ENERGY TRANSFORMATIONS IN THE SYNCHRONOUS CONVERTER.

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CONTENTS.

1. Introduction.
2. The synchronous converter compared with the auto-transformer. Statement of object of this paper.
3. The many-ring converter.
Derivation of armature current.
Discussion of energy transformations.
4. The two-ring converter. Discussion of energy transformations.
5. The three-ring converter. Discussion of energy transformations.
6. The four-ring converter. Discussion of energy transformations.
7. Summary of results as to energy transformations.
8. Armature reaction of many-ring converter. Influence of load upon electromotive force curve. Cutting away of pole faces to annul distortion of electromotive force curve at full load.
9. The criterion of reality of component parts into which a physical aggregate is resolved by mathematical analysis.

1. INTRODUCTION.

The engineer ordinarily approaches the study of the synchronous converter with ready made knowledge of the direct current dynamo and of the alternating current dynamo. The result is that the action of the synchronous converter is usually described in terms of the activities of the synchronous alternating current motor and the direct current generator, and the

theory of the synchronous converter is usually developed in terms of those notions which apply primarily to the synchronous alternating current motor, on the one hand, and to the direct current generator on the other. As a matter of fact, the *external electrical relations* of the synchronous converter may be completely represented by this method of attack. When, however, the synchronous converter itself—its *internal activities*, as it were—is the object of study, the notions of the synchronous alternating current motor and of the direct current generator do not suffice. There are certain elements of behavior of the synchronous converter which are foreign both to the synchronous alternating current motor and to the direct current generator; to examine into these elements we must attend to the actual physical activities of the synchronous converter without regard to the mathematical analysis of these activities into their more or less fictitious and unreal component parts, namely: synchronous motor activities and direct current generator activities. I give in a subsequent section of this paper a statement of the criterion which enables one to judge of the physical reality or unreality of the component parts into which a physical aggregate is resolved by mathematical analysis, and it suffices here to state that this criterion shows the physical unreality of synchronous motor activity and direct current generator activity as component parts of the activity of the synchronous converter.

2. THE SYNCHRONOUS CONVERTER COMPARED WITH THE AUTOTRANSFORMER. STATEMENT OF OBJECT OF THIS PAPER.

Let b Fig. 1 represent one of the d.c. brushes of a two-ring converter. Let this brush be chosen as the zero or reference point of potential, and let a represent the other d.c. brush. Let r be the position of the a.c. tap at a given instant. This tap r is at a potential which is between the potentials of b and a . Let i be the instantaneous value of the alternating current entering at r . A portion, A , of this current i flows up hill, as it were, from r to a , receiving energy because of assistance due to the induced electromotive forces in the windings between r and a . Another portion, B , of the current i flows down hill from r to b , giving up energy because it is opposed by induced electromotive forces in the windings between r and b .

Let b , Fig. 2, represent one of the service mains to which alternating current is supplied by an autotransformer. Let this main be chosen as the zero or reference point of potential, and let a represent the other service main. Let r and r' be the two supply mains. The main r is at a given instant at a potential which is between the potentials of b and a , and the current i , which enters at r at the given instant divides into two parts, A and B . The portion A flows up hill, as it were, to a , receiving energy because of assistance due to the induced electromotive forces in the windings between r and a . The other portion, B , flows down hill, from r to b , giving up energy because it is opposed by induced electromotive forces in the windings between r and b .

The energy received by the current A as it is boosted up from r to a is in general equal to the energy which is delivered by the current B which flows down from r to b .

The energy received by the current A as it is boosted up from r to a is equal to the energy which is delivered by the current B which flows down from r to b .

In the auto-transformer a definite portion of the energy which is transferred from the supply mains to the service mains is transferred by virtue of the conductive connections between supply mains and service mains through the windings of the transformer. In the synchronous converter, also, a definite portion of the energy which is transferred from the supply mains to the service mains is transferred by virtue of the conductive connections through the armature windings of the converter.

The Synchronous Converter

The Autotransformer

In the auto-transformer a definite portion of the energy which is transferred from the supply mains to the service mains is transferred by being inductively transformed from winding B, Fig. 2, which acts as a primary coil, to winding A, which acts as a secondary coil. In the synchronous converter a definite portion of the energy which is transferred from the supply mains to the service mains, is transferred by being inductively transformed from the portion B, Fig. 1, of the armature winding, which at the given instant acts as a motor, to the portion A of the armature winding, which at the given instant acts as a generator.

