280. On the Kinetic Measure of a Force

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It is required to prove that Pa passes through X , the point where the arc centre β and radius $\beta\gamma$ cuts the circle BXC , Y being the other point of intersection of these two circles.

Produce Pa to cut AC at N .

Produce $A\beta$ to cut Pa at M .

XY is the radical axis of the two circles $F\gamma X$, BXC

and $\therefore$ passes through a , since aC , aQ are tangents to these two $\odot^*$ and equal.

If $AB = 2d$, then $\beta E = \frac{2d}{\sqrt{3}}$, $AE = d$. Hence $\frac{\beta E}{EA} = \frac{2}{\sqrt{3}}$.

$$\left. \begin{aligned} CQ \text{ and } AP \text{ are parallel and } Ca = \frac{1}{4}AP = \frac{d}{\sqrt{3}} \\ \text{and } CN = \frac{1}{4}AN = \frac{1}{3}AC = \frac{2d}{3} \end{aligned} \right\};$$

$$\therefore \frac{aC}{CN} = \frac{\sqrt{3}}{2} = \frac{EA}{E\beta}.$$

Hence angle $\beta AE = \text{angle } CaN = \text{angle } QaP$.

Hence angle $QPM = \text{angle } BAM$.

Hence $BPAM$ are concyclic.

$\therefore$ Angle $PMA = \text{angle } PBA = \text{a right angle}$.

$\therefore A\beta$ is perpendicular to Pa .

But $A\beta$ (line of centres) is perpendicular to radical axis YXa ,

$\therefore YXa$ and Pa coincide,

i.e., Pa passes through X . S. DE J. LENFESTEY.

280. [R. 6. a. β .] *On the Kinetic Measure of a Force.*

That the force on a particle is proportional to the time-rate of change of its momentum is, of course, an article of faith; but that the same statement may be applied to a body without any reservation as to what is meant by this term appears to be open to discussion.

To take an example: Suppose a railway train, moving uniformly, to enter a tunnel; the part within the tunnel at any instant is clearly a "body" and is acquiring momentum so long as the train is only partly within the tunnel. But we do not say that this fact points to the existence of unbalanced force on this portion of the train, for the change of momentum arises here from a flux of mass into the tunnel, and our "body" does not consist from time to time of the same particles.

Let μ be the mass per unit length of the train, v its velocity. In the time dt the momentum brought into the tunnel is clearly $\mu v dt \times v$. We must then subtract μv^2 from the rate at which momentum is accumulating in the "body" to arrive at the true kinetic measure of the force upon it. Similarly, if we fix attention on the part left within the tunnel as the train is emerging we must add μv^2 to the rate of change of its momentum in order to form a just estimate of the resultant force upon it; which force should be zero in both cases, as the motion is uniform. (Cf. Lamb's *Motion of Fluids*, 1st ed., § 12.)

The problem of snow-sliding down a roof, alluded to by Prof. G. H. Bryan in our May number, is, in my opinion, incorrectly solved in one of our textbooks. No friction or adhesion are allowed for at all, and yet, wonderful to say, the acceleration of the portion left on the roof is only $\frac{1}{3}$ that of a freely-sliding particle!

This extraordinary result is obtained thus: Let b be the breadth of this portion, x its length, μ the mass of snow per unit area, a the slope of the roof.

$$-\frac{d}{dt}(\mu b x \dot{x}) = \mu b x g \sin a; \dots\dots\dots(1)$$

$$\therefore \frac{d}{dt}(x\dot{x}) = -xg \sin a,$$

$$x\dot{x} \frac{d}{dx}(x\dot{x}) = -x^2 g \sin a,$$

$$\frac{1}{2}(x\dot{x})^2 = -\frac{1}{3}x^3 g \sin a,$$

$$\frac{1}{2}\dot{x}^2 = -\frac{1}{3}xg \sin a,$$

$$\ddot{x} = -\frac{1}{3}g \sin a,$$

whereas the equation of motion should, I think, have been

$$-\mu b x \ddot{x} = \mu b x g \sin a, \dots\dots\dots(2)$$

whence

$$\ddot{x} = -g \sin a.$$

It will be seen that the term $\mu b \dot{x}^2$, representing the rate of efflux of momentum across the spouting, must be added to the first member of (1) to make the equations (1) and (2) agree, as the tunnel theory would indicate. That (2) gives a result less in accordance with observation than (1) may easily be accounted for by the fact that the friction as compared with the weight of the snow is by no means negligible. (Discussion invited.)

C. E. M'VICKER.

281. [D. c.]. *Note on Taylor's Theorem.*

Bertrand's proof of Taylor's Theorem with Lagrange's remainder is given in Williamson's *Differential Calculus*; other text-books give very similar proofs. I have been in the habit of using the following modification of Bertrand's proof for over 25 years, but thought it too small a thing to print. Now I see an appeal to print any *little thing* in the *Gazette*; and I further notice that Prof. Love gives two series in expanding e^x , which are only particular cases of my general series.

With the usual limitations as to continuity, which need not be repeated here, we have to find R if

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \dots + \frac{h^{n-1}}{[n-1]}f^{(n-1)}(x) + \frac{h^n}{[n]}R.$$

Now form a new function of z , from the right-hand side of the above, by writing $x+z$ for x and $h-z$ for h ; or let

$$\begin{aligned} \phi(z) = & f(x+z) + (h-z)f'(x+z) \\ & + \frac{(h-z)^2}{2}f''(x+z) + \dots + \frac{(h-z)^{n-1}}{[n-1]}f^{(n-1)}(x+z) + \frac{(h-z)^n}{[n]}R. \end{aligned}$$

Then $\phi(z)$ takes the value $f(x+h)$ when $z=0$ and when $z=h$.

Hence $\frac{d\phi(z)}{dz}$ must vanish for some value of z between 0 and h , say when $z=\theta h$.

$$\text{Now} \quad \frac{d\phi(z)}{dz} = \frac{(h-z)^{n-1}}{[n-1]} \{f^{(n)}(x+z) - R\};$$

$$\therefore f^{(n)}(x+z) - R \text{ vanishes when } z = \theta h, \text{ or } R = f^{(n)}(x + \theta h).$$

When I first used this method I included the term $f(x+h)$ in $\phi(z)$, giving either this term or all the others the minus sign. The introduction of z