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THE TEACHING OF MATHEMATICS FOR ENGINEERS[1]

MATHEMATICS, from the standpoint of the engineer, is a means, and not an end. It is an instrument or tool by which he may determine the value and relations of forces and materials.

The usefulness of tools depends upon the sort of work which is to be done, upon the kinds of tools which are available and upon the skill of the man who uses them. We may inquire, therefore, what are the uses to which the engineer may apply mathematics? What kind of mathematics does he need? And what skill should he possess in their use?

First, then, what work is to be done by the young men who are now taking engineering courses? A few—and only a few—will be original investigators or designers who will need mathematics as an instrument of research. A considerable number will regularly employ elementary mathematics in more or less routine calculations. Many will have little use for mathematics, as engineering courses are recognized as affording excellent training for various business, executive and other non-technical positions, particularly in connection with manufacturing and operating companies. It has been stated by the vice-president of a large electric manufacturing company that not over ten per cent. of the technical graduates employed by that

[1] Read before Sections A and D of the American Association for the Advancement of Science and the Chicago Section of the American Mathematical Society, at the Chicago meeting, December 30, 1907.

company are fitted by temperament or by education to take up with success the work of pure engineering. A recent classification of the graduates of Sibley College, Cornell University, shows that about half are in occupations which require no advanced mathematics and it is probable that many of the 36 per cent. classed as mechanical and electrical engineers seldom go beyond the rules of arithmetic. Hence a goodly proportion of engineering graduates do not need to be mathematical experts. Their mathematical studies need not aim to produce experts, but should have as a principal object the mathematical training which is a most efficient kind of training in an engineering course. On the other hand, the engineers who will have practical use for the higher mathematics will find their ability as engineers is in a large measure determined by their ability as mathematicians.

Second, the question, what kinds of mathematics does the engineer need? is closely related to the class of work he is to do. In general a great deal of engineering work is done with much less use of higher mathematics than most professors probably imagine; and furthermore, it may be remarked, with much less than could profitably be employed. Engineers are apt to use ordinarily the mathematical methods with which they are most familiar and which will bring the result with the least effort. One man employs calculus, another draws a diagram, another writes out formulæ, while another gets his results by mental arithmetic. The object is to get the result.

The fundamental idea that mathematics is something for the engineer to use finds many illustrative analogies in ordinary tools. Adaptation is the first requisite. Tools should be suited to the work to be done. An expensive machine tool with its refined adjustments is quite unnecessary for executing a piece of work which can be done with sufficient accuracy by a few minutes' application of a file. An ordinary calculating slide rule is infinitely better than a table of seven-piece logarithms in every-day work.

On the other hand, it is particularly wasteful to attempt to execute a difficult and intricate piece of work with inadequate tools. But more important than the tool is the skill of the man who uses it. A skillful workman can accomplish results with a few simple tools which others can not get with the most elaborate special equipment.

Third, therefore, skill in the use of mathematics is the really essential thing. A judicious use of arithmetic with a little algebra or a simple diagram often leads to more satisfactory results than others secure through elaborate processes involving lengthy equations and complicated operations. In the latter, errors are liable to occur, the common-sense import of the problem is apt to be overlooked, assumptions may be made to facilitate calculations which are physically unwarranted as one loses sight of the physical problem in the intricacy of the mathematical solution. Abstract mathematical studies, if pursued as a kind of intellectual calisthenics, may produce a pure mathematician, but they may unfit a man for practical engineering. A mathematician is not necessarily an engineer; nor is an elocutionist necessarily a good lecturer, nor is a tool expert a successful manufacturer.

Mathematics is used in engineering to express the quantitative relations of natural phenomena. The mathematician delights in the relations: he divorces them from the phenomena and gives them abstract expression, while the engineer is concerned with the natural phenomena; he demands the physical conception; the me-

dium of expressing these relations is of secondary consequence.

The mathematician evolves the equation for a parabola and finds a convenient illustration in the law of projectiles. The engineer finds that a physical result follows from the application of certain forces, and uses the formula merely as a convenient method of expressing the law. The analogue in the case of mechanical tools is found by regarding a set of drawing instruments or a transit or a lathe, as something intelligently designed, properly proportioned, accurately made and finely finished, the merit of which lies in its own inherent excellence; or, on the other hand, by considering them as tools adapted for doing a certain range and character of work with a sufficient degree of accuracy and at low cost.

A manual-training school gives familiarity with mechanical tools and mathematical study gives familiarity with intellectual tools. In work with the manual tool the boy uses it for making something —he learns the principle on which it operates and the way to use it, by making something; if it is something useful it awakens a higher interest than does some fancy device. Likewise training of engineers in mathematics should be by doing something, by the solving of problems, by dealing with real rather than abstract conditions. Let this training be secured while applying mathematics to its normal and legitimate purpose as an auxiliary in the study of other branches.