The object of this paper is to determine, of the total energy which flows through a synchronous converter, the fractional part which is conductively transferred from supply mains to service mains, and the fractional part which is inductively trans-

formed. In case of the two-ring converter, the inductively transformed energy consists of two distinct parts, namely, (a) The energy which is transformed by simultaneous and balanced generator-motor action, and (b) The energy which is transformed by successive and unbalanced generator-motor action. These parts will be more fully discussed when we consider the problem of the two-ring converter.

3. THE MANY-RING CONVERTER.

Fig. 3 represents one side of a two-pole, many-ring converter. E is the electromotive force between the d.c. brushes a and b , I is the direct-current entering at a and leaving at b , i is the current in the armature wires, at a point on the armature which

is at angular distance β from brush a , and di is the value of the alternating-current flowing out of all the collecting rings connected to commutator bars in the angular element $d\beta$ at the instant that these bars are at angular distance β from brush a . The arrows represent the directions in which the various electromotive forces and currents are considered as positive.

We shall assume here, as also in the discussion of the two-ring, the three-ring and the four-ring converters, that the converter receives alternating-currents at unity power-factor, that alternating electromotive forces and currents are harmonic, and that the losses of energy in the converter are negligible.

Before proceeding to the consideration of energy transformations, it is necessary to determine the distribution of current in

the armature for given direct current output. From the above assumptions, we have:

$$di = -A \cos \beta \cdot d\beta \quad (1)$$

in which A is an undetermined constant. Integrating this equation we have

$$i = -A \sin \beta + K \quad (2)$$

in which K is another undetermined constant. These constants A and K are determined by the following two conditions: 1st. The generator and motor activities are at each instant balanced in the many-ring converter and 2d, The alternating current input of power is equal to the direct current output of power. In order to formulate these two conditions it is necessary to find an expression for the generator (or motor) action in the element $d\beta$. Let de be the induced electromotive force in the element $d\beta$. Then it is easily shown on the basis of the above assumptions that

$$de = \frac{1}{2} E \sin \beta \cdot d\beta \quad (3)$$

Let dG be the generator action in watts (motor action if negative) in the element $d\beta$. Then $dG = i \cdot de$, or

$$dG = -\frac{EA}{2} \sin^2 \beta \cdot d\beta + \frac{EK}{2} \cdot \sin \beta \cdot d\beta \quad (4)$$

The first condition above mentioned requires that the integral of dG from $\beta = 0$ to $\beta = \pi$ shall be equal to zero. That is:

$$\int_0^{\pi} -A \sin^2 \beta \cdot d\beta + K \sin \beta \cdot d\beta = 0 \quad (5)$$

which gives

$$\frac{\pi A}{2} - 2K = 0 \quad (6)$$

The alternating current intake of power in the half armature is

$$\frac{1}{2} E \int_0^{\pi} (1 - \cos \beta) di$$

and the direct current output of power in the half armature is

$$\frac{1}{2} EI$$

and these are equal, so that, using the value of di from equation (1), we have

$$I - \int_0^{\pi} A \cos^2 \beta \cdot d\beta - A \cos \beta \cdot d\beta = 0 \quad (7)$$

which gives

$$\frac{I - \pi A}{2} = 0 \quad (8)$$

Equations (6) and (8) determine the values of A and K , giving

$$A = \frac{2I}{\pi} \quad (9)$$

$$K = \frac{I}{2} \quad (10)$$

Using these values of A and K in equation (2) we have

$$i = \frac{I}{2} - \frac{2I}{\pi} \sin \beta \quad (11)$$

It is worthy of note that the problem of the determination of armature currents in a synchronous converter is one which is properly based in every case on energy considerations, as in the above discussion.

