate used in this case had a focal length of about five feet; but plates of shorter focus show the principle equally well, and make more convenient instruments to handle. If we use an ordinary low-power eyepiece in the principal focus of the zone-plate, we obtain a highly magnified inverted image. If now we push the eyepiece in, the image becomes blurred, vanishes, and a second smaller inverted image appears; and on continuing to diminish the distance, we get images successively smaller and smaller as the eyepiece passes through the different foci of the plate. The best object to view is an incandescent electric lamp at a distance of six or eight metres. With the Soret form of plate, I have never been able to find more than three foci, while the phase-reversal plates easily show six. If instead of the glass eyepiece we use one of the micro-zoneplates, we obtain both erect and inverted images; the inverted when the eyepiece is in the focus, and the erect when it is between the focus and the plate—the former image being due to the action of the micro-plate as a convex lens, the latter to its action as a concave lens, as in the Galilean telescope. In spite of the fineness of the lines in the micro-plates, I find that they can be made to yield very excellent phase-reversal plates by contact printing, so that by making both objective and eyepiece on the new principle, we have a diffraction-telescope wholly without lenses or mirrors, which is fairly efficient for viewing small brightly-illuminated objects.

Starting with the eyepiece close to the objective and moving it slowly back, we get a rapid alternation of erect and inverted images, to the number of eight or ten in all. As the eyepiece moves the colour of the image changes, the foci being differently situated for the different colours.

I have tried making landscape-photographs, using one of these short-focus retardation-plates instead of a lens. The results far surpassed my expectations. There are good qualities in the zone-plate picture that almost offset the deficiencies. Very soft and artistic effects can be produced. Where there is a mass of twigs against the sky the zone-plate rubs in a little extra shade for a background instead of cutting each stem sharp and black against the white sky. The shadows are massed to a great extent, and yet there is definition enough to prevent the picture from appearing out of focus. The landscape reproduced (fig. 1) was made with a plate of 4 millim. aperture and 14 centim. focus, with an exposure of half a second, which is certainly a great improvement on pin-hole photography. The picture has lost much in reproduction. The actinic focus of the plate must be found and the image on the ground-glass is never sharp, owing to the chromatic aberration. I have used a blue glass to advantage in finding the focus; and a low sun in thin clouds makes an excellent object to focus upon.

Fig. 1.

A very simple elementary way of explaining the multiple foci, both real and virtual, of the zone-plate which I have not seen given anywhere, probably because of its obvious nature, is the following:—We may regard the zone-plate as a circular grating in which the grating space becomes less and less as we proceed outward from the centre, consequently the bending or deviation of the diffracted ray from the normal becomes greater as we near the edge, and the change in the grating space is such as to bring all the deviated rays of a single colour together in a point. The principal real focus of the plate corresponds to the superimposed spectra of the first order; the second order spectra being bent more come together at a point nearer the plate, forming a second focus, and so on—the different foci corresponding to spectra of different orders. There are also spectra of the first order bent outwards, or away from the centre, and these rays projected backwards behind the plate will meet, forming a virtual focus in a position corresponding to that of the real focus on the opposite side of the plate; the second order spectra are bent out more, consequently the virtual focus of these rays is nearer the plate, and we thus see, that for every real focus on the one side of the plate, there is a corresponding virtual focus on the other.

Plates of very long focus are also useful for demonstration. I have one 7 centim. in diameter, with a focal length of about twelve metres. This plate will project a very good image of the sun, about 11 centim. in diameter, on a screen placed at the focus. The effect is especially fine when the sun is behind the leafless branches of distant trees, for then each twig stands out sharp and distinct on the image. When this plate is set up at a distance of 3 metres from an arc-light and the eye is brought into one of the nearer foci, 3 or 4 metres behind the plate, the effect is very fine; the whole area filling up with a dazzling white light of almost insupportable brilliancy.

