ART. XL.—*A method for determining the flexure of a Telescope Tube for all positions of the Instrument*; by J. M. SCHAEBERLE.

IN all observations with the meridian circle only the maximum *astronomical* flexure of the telescope tube is determined directly from observations, two horizontal collimators being used for this purpose. This maximum flexure is then assumed to vary as the cosine of the altitude. I propose in this paper to give a very simple method by means of which the accuracy of this law can be tested directly for all positions of the tube, nothing but a small plane mirror, silvered by Draper's process, being required for this purpose. To this mirror is cemented a circular ring of cardboard of just sufficient thickness to prevent

the mirror from touching the object-glass of the telescope to which it is cemented with a solution of gum arabic. The centering is quickly done by shifting the mirror upon the spherical surface of the lens until the reflected image of the horizontal wire is nearly in coincidence with the wire itself.

In experimenting I have used a piece of plate glass: to test the surface of the same before silvering, the image of a distant brick wall, reflected obliquely from the glass surface, was examined with the aid of an excellent $2\frac{1}{2}$ inch achromatic; from a number of pieces one was selected which gave a sharp image of both the vertical and horizontal joints for the same focus. A beautifully sharp and fine image of the wires will be given by such a mirror (silvered on the side toward the object-glass) although less than two inches in diameter.*

Let m_1 be the reading when the micrometer wire is in coincidence with the reflected image of the fixed horizontal wire, the telescope being pointed to the zenith, and let m be the reading for any other pointing. Let x and y be respectively the deflections of the object- and eye-ends of the tube from the positions these ends would have if there were no flexure.

L = focal length of objective.

l = distance of the object-glass from the line about which the flexure causes it to revolve. Placing the sines and tangents of very small angles equal to their arcs, the linear distance of the fixed wire from its reflected image, can be expressed by a very simple equation. For we have,†

$$\frac{2}{\sin 1''} \left(\frac{x}{l} - \frac{x-y}{L} \right) = \frac{m-m_1}{L \sin 1''} \quad \text{therefore}$$

$$x \left(\frac{L}{l} - 1 \right) + y = \frac{m-m_1}{2}.$$

Letting $\left(\frac{L}{l} - 1 \right) = c$ and $\frac{m-m_1}{2} = F$, we can write

$$\begin{aligned} cx + y &= F \\ cx' + y' &= F' \\ cx'' + y'' &= F'', \\ &\text{etc., etc.} \end{aligned}$$

If f_0 represents the maximum *astronomical* flexure determined by means of two horizontal collimators, we have

* The focal length of the objective used is eight feet, and the wires made visible in precisely the same way as in taking nadir observations.

† The flexure really gives the tube a curved outline. In assuming that the change in the inclination of the mirror (due to flexure) is equal to $\frac{x}{l \sin 1''}$ (where l can usually be taken equal to the distance of the objective from the nearest face of the cube, to which the tube is bolted) only very small terms are omitted which can generally be neglected as inappreciable,

$$\begin{aligned} f_0 &= x_0 - y_0 \\ F_0 &= cx_0 + y_0 \end{aligned}$$

hence we obtain

$$x_0 = \frac{f_0 + F_0}{c + 1} \quad y_0 = \frac{F_0 - cf_0}{c + 1}.$$

Representing the zenith distance of the pointing by z we should have

$$\begin{aligned} (cx_0 + y_0) \sin z &= cx + y = F \\ (cx_0 + y_0) \sin z' &= cx' + y' = F', \\ &\text{etc., etc.} \end{aligned}$$

provided the law, that the flexure varies as the cosine of the altitude, is correct.

If the measured values F , F' , etc., are not satisfied by the above equations, the residuals will furnish data for obtaining the form of the function which, when substituted in the place of $\sin z$, will best represent the observed readings. Any sudden *apparent* change in the flexure, as for instance the shifting of the object-glass in its cell, could be observed directly by keeping the eye at the instrument while it is being revolved. In some cases it might be well to fasten the mirror to the cell instead of to the objective. After the flexure has been determined, a thin strip of metal, the weight of which is exactly equal to the weight of the mirror, can be wound around the object end of the tube.