We are now prepared to discuss the problem of energy transformations in the many-ring converter. Since the total generator action which is taking place at each instant in the many-ring converter is equal to the total motor action, therefore the inductively transformed power is equal to the total generator action (or to the total motor action). That is: the inductively transformed power, P , is equal to the integral of (4) over those portions of the armature where dG is positive. Now dG is positive where i is positive, since de is everywhere positive, and from equation (11) we find that i is *negative* from $\beta = 0$ to $\beta = 51^\circ.75$, *positive* from $\beta = 51^\circ.75$ to $\beta = 128^\circ.25$, and *negative* from $\beta = 128^\circ.25$ to $\beta = 180^\circ$. Therefore, attending to one-half of the armature and multiplying the result by two, we have:

$$P = EI \int_{\beta = 51^\circ.75}^{\beta = 128^\circ.25} -\frac{2}{\pi} \sin^2 \beta \cdot d\beta + \frac{1}{2} \sin \beta \cdot d\beta \quad (12)$$

which gives

$$P = 0.1153 EI \quad (13)$$

That is, $11\frac{1}{2}$ per cent. of the energy which flows through a many-ring converter is inductively transformed and $88\frac{1}{2}$ per

cent. is conductively transferred from the alternating current supply mains to the direct current service mains.

REMARK:—The activities of the two-ring, three-ring and four-ring converters are much more complicated than those of the many-ring converter. The wide separation of the alternating current taps in the few-ring converter introduces pulsations in value of current and power in the various windings, as they pass a given point, so that in considering the internal actions of the few-ring converter, everything must be expressed as a function of time; and in the summation of generator and motor actions time integrations are involved.

4. THE TWO-RING CONVERTER.

Fig. 4 represents one side of the armature of a two-pole two-ring converter; r is one of the a.c. taps, a and b are the d.c.

Fig. 4

brushes, E is the electromotive force between d.c. brushes, I is the direct current flowing in at a and out at b , and i_a and i_b are the values of the currents in the armature parts as shown at instant t . The arrows show the directions in which the various electromotive forces and currents are considered as positive.

The algebraic expressions for i_a and i_b are determined fundamentally by the same consideration of energy relations as in case of the many-ring converter. These expressions are:

$$\left. \begin{aligned} i_a &= \frac{1}{2} I + I \cos \omega t \\ i_b &= \frac{1}{2} I - I \cos \omega t \end{aligned} \right\} \quad (14)$$

Furthermore,

$$\left. \begin{aligned} \text{e.m.f. between } a \text{ and } r &= \frac{1}{2} E (1 - \cos \omega t) \\ \text{e.m.f. between } r \text{ and } b &= \frac{1}{2} E (1 + \cos \omega t) \end{aligned} \right\} \quad (15)$$

Let A be the generator action (motor action when negative), in watts between a and r ; and let B be the generator action (motor action when negative), between r and b . Then

$$A = \frac{1}{2} E I (\frac{1}{2} + \frac{1}{2} \cos \omega t - \cos^2 \omega t) \quad (16)$$

$$B = \frac{1}{2} E I (\frac{1}{2} - \frac{1}{2} \cos \omega t - \cos^2 \omega t) \quad (17)$$

In case of the two-ring converter, the total generator action does not balance the total motor action at each instant and we must distinguish two parts in the inductively transformed power, namely: the power P which is transformed by simultaneous and balanced generator and motor action, and the power Q , which is first converted into kinetic energy as the armature is accelerated and afterwards converted back into electrical energy as the armature is retarded.

The power $P/2$ (the division by two is on account of our considering one-half only of the armature) is equal to the integral of A during the time that A is positive, B negative, and A numerically less than B ; *plus* the integral of B during the time that B is positive, A negative and B numerically less than A ; *minus* the integral of A during the time that A is negative, B positive, and A numerically less than B ; *minus* the integral of B during the time that B is negative, A positive, and B numerically less than A ,—these integrals being extended over half a cycle, and the final result divided by the duration of half a cycle. The value of P so found when divided by $E I$ gives, of the total power which flows through the machine, the fractional part which is inductively transformed without passing through the intermediate stage of kinetic energy.

From equations (16) and (17) we find:

1. Generator action in ar is *less* than motor action in rb from $\omega t = 0$ to $\omega t = 45^\circ$.
2. Generator action ar is *greater* than motor action rb from $\omega t = 45^\circ$ to $\omega t = 60^\circ$.
3. Generator action takes place in both ar and rb from $\omega t = 60^\circ$ to $\omega t = 120^\circ$.