In the teaching of mathematics for its own sake stress is apt to be laid upon the processes of deriving results rather than the real meaning of the results themselves. An engineer who uses logarithms has no more concern regarding their derivation than the ordinary user of the dictionary for finding the pronunciation of words has in their etymological derivation. The ability to reproduce demonstrations in higher mathematics from memory with the book shut is often not as important as it is to understand them with the book open. In general an engineer, who has occasion to use higher mathematics, will not be interested in evolving difficult equations, nor will he appeal to his memory, but with text-book or reference before him he will seek the things he wants to use. He should know where to find them and how to use them.

In emphasizing what a skilled mechanic can make with very ordinary tools, or the true engineer can accomplish with the parallelogram of forces and the rule of three, there is no intention of discrediting the value of fine equipments, either mechanical or mathematical, if there be the ability to use them.

Possibly the practical utility of mathematics may appear to be urged too strongly, particularly as the writer really believes in thorough mathematical training, but he has seen so many cases in which mathematical instruction has never been digested and assimilated, he has seen simple problems confused by unnecessary mathematical complications, he has seen men satisfied with results which are absurd because of some mathematical equations—sometimes quite unnecessary—which seem to obliterate common-sense perspective, and he recalls the new insight into mathematics which came through "Analytic Mechanics" under Professor S. W. Robinson at the Ohio State University, and "Problems in Mechanics," under Dr. Fabian Franklin at Johns Hopkins University, that he feels there is little danger in over-emphasizing the importance of concrete training in mathematical study.[2]

[2] Both of these teachers of mathematics had been trained as engineers and had practised the profession.

The practical questions which the discussion of this subject presents are these:

What mathematical subject-matter should be covered? And,

How should it be taught?

The first difficulty is that there is not, and can not be, a differentiation in technical education which is at all comparable with the wide range of occupations into which graduates will enter. We may assume, therefore, that we are considering the case of the average engineering student, taking for granted that options may be used by the best students for enabling them to take up the more advanced and difficult mathematics. Obviously the student should have enough mathematics to enable him to demonstrate the important engineering laws and formulas and to read intelligently mathematically written engineering literature. While only the relatively simple mathematics is commonly used by engineers, yet the ability to handle new problems with confidence requires a thorough understanding and appreciation of the significance of the mathematical and physical basis of the laws and phenomena he is to use. A man who is a thorough mathematician and knows how to apply his knowledge has a great advantage over the pure mathematician or the man without mathematical equipment. The better knowledge one has of the complex, the more certainty he has in applying the simple. A student should understand something of the power of the advanced mathematics and the field of its efficient application. Although he may not be expert in using it himself, he will know when to call for a mathematical expert.

An engineer of fairly wide experience remarked a short time ago: "The ordinary engineer does not use higher mathematics because he doesn't know how. He does not have the proper conception of the fundamental principles of the calculus because the subject has been taught by men whose ideals are those of pure mathematics."

If mathematics is something for engineers to use, let its use be taught to engineering students. After the fundamentals are learned, the students should attack the engineering problem at once and bring in mathematics as a means of solving it. Mathematics is often advocated for developing the reasoning powers and the ability to reason from cause to effect. There is danger, however, that mathematical machinery may make the mere process obscure the cause and the effect. Let them be foremost, with the process secondary or auxiliary to them.

The way mathematics is brought to bear on some engineering problems reminds one of the story of the old lady who greatly admired her preacher because he could take a simple text and make it so very complicated.

Old traditions have not wholly disappeared, the fear of degrading the pure science of mathematics by applying it to useful things still lingers—in influence, if not in precept. We must go further and adapt mathematics to engineering, not only in subject matter, but in method. A mathematical teacher with no patience for anything except mathematics will probably teach a kind of mathematics which has no connection with anything except mathematics. Engineering mathematics may be better taught as a part of engineering by an engineer, than as a part of mathematics by a pure mathematician. The maker of levels and transits who is expert in the construction of the instruments and an enthusiast over the accuracy of the surfaces, the excellence of the bearings, the near approach to perfection in the graduation and the general refinement and beauty of workmanship, may make a good in-

structor on instruments, but a poor teacher of civil engineering.

After all, it is not so much abstract courses as it is personal men with which we have to do, it is not mere knowledge of facts or facility in mathematical manipulation, but it is training. The young man is to be developed, his native individuality is to be the basis, he is to increase not only his knowledge, but his powers and the ability to use them. It is not mathematical skill so much as a mathematical sense, or mathematical common-sense, which is wanted. With pure mathematics as a science we have no quarrel—and little affiliation.

