SOME LECTURE ROOM METHODS IN THE ELEMENTARY THEORY OF ELASTICITY.

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NEXT to thermodynamics the elementary theory of elasticity is perhaps the most difficult branch of physics to present to a class. The importance of this subject to the engineer, however, demands a more or less adequate treatment of it in our technical schools and every possible means should be employed to render the details of the theory concrete and to develop the geometric-mental images which form the basis of any understanding of the subject. The object of this paper is to describe some lecture experiments which have been found by the writer to be helpful in the presentation of this abstruse subject in the lecture room.

The elementary theory of elasticity may be conveniently divided into four parts, (*a*) Strains, (*b*) Stresses, (*c*) Stress-strain relations, (*d*) Particular cases such as the bent beam and twisted rod. Part (*c*) is not satisfactorily treated in any of our text-books, as it seems to the writer, and at the end of this paper an outline is given of what seems to be the most satisfactory elementary development of stress-strain relations.

Concerning strains it seems to the writer to be exceedingly important that the student get a clear idea of the character of homogeneous strain, that he understand the significance of the elements of a homogeneous strain—namely three mutually perpendicular stretches, and that he see the applicability of the notion of homogeneous strain to the small parts of a body non-homogeneously strained. In the attainment of these objects the notion of the strain ellipsoid is of the greatest usefulness. Now most students are not sufficiently familiar with solid geometry to get a clear idea of the strain ellipsoid and the writer finds it helpful to use the distortion of an elastic sheet as an illustration in two dimensions, where the relation of the ellipse to the circle gives the geometrical picture of the

strain. An extremely thin sheet of rubber, slightly oiled with a non-mineral oil to make it semitransparent, is stretched over a light frame fifteen or twenty centimeters square. This state of the sheet is taken as the normal or unstrained state and small circles (also a large circle) are marked upon it. The small circles may be projected by means of a lantern. The sheet may now be stretched homogeneously, the sheet being fastened to the frame by a number of spring clips, and the circles are seen to be changed to similar and similarly placed ellipses. The student's knowledge of the relation of the ellipse to the circle, gotten perhaps from his work in projection drawing, enables him to realize that the distortion of the sheet at each point consists of two mutually perpendicular stretches, while the similarity of the ellipses shows that the strain is homogeneous. Stretching the sheet non-homogeneously the small circles are changed to dissimilar and dissimilarly placed ellipses. The attention of the student may then be directed to the fact that the large circle seen directly differs sensibly from an ellipse while the small circles *even when greatly magnified on the screen* are sensibly elliptical. It is also instructive to show the character of the distortion of square figures marked upon the sheet.

Concerning stresses, it is very important that the student get a clear idea of homogeneous stress, that he understand the significance of the elements of a homogeneous stress, namely, three mutually perpendicular pulls, and that he see the applicability of the notion of homogeneous stress to the small parts of the body, subjected to a non-homogeneous stress. These things cannot, perhaps, be treated experimentally in the lecture room, and, therefore, it seems advisable to return to the experiments with the rubber sheet after a statement of the fundamental fact of the stress-strain relation for isotropic substances, namely, that the axes of stress and of strain are parallel, the student being led to infer the stresses from the observed strains.

In the discussion of shearing strain, the writer uses a large, square, hinged frame, over which muslin cloth is stretched, the threads of the cloth being parallel to the sides of the frame. The reduction of a shearing strain to two diagonal stretches (mutually perpendicular, of course) is most clearly shown by drawing a

large circle upon the undistorted muslin. This circle is changed by the distortion to an ellipse with its axes diagonal to the frame.

In the discussion of the strains (and stresses) in the twisted cylinder, the writer uses the following model: A large tin cylinder is covered with muslin cloth, which is fastened to the cylinder at one end and to a loose circular disk at the other end. Squares and circles marked upon this cloth show very clearly the strains (and stresses) produced when the circular head is slightly turned. In particular, the student can see that the cylindrical shell of muslin is lengthened along a set of helical lines which are inclined at an angle of 45° to the straight elements of the cylindrical surface, and shortened along another set of helical lines, which are everywhere at right angles to the first set. The student can infer the existence of tension along one set of helical lines, and the existence of compression along the other set of helices, and he can infer from a previous discussion of shear that tangential stresses act across all horizontal lines and across all vertical lines.

STRESS-STRAIN RELATIONS.

Considerations of symmetry show that axes of stress and axes of strain coincide in isotropic substances. This is the first fact to be brought out in the discussion of stress-strain relations. Further development should, it seems, be based upon the experimental study of longitudinal stress and the strain produced thereby. Let P be the longitudinal pull, and let p' , q' and r' be the corresponding stretches.

Experiment shows that when p' is small, it is proportional to P or equal to aP .

Also experiment shows that q' and r' are equal to each other and negative (*i. e.*, contractions), and that they are proportional to P or equal to bP . Therefore

$$\begin{aligned} p' &= aP \\ q' &= -bP \\ r' &= -bP \end{aligned} \tag{1}$$

Similarly, a pull Q (parallel to the stretch q' above) produces the stretches:

$$\begin{aligned}
 p'' &= -bQ \\
 q'' &= aQ \\
 r'' &= -bQ
 \end{aligned}
 \tag{2}$$

And a pull R (parallel to the stretches r' and r'' above) produces the stretches :

$$\begin{aligned}
 p''' &= -bR \\
 q''' &= -bR \\
 r''' &= aR
 \end{aligned}
 \tag{3}$$

Experiment shows that the stretches (small) produced by a number of pulls acting together are equal to the sums of the respective stretches produced by the pulls acting separately so that the stretches P , Q and R acting together produce the stretches :

$$\begin{aligned}
 p &= aP - bQ - bR \\
 q &= -bP + aQ - bR \\
 r &= -bP - bQ + aR
 \end{aligned}
 \tag{4}$$

The equations express the stress-strain relation for isotropic substances. They are based upon a semi-axiomatic fact and three experimental facts. The first experimental fact may be readily demonstrated experimentally, and the second may be illustrated by a rubber band, but the third is scarcely within reach of the hasty experimental methods of the lecture room.

SOUTH BETHLEHEM, March 13, 1900.