4. Motor action in $a r$ is *less* than generator action in $r b$ from $\omega t = 120^\circ$ to $\omega t = 135^\circ$.

5. Motor action in $a r$ is *greater* than generator action in $r b$ from $\omega t = 135^\circ$ to $\omega t = 180^\circ$.

Therefore,

$$P = \frac{2}{\pi} \int_0^{45^\circ} A \cdot dx + \frac{2}{\pi} \int_{135^\circ}^{180^\circ} B \cdot dx - \frac{2}{\pi} \int_{120^\circ}^{135^\circ} A \cdot dx - \frac{2}{\pi} \int_{45^\circ}^{60^\circ} B \cdot dx \quad (18)$$

in which x is written for ωt .

The numerical evaluation of the integrals in equation (18) gives:

$$\frac{P}{EI} = 0.09517 \quad (19)$$

That is; $9\frac{1}{2}$ percent of the total energy which flows through a two-ring converter is inductively transformed by simultaneous and balanced generator and motor action.

The power $Q/2$ (division by two is on account of considering one-half only of the armature) is found by taking the integral of $A+B$ during the time that this sum is positive, namely; from $\omega t = 45^\circ$ to $\omega t = 135^\circ$, and dividing the result by the duration of half a cycle. This gives

$$\frac{Q}{EI} = \frac{1}{\pi} \int_{45^\circ}^{135^\circ} (1 - 2 \cos^2 x) dx = 0.3183 \quad (20)$$

That is; 31.8 percent of the energy which flows through a two-ring converter is inductively transformed by successive and unbalanced generator and motor action.

From the above results, it follows that 58.65 percent of the energy which flows through a two-ring converter is transferred from the alternating-current supply mains to the direct-current service mains by virtue of the conductive connections through the converter armature.

5. THE THREE-RING CONVERTER.

Fig. 5 shows one side of the armature of a two-pole three-ring converter:— r' and r'' are two of the three a.c. taps; a and b are the d.c. brushes; i_a , i_i , and i_c are the values of the currents in the

armature parts, as shown at instant t ; E is the electromotive force between the d.c. brushes. and I is the steady current flowing in at a and out at b . The arrows show the direction in which the various quantities are considered positive.

The algebraic expressions for i_a , i_b and i_c are

$$\left. \begin{aligned} i_a &= \frac{I}{2} - \frac{4I}{3\sqrt{3}} \cos(\omega t - 150^\circ) \\ i_b &= \frac{I}{2} - \frac{4I}{3\sqrt{3}} \cos(\omega t - 30^\circ) \\ i_c &= \frac{I}{2} - \frac{4I}{3\sqrt{3}} \cos(\omega t + 90^\circ) \end{aligned} \right\} \quad (21)$$

For present purposes, we need consider only the generator and motor actions which occur in the three armature parts $a r'$,

$r'r''$ and $r''b$, during one-third of a cycle, namely, from $\omega t = 0$ to $\omega t = 120^\circ$. The integral of all the generator actions in these armature parts during this third of a cycle divided by the duration of the third of a cycle, gives one-half of the inductively transformed power P .

The electromotive forces involved are as follows:

$$\text{e.m.f. between } a \text{ and } r' = \frac{1}{2} E (1 - \cos \omega t) \quad (22)$$

$$\text{e.m.f. between } r' \text{ and } r'' = \frac{\sqrt{3} E}{2} \cos(\omega t - 30^\circ) \quad (23)$$

This equation is available only from $\omega t = 0$ to $\omega t = 60^\circ$ since at the instant $\omega t = 60^\circ$, r'' coincides with b and after this

instant a portion of the section $r' r''$ is in the half armature which is out of consideration.

$$\text{e.m.f. between } r' \text{ and } b = \frac{1}{2} E (1 + \cos \omega t) \quad (24)$$

This equation is used only during the interval from $\omega t = 60^\circ$ to $\omega t = 120^\circ$.

$$\text{e.m.f. between } r'' \text{ and } b = \frac{1}{2} E \left[1 - \cos (\omega t - 60^\circ) \right] \quad (25)$$