The diffraction phenomena produced by openings in thin transparent films were studied by Quincke and published in Poggendorff's *Annalen*, 1867. Quincke, moreover, prepared gratings with narrow strips of a thin transparent lamina instead of the usual dark spaces: these, by retarding the light, threw the waves out of phase with those which passed between them. These gratings he prepared by ruling the silvered surface of a piece of glass on a dividing-engine, forming an ordinary grating in which the dark lines were narrow strips of silver equal in width to the clear spaces. By covering the plate with iodine the silver was changed into transparent iodide. He found that these transparent gratings gave spectra very similar to ordinary gratings, save in one respect, namely, that with films of certain thickness the central direct image disappeared entirely when monochromatic light was used, and passed through a succession of colours when white light was used. By making the films slightly wedge-shaped and ruling the lines at right angles to the thin edge, he obtained a grating in which the thickness increased continually towards the bottom. On looking at an illuminated slit through the top of this grating where the thickness of the film was zero, the central image alone was observed. On moving the grating up, the film began to have an appreciable thickness, and side or diffracted images appeared on either side of the central one. These increased in brightness with increasing thickness of the film, while the central image became fainter, and finally disappeared when the thickness was such as to give to the light passing through it a retardation of one half wave-length on the light passing by it. We thus have a transparent plate which will not allow a ray to pass directly through it, or, in other words, a transparent grating which gives no central image. I find that very satisfactory lamina gratings can be prepared in the same way as the zone-plates; by drawing black lines of uniform width and with uniform spaces between them on white paper, reducing them by photography, and printing on the sensitized gelatine. The same effects can be observed by viewing an illuminated slit through the outer edge of the lamina zone-plate held close to the eye.

Quincke found that if a portion of the hypothenuse surface of a right-angle prism be silvered, a ray reflected from the metal suffers a phase displacement relative to a ray which is totally reflected from the boundary between glass and air. Whether or not this change is a complete phase-reversal or not depends on a number of conditions. If polarized light is used, the change of phase depends on whether the plane of polarization is parallel or perpendicular to the plane of incidence, on the angle of incidence, and on the thickness of the film. It appeared probable that a silver zone-plate, formed on the surface of a reflecting prism, would give excellent results under certain conditions. Such a plate could be ruled on a slowly revolving prism, but the process of advancing the knife-point over the right distances would be tedious in the extreme, and I accordingly hunted about for a photomechanical process. After a little experimenting I devised a method which yielded beautiful results. Plate-glass, carefully cleaned and silvered by the chemical method, is coated with an exceedingly thin film of gelatine: the solution should be just too weak to set into a jelly on cooling, merely thickening a little. It should be poured while warm over the silver surface, and the plate set on edge to dry. This takes place in five or ten minutes, and the surface now shows interference colours. The film is now sensitized by a ten-second immersion in a very dilute solution of bichromate of potash (plate dipped in it should be coloured a very pale yellow only) and dried in the dark *.

An impression is taken by exposure under a negative, the plate is held for a moment in cold water and then washed in a stream of fairly hot water. This is the fussy part of the operation ; for if the stream is too violent or the water too hot it will wash away everything, and if it is too feeble some gelatine will be left between the undissolved zones. I find that pouring from a beaker held at a height of 10 centim. gives a stream of the proper strength, and the temperature should be about what the hand can bear comfortably. A half minute's washing is enough, and the plate must now be flooded with alcohol and then with ether, to prevent the slow solution of gelatine during the drying, which always coats the clear zones with a thin film. The plate will dry in a few seconds, and if it has been properly prepared the odd zones will be clean bright silver, and the even ones covered with a thin film which appears white by reflected light. The plate is now covered with iodine crystals, which convert the exposed silver into iodide, and the process can be watched by holding the plate above the head and looking at the under side. The iodide zones will appear white, and on shaking off the iodine transparent by transmitted light.

A strong solution of sodium thiosulphate is now applied, which dissolves the silver iodide, and the plate is then rubbed with the fingers under warm water until the gelatine is all removed. If everything goes well a most beautiful plate is the result. The silver rings are as sharp as if cut with a knife, and the spaces between them are quite clean, even where there are fifty zones to the millimetre. Such a plate used with transmitted light is much better than the usual photographic type, in that the clear spaces are perfectly clean and transparent, and the dark ones perfectly opaque, which, as I said before, is usually not the case.