If you ask men who use engineering graduates what qualities they should possess, you will find that special prominence is given to "common-sense" and "the ability to do things." In mathematical training it is quality rather than quantity which is of first consequence. It should develop the facility for systematic and logical reasoning, thus furnishing a general method as well as a specific means of getting results.

We are concerned with applied mathematics. The ability to state a problem; to recognize the elements which enter into it; to see the whole problem without overlooking some important factor; to use good judgment as to the reliability or accuracy of the data or measurements which are involved; and, on the other hand, the ability to interpret the result; to recognize its physical significance; to get a common-sense perspective view of its meaning and the consequences which may follow; to note the bearing of the various data upon the final result; to determine what changes in original conditions may change a bad result into one which is practical and efficient—such abilities as these are of a higher order than the ability to take a stated problem and work out the answer. It may be urged that all this is not strictly mathematics. But it is just this sort of judgment and insight which makes mathematics really useful, and without them there is danger that they may be neither safe nor sane.

The trend in education is to a closer relation to the affairs of life. Science and applied science, scientific and engineering laboratories, are overcoming old ideas and prejudices. Modern engineering development brings its transforming influence to bear upon education as well as the utilities of modern life. The engineering school has had a phenomenal growth within the lifetime of the recent graduate—a growth in ideals and methods as well as students and equipment. It has raised and agitated broad questions as to what constitutes efficient education for producing effective men. It has aimed to combine not only the abstract with the concrete, the lecture room with the laboratory, and the scientific experiment with the practical test; but it has sought by various means to bring the work of the school into close relation with active professional and commercial practise. It has a definiteness of aim and purpose which other educational courses are apt to lack. It sets out to produce men who can deal with forces and materials according to scientific principles. It develops men whose contact with physical facts and natural laws at first hand and whose ability to reason logically fit them for dealing with new problems. The training which fits men for handling engineering problems is the kind that is needed for dealing with the organization and directing of men. The sphere of the engineer is one the scope of which will continue to increase as engineering education and training produce men whose contact with natural phenomena gives them an inherent respect for facts as their premises, who are able to think straight to logical

and common-sense conclusions, who have an equipment of technical knowledge and who can produce results.

In discussing the teaching of mathematics to engineers, we should emphasize not the mathematics nor the engineers, but the teaching. Aside from the imparting of knowledge and technical ability, the teaching of mathematics gives opportunity for training in the use of logical methods and in the drawing of intelligent conclusions from unorganized data which will make efficient men, whether they follow pure engineering, or semi-technical, or business pursuits. Such teaching does not come from the text-book; it must be personal—it comes from the teacher. He must be in sympathy with engineering work and have a just appreciation of its problems and its methods. He must be imbued with the spirit and the ideals of the engineer.

Chas. F. Scott

THE POINT OF VIEW IN TEACHING ENGINEERING MATHEMATICS[1]

I hardly know why I should have been asked to address you at this conference. Possibly, however, the fact that I am a civil engineer by profession, without having been permitted ever to practise this profession, and the additional fact that I have been a professional teacher of mathematical physics, without having been permitted to continue in this work, have led your committee to think that I might furnish a conspicuous illustration of the failures to which colleges and universities may lead in these lines of endeavor.

Having listened attentively to the three formal papers just read, I find it essential to revise my program and instead of following similar lines to those of the preceding speakers, it seems essential to take direct issue with them. This I am disposed to do, not so much because I differ wholly from the views they have set forth, as because it seems necessary to have other sides of the questions they have discussed represented. The preceding speakers appear to me to have taken themselves somewhat too seriously. This is a general fault of both theoretical and practical educationalists. My own experience leads me to conclude that in educational affairs the teacher, the school, the college and the university play a much less important rôle than we commonly suppose. In fact, I have reached the provisional conclusion that the majority of our students turn out fairly well in the world not so much by reason of the academic instruction they receive as in spite of it.

My impression also is that in taking ourselves too seriously as teachers of one subject or another, we have, as a rule, quite underestimated the magnitude and the difficulty of the psychological problems with which we have to deal. We have, as a rule, quite overestimated the capacity of our average student, and have thus usually expected too much from him. It is, of course, desirable to set our ideal high and try to rise to an elevated intellectual level; but in doing so we have commonly neglected the influence of heredity as well as of environment. I am inclined to think Dr. Holmes was right when he said that it is essential in the generation of a gentleman to begin four hundred years before he is born. So also is it necessary, if we wish to develop a student into a first-class scholar, to begin back some generations before we take up the formal work of training in our colleges or schools of engineering. It is an important fact, also too commonly over-

[1] Extempore remarks before Sections A and D of the American Association for the Advancement of Science and the Chicago Section of the American Mathematical Society, at the Chicago meeting, December 30, 1907.