Let A , B' , B'' and C be the generator actions (motor actions when negative) in the armature parts $a r'$, $r' r''$, $r' b$ and $r'' b$ respectively at instant t . Then we have:

$$A = \frac{1}{2} E (1 - \cos \omega t) \cdot i_a \quad (26)$$

$$B' = \frac{\sqrt{3} E}{2} \cdot \cos (\omega t - 30^\circ) \cdot i_b \quad (27)$$

$$B'' = \frac{1}{2} E (1 + \cos \omega t) \cdot i_b \quad (28)$$

$$C = \frac{1}{2} E \left[1 - \cos (\omega t - 60^\circ) \right] \cdot i_c \quad (29)$$

Equation (27) applies from $\omega t = 0$ to $\omega t = 60^\circ$, and equation (28) applies from $\omega t = 60^\circ$ to $\omega t = 120^\circ$.

From equations (26) to (29) we find:

A is *positive* from $\omega t = 0$ to $\omega t = 100^\circ.5$

A is *negative* from $\omega t = 100^\circ.5$ to $\omega t = 120^\circ$

B' is *negative* all the time during which it applies

B'' is *negative* from $\omega t = 60^\circ$ to $\omega t = 79^\circ.5$

B'' is *positive* from $\omega t = 79^\circ.5$ to $\omega t = 120^\circ$.

C is *positive* from $\omega t = 0$ to $\omega t = 60^\circ$

Therefore

$$P = \frac{3}{\pi} \int_0^{100^\circ.5} A \cdot dx + \frac{3}{\pi} \int_{79^\circ.5}^{120^\circ} B'' \cdot dx + \frac{3}{\pi} \int_0^{60^\circ} C \cdot dx \quad (30)$$

Substituting the values of i_a , i_b and i_c from equation (21) in equations (26), (28) and (29), and the resulting values of A , B'' and C in equation (30) and integrating, we find:

$$P = 0.2358 E I \quad (31)$$

That is, 23.6 percent of the energy which flows through a three-ring converter is inductively transformed by simultaneous

and balanced generator and motor action, and the remainder, 76.4 percent is transferred from the supply mains to the service mains by virtue of the conductive connections through the converter armature.

6. THE FOUR-RING CONVERTER.

Fig. 6 shows one side of the armature of a two-pole four-ring converter; r' and r'' are two of the a.c. taps; a and b the d.c. brushes; i_a , i_b and i_c are the values of the currents in the armature parts, as shown at the instant t ; E is the electromotive force between the d.c. brushes, and I is the current flowing in at a and

Fig. 6

out at b . The arrows show the direction in which the various quantities are considered as positive.

The algebraic expressions for i_a , i_b and i_c are as follows:

$$\left. \begin{aligned} i_a &= \frac{I}{2} - \frac{I}{\sqrt{2}} \cdot \cos(\omega t - 135^\circ) \\ i_b &= \frac{I}{2} - \frac{I}{\sqrt{2}} \cdot \cos(\omega t - 45^\circ) \\ i_c &= \frac{I}{2} - \frac{I}{\sqrt{2}} \cdot \cos(\omega t + 45^\circ) \end{aligned} \right\} \quad (32)$$

$$\text{Furthermore, e.m.f. between } a \text{ and } r' = \frac{1}{2} E (1 - \cos \omega t) \quad (33)$$

$$\text{e.m.f. between } r' \text{ and } r'' = E \cos(\omega t - 45^\circ) \quad (34)$$

$$\text{e.m.f. between } r'' \text{ and } b = \frac{1}{2} E (1 - \sin \omega t) \quad (35)$$

In the present case we need only consider the generator and motor actions which occur in the half armature during a quarter of a cycle, namely, from $\omega t = 0$ to $\omega t = 90^\circ$. The integral of the generator actions in the three armature parts $a r'$, $r' r''$, and $r'' b$ during the specified quarter of a cycle divided by the duration of a quarter cycle, gives one-half of the inductively transformed power P .