It is not necessary to make the zones directly on the prism, though I tried this first. It is essentially the same thing if we make them on a plate of glass and fasten this to the prism face with Canada balsam, the silver side out of course. The brilliancy of the image produced by this arrangement is very great, but since the reflecting surface is at an angle of

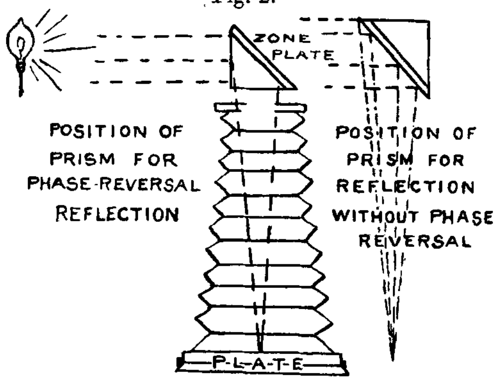
* Lately I have added the bichromate to the gelatine before flowing the plate, and find the process quicker and the results better.

45° the focus instead of being a point is a line. When the screen is near the prism we get a very bright vertical line, which fades away as the screen is moved off, being replaced with a horizontal line. This is exactly what we get when we hold a transmission plate at an angle with the screen, and the greater the obliquity the shorter the focus, the increasing obliquity decreasing the grating space on the sides of the plate.

To get a point-focus from a reflecting plate on a prism the zones must be elliptical, the major axes being double the minor, the whole being so placed that its projection on each of the other two faces of the prism is a circle. To make an elliptical zone-plate we have only to photograph the original drawing in an oblique position. A slight error due to perspective will of course be introduced, but it is insufficient to materially affect the results.

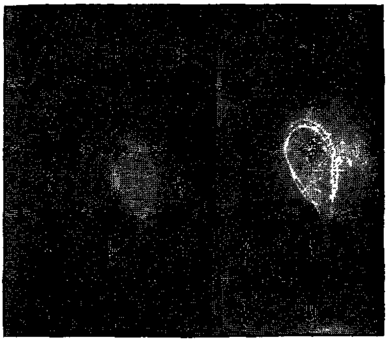
From a negative made from the large drawing set at an angle of 45° I prepared a silver-on-glass elliptical plate, and cemented it onto the prism with balsam. This combination fulfilled every expectation, and yielded a very brilliant and sharp image. It furnished, moreover, a very nice means of comparing the effect of an ordinary plate with a phase-reversal one; for by turning it round a focal image was formed by the reflexion from the outer surface of the silver zones alone, which was many times fainter than that produced by the internal reflexion. In making this comparison I noted the curious circumstance that the focus from internal reflexion is longer than that from external reflexion, but can as yet offer no explanation of the fact.

Fig. 2.



In order to give some idea of the relative intensities of the focal images produced by the plate with and without a phase-reversal, I photographed an incandescent lamp-filament, first from the outside surface of the zones and then from the inside, by the arrangement shown in fig. 2. The time of exposure and development was the same, and both were taken on the same plate. Reproductions of the two images are shown in fig. 3. It will be noted that the brilliant one

Fig. 3.



(or phase-reversal image) is larger than the other. This is due to the increase in the focal length to which I have alluded, and, other things being equal, would make the illumination less. The image obtained from the outer surface of the plate is so faint that it is hardly perceptible.

I have made a very cursory examination of these phase-reversal reflecting plates with polarized light, and find that if the light falling on the prism be plane-polarized, rotation of the nicol causes fluctuations in the brilliancy of the image. The effect, however, varies with the angle of incidence; at an angle of 40° with the normal of the reflecting surface no effect is produced by rotating the nicol. When the angle is greater than this, partial extinction of the image is produced when the plane of polarization is perpendicular to the plane of reflexion; when the angle is less than 40°, the reverse is the case. The field about the image grows bright when the central image weakens, and *vice versâ*. This requires a much more complete investigation than I have had time to give it yet, but is doubtless caused by the variations in the phase-change due to variations in the angle of incidence which Quincke observed.

Lord Rayleigh has drawn attention to the fact that if it were possible to construct a lamina-grating in which an arbitrary retardation could be introduced at every part of the aperture all the light could be concentrated in any desired spectrum. This might be accomplished, as Prof. Crew has suggested to me, by shading the original drawing: possibly photographing a wire-grating on a white ground, illuminated by oblique light (the shading being produced by penumbræ). Or a series of wedge-shaped strips of tinted glass or gelatine in transmitted light might yield results. The gelatine films would have to receive the impression through the glass, I imagine, if variations in the thickness were necessary.

