

JOURNAL
OF THE
INSTITUTE OF ACTUARIES
AND
ASSURANCE MAGAZINE.

On the Integral of Gompertz's Function for expressing the Values of Sums depending upon the contingency of life. By W. M. MAKEHAM, Fellow of the Institute of Actuaries.

GOMPERTZ'S Function for expressing the probability of a person surviving any given period, x , is $\frac{1}{g} \cdot g^{ax}$. For the purpose of adapting this expression to the entire period of adult life I have added the factor ϵ^{-ax} . The two expressions become identical in form when the factor $\epsilon^{-\delta x}$ representing the effect of discount is introduced, and both may therefore be correctly designated by the name of "Gompertz's Function."

The interest excited by the publication of Gompertz's celebrated hypothesis, and the attention which has in consequence been bestowed upon it, have already resulted in the discovery of some curious and important properties of the function in question; and, as Mr. Woolhouse has pointed out, it is not improbable that the disclosure of others may reward the industry of those who choose to cultivate the field. The principal of those already discovered—viz., the property of uniform seniority—we owe to De Morgan. By the aid of this important property the number of

tables required in the computation of Life Contingencies became (as I have shown on a former occasion) materially reduced. For each mortality table, and each rate of interest, a complete set, from a single life up to any required number of joint lives, could be comprised in a comparatively moderate space, while under the old system the task would be attended with insuperable difficulties.

A further step in the same direction will be found in a paper by Mr. Woolhouse (see *Journal of the Institute of Actuaries* for July 1870), who suggested a contrivance by which the number of tables for any given rate of mortality could be reduced to a single series for successive rates of interest,—the annuity values for any given combination of ages being found by a process of *double* interpolation. It will be seen also, from the Editor's note at the end of the paper in question, that a similar idea had occurred to Mr. Meech, an eminent American actuary, and was communicated to me by that gentleman in December 1868.

By the last mentioned improvement we should be able to deduce the value of an annuity on any given combination of lives and at any given rate of interest, by means of a single table of double entry calculated for specific values of the constants g and q of the formula. But the table in question would be of no use for deducing the values of annuities according to a mortality table derived from other values of these constants. The table, however, which I now submit, and which, like that last referred to, is a single table of double entry, has this further important advantage, viz., that it is applicable to *all* values of the constants g and q whatever, and is therefore available for determining the values of annuities, not only for any given combination of lives and for any given rate of interest, *but also for any given rate of mortality*. That is to say, the advantage obtained by the law of uniform seniority (as modified according to the idea of Messrs. Meech and Woolhouse) in reference to a *single given mortality table*, is, by means of a contrivance which I have now to explain, extended so as to comprise all mortality tables whatever constructed according to Gompertz's Function.

The value of a continuous annuity on any given life or any combination of joint lives expressed in terms of Gompertz's Function, is

$$\frac{1}{g^{qx} e^{-(a+\delta)x}} \int_x^{\infty} g^{qx} \cdot e^{-(a+\delta)x} dx.$$

Now it will be seen that in order to tabulate this function

completely we should require a table of no less than *four* variables—corresponding to the three constants $g, q,$ and $(a + \delta),$ together with $x,$ the inferior limit of integration. *Three* variables, altho' a somewhat troublesome matter, are at least *practicable* (as I have shown in a paper on this subject in vol. 16 of the *Journal*) by reason of the fact that space has *three* dimensions; but *four* involve a difficulty which would puzzle the genius of Euclid himself to surmount.

Among the contrivances available for facilitating the tabulation of definite integrals none are of more important use than that of changing the independent variable. In the case in hand, I put

$$z = x \log q + \log^2 \frac{1}{g} \dots \dots \dots (1)$$

$$n = \frac{a + \delta}{\log q} \dots \dots \dots (2)$$

the logarithms being common logarithms.