Let A , B and C be the generator actions (motor actions when negative) in the armature parts $a r'$, $r' r''$, and $r'' b$ respectively at instant t . Then we have:

$$A = \frac{1}{2} E (1 - \cos \omega t) \cdot i_a \quad (36)$$

$$B = \frac{E}{\sqrt{2}} \cdot \cos (\omega t - 45^\circ) \cdot i_b \quad (37)$$

$$C = \frac{1}{2} E (1 - \sin \omega t) \cdot i_c \quad (38)$$

From these equations we find

A is *positive* from $\omega t = 0$ to $\omega t = 90^\circ$.

B is *negative* from $\omega t = 0$ to $\omega t = 90^\circ$.

C is *positive* from $\omega t = 0$ to $\omega t = 90^\circ$.

Therefore,

$$P = \frac{4}{\pi} \int_0^{90^\circ} A \cdot dx + \frac{4}{\pi} \int_0^{90^\circ} C \cdot dx \quad (39)$$

in which x is written for ωt .

Substituting the values of i_a and i_c from equations (32) in equations (36) and (38) and substituting the resulting values of A and C in equation (39) and integrating, we find

$$P = 0.1285 E I \quad (40)$$

That is, 12.9 percent of the energy which flows through a four-ring converter is inductively transformed by simultaneous and balanced generator and motor action, and 87.1 percent is conductively transferred from the supply mains to the service mains.

7. SUMMARY.

Energy Transformations in the Synchronous Converter.

Number of rings.	Percent of energy transformed by successive and unbalanced generator and motor action.	Percent of energy transformed by simultaneous and balanced generator and motor action.	Percent of energy conductively transferred.
2	31.83	9.517	58.65
3	0	23.58	76.42
4	0	12.85	87.15
Infinite	0	11.53	88.47

The calculation of these results demanded a great deal of painstaking computation; in fact, an amount which is very unusual in problems of this kind. These computations were done chiefly by Professor Wm. Esty, to whom the author wishes to express his thanks.

8. ARMATURE REACTION OF THE MANY-RING CONVERTER AND ITS EFFECT IN DISTORTING THE ELECTROMOTIVE FORCE CURVE OF THE MACHINE.

Consider the distribution of the magnetic field in the gap-space when the load on the converter is zero, and consider the distribution of field in the gap-space when the machine is delivering an amount of direct-current I . Let f be the difference in value of these two fields at a point at angular distance β from the direct current brush a , Fig. 3. Let df be the variation of f in the element of angle $d\beta$. Let l be the magnetic length of the gap-space in centimetres, n the number of armature conductors per centimetre of armature circumference, and let i be the current in the armature conductors at the element $d\beta$. Then from the fundamental relation between magnetomotive force and current-turns, we have:

$$l \cdot df = \frac{4\pi}{10} n i \cdot d\beta$$

$$\text{or} \quad df = \frac{4\pi n i}{10l} \cdot d\beta \quad (41)$$

Substituting the value of i from equation (11) we have

$$df = \frac{2\pi n I}{10l} \cdot d\beta - \frac{8n I}{10l} \cdot \sin \beta \cdot d\beta \quad (42)$$

Integrating

$$f = \frac{2\pi n I \beta}{10l} + \frac{8n I \cos \beta}{10l} + C \quad (43)$$

In which C is a constant to be determined. By symmetry of winding and symmetry of distribution of current in two halves of armature, the value of f when $\beta = 0$ must be equal and opposite to the value of f , when $\beta = \pi$. Applying this condition to equation (43) we find,

$$C = -\frac{\pi^2 n I}{10 l} \quad (44)$$

which, substituted in equation (43) gives:

$$f = \frac{n I}{10 l} (2 \pi \beta + 8 \cos \beta - \pi^2) \quad (45)$$

This equation holds good on one side only of the armature. For expressing the values of f on the other side of the armature, the angle β must be measured from the brush b , Fig. 3.

If the electromotive force curve of the converter is determined by using an auxiliary concentrated winding of T turns of wire, then the amount by which the ordinate of the full load electromotive force curve exceeds the corresponding ordinate of the zero load electromotive force curve at the point β in volts is

$$e' = \frac{2 L T v n I}{l} (2 \pi \beta + 8 \cos \beta - \pi^2) \div 10^9 \quad (46)$$

in which L is the length of the armature in centimetres and v is the peripheral velocity of the armature in centimetres per second.