If this could be accomplished, the same principle could be applied to the zone-plate, all the light being brought to a single focus.

Experiments in this direction are now in progress.

Physical Laboratory of the University of Wisconsin.
Madison, February 1898.

LIV. *Note on the Pressure of Radiation, showing an Apparent Failure of the usual Electromagnetic Equations.* By Lord RAYLEIGH, *F.R.S.**

FOLLOWING a suggestion of Bartoli, Boltzmann † and W. Wien ‡ have arrived at the remarkable conclusion that that part of the energy of radiation from a black body at absolute temperature θ, which lies between wave-lengths λ and $\lambda + d\lambda$, has the expression

$$\theta^5 \phi(\theta\lambda)\, d\lambda, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where ϕ is an arbitrary function of the *single* variable $\theta\lambda$. The law of Stefan, according to which the total radiation is as θ^4, is thererein included. The argument employed by these authors is very ingenious, and I think convincing when the postulates are once admitted. The most important of them relates to the *pressure* of radiation, supposed to be operative upon the walls within which the radiation is confined, and estimated at one-third of the *density* of the energy in the case when the radiation is alike in all directions. The argument by which Maxwell originally deduced the pressure of radiation not being clear to me, I was led to look into the question a little more closely, with the result that certain discrepancies have presented themselves which I desire to lay before those who have made a special study of the electric equations. The criticism which appears to be called for extends indeed much beyond the occasion which gave rise to it.

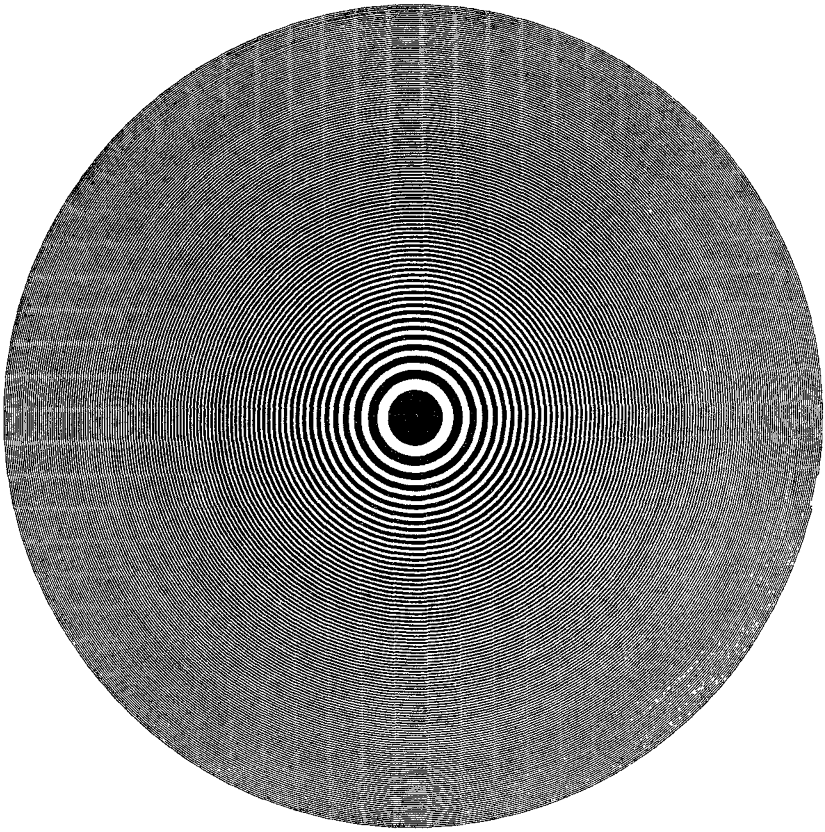
A straightforward calculation of the pressure exercised by plane electric waves incident perpendicularly upon a metallic reflector is given by Prof. J. J. Thomson §. The face of the reflector coincides with $x=0$, and in the vibrations under

* Communicated by the Author.

† Wied. *Ann.* vol. xxii. pp. 31, 291 (1884).

‡ *Berlin. Sitzungsber.* Feb. 1893.

§ 'Elements of Electricity and Magnetism,' Cambridge, 1895, § 241.



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