If it is desired to cut away a layer of thickness x from the pole face, so as to compensate for the armature reaction at full load, causing the machine to give the same electromotive force curve at full load as it previously did at zero load, this may be done and the thickness x is a function of β . In fact

$$x = \frac{f}{B} \cdot l$$

in which B is the field intensity in the gap-space at zero load. Therefore

$$x = \frac{n I}{10 B} (2 \pi \beta + 8 \cos \beta - \pi^2) \quad (47)$$

9. THE CRITERION OF REALITY OF THE COMPONENT PARTS INTO WHICH A PHYSICAL AGGREGATE IS RESOLVED BY MATHEMATICAL ANALYSIS.

In the derivation of an algebraic expression for a determinate physical variable, two sets of conditions in general apply:—1. The condition or conditions which must be satisfied always and everywhere; that is, throughout the interval of time and throughout the region of space over which the expression is to apply. These conditions are in most cases the requirements of the principle of the conservation of energy. 2a, The condition or conditions which must be satisfied at the boundary of the region and 2b, The condition or conditions which must be satisfied at the beginning of the time. Conditions 1 we will call *persistent* conditions. Conditions 2a may be *persistent*.

Consider as an example the problem of the vibration of an elastic string. The energy of the string is repeatedly transformed from kinetic to potential and back again, and the constancy of total energy is expressed by the well-known differential equation:

$$\frac{d^2 y}{dt^2} = -v^2 \frac{d^2 y}{dx^2} \quad (a)$$

which must be always and everywhere satisfied.

At the ends of the string we have the persistent conditions,

$$y = 0 \text{ when } \begin{cases} x = 0 \\ \text{or} \\ x = l \end{cases} \quad (b)$$

Any motion of the string is possible which satisfies these persistent conditions. Suppose that at the instant $t = 0$, the string is distorted into the shape represented by the equation,

$$y = f(x) \quad (c)$$

The solution of the problem is

$$y = A \sin \frac{2\pi x}{l} + B \sin \frac{4\pi x}{l} + C \sin \frac{6\pi x}{l} + \dots \quad (d)$$

The various terms in this equation represent the component parts into which the actual motion of the string is resolved by the mathematical analysis of the problem. Each of these component parts satisfies all the persistent conditions of the problem and may therefore actually exist by itself, and there is no reason why we should not look upon each of these parts as real. *In general, the component parts into which a physical aggregate is resolved by*

mathematical analysis may be considered as real when each of these parts satisfies all of the persistent conditions of the problem and may exist alone.

Consider, as another example, the problem of the decay of current in an inductive circuit when the driving electromotive force is removed. Placing the rate of disappearance of electrokinetic energy equal to the rate of generation of heat in the circuit, we have as a result the differential equation:

$$R i + L \frac{di}{dt} = 0 \quad (e)$$

which equation must be always satisfied. Furthermore, at the beginning, the current has a certain prescribed value I . That is:

$$i = I \text{ when } t = 0 \quad (f)$$

The solution of this problem is

$$i = I \left(1 - t + \frac{t^2}{1'2} - \frac{t^3}{1'2'3} + \frac{t^4}{1'2'3'4} - \dots \right) \quad (g)$$

in which R/L has for brevity been placed equal to unity. The various terms in equation (g) represent component parts of the current i ; these parts do not satisfy the conditions of the problem; they cannot exist alone under the conditions of the problem, and therefore these parts cannot be looked upon as physically *real*. *In general, the component parts into which a physical aggregate is resolved by mathematical analysis cannot be considered as real when they do not separately satisfy the persistent conditions of the problem and when these parts cannot exist individually and separately under the conditions of the problem.*

Consider the case of the polyphase synchronous converter after it is in full operation. The persistent condition is equality of inflow and outflow of electrical energy and constancy of kinetic energy of armature. The problem of the determination of armature current under these conditions leads to expressions [see equations (11), (14), (21) and (32), which involve two terms. That is: the armature current is resolved into two parts and these parts correspond to inflowing alternating current and outflowing direct current respectively. This correspondence strongly suggests the physical reality of these two parts of the armature current; but as a matter of fact neither part of the armature current satisfies the conditions of the problem, neither part can exist alone under the conditions of the problem and therefore these parts are mathematical fictions and not physical realities.