From (1) we get

$$dz = \log q \cdot dx,$$

and substituting these values in the given function, the integral becomes

$$\frac{1}{\log q} \cdot \frac{1}{10^{-10^2} \cdot \epsilon^{-nz}} \int_z^\infty 10^{-10^2} \cdot \epsilon^{-nz} \cdot dz,$$

in which it will be seen that by a very simple transformation we have succeeded in eliminating *two* of the arbitrary constants, and thus reducing the process of tabulation to one of *double entry* only.

The following is a proof of the transformation just effected.

$$\begin{aligned} g^{q^x} &= 10^{\log g \cdot 10^{\log q \cdot x}} \\ &= 10^{\log g \cdot 10^2 \cdot \log^2 \frac{1}{g}}, \text{ from (1).} \end{aligned}$$

But $10^{-\log^2 \frac{1}{g}} = 10^{-\log \log \frac{1}{g}} = \frac{1}{10^{\log \cdot \log \frac{1}{g}}} = \frac{1}{\log \frac{1}{g}} = -\frac{1}{\log g}$

$\therefore g^{q^x} = 10^{\log g \cdot 10^2 \times -\frac{1}{\log g}}$
 $= 10^{-10^2}$

Also $\epsilon^{-(a+\delta)x} = \epsilon^{-n \log q \left(\frac{z - \log^2 \frac{1}{g}}{\log q} \right)}$ from (1) and (2)
 $= \epsilon^{-nz} \epsilon^{n \log^2 \frac{1}{g}}$
 $= \epsilon^{-nz} c$ [putting $\epsilon^{n \log^2 \frac{1}{g}} = c$]

Hence we have

$$g^{qx} \epsilon^{-(a+\delta)x} = 10^{-10^x} \cdot \epsilon^{-nx} \cdot c$$

and as $dx = \frac{dz}{\log q}$, the integral becomes

$$\frac{1}{\log q} \cdot \frac{1}{10^{-10^z} \cdot \epsilon^{-nz} \cdot c} \int_z^\infty 10^{-10^z} \cdot \epsilon^{-nz} \cdot c \cdot dz$$

from which we get the transformed integral above given, as the constant c from the way in which it is involved evidently disappears.

On a former occasion (see *Journal* vol. xiii, p. 349) I proposed the following transformation of the same integral, which would answer all the purposes of that just described, but which I think would be found more troublesome to calculate.

Put
$$v = \left(\log_e \frac{1}{g} \right) \cdot q^x$$

and
$$m = -\frac{a + d}{\log_e q}$$

whence $g^{qx} (= \epsilon^{(\log_e g)qx}) = \epsilon^{-v}$. And observing that $-(a + \delta) = m \log_e q$, we have $\epsilon^{-(a+\delta)x} = \epsilon^{(\log_e q)mx} = q^{mx} = \frac{v^m}{(\log_e \frac{1}{g})^m}$. Again,

$\frac{dv}{dx} = (\log_e \frac{1}{g}) \cdot q^x \cdot \log_e q = v \cdot \log_e q$. Hence, substituting, we have:

$$g^{qx} \epsilon^{-(a+\delta)x} = \frac{\epsilon^{-v} \cdot v^m}{(\log_e \frac{1}{g})^m}$$

and

$$\frac{1}{g^{qx} \epsilon^{-(a+\delta)x}} \int_x^\infty g^{qx} \epsilon^{-(a+\delta)x} dx = \frac{1}{\epsilon^{-v} \cdot v^m \cdot \log_e q} \int_v^\infty \epsilon^{-v} \cdot v^{m-1} \cdot dv$$

the second member of which equation (omitting the factors outside the symbol of integration) is the form of the well known gamma-function, or second Eulerian Integral.

Now it is a well known property of the last named integral that

$$\int \epsilon^{-v} \cdot v^m \cdot dv = -\epsilon^{-v} v^m + m \int \epsilon^{-v} v^{m-1} dv$$

whence the following equation is easily deduced:

$$\frac{v}{\epsilon^{-v} \cdot v^{m+1} \log_e q} \int_v^\infty \epsilon^{-v} \cdot v^m \cdot dv = \frac{1}{\log_e q} + \frac{m}{\epsilon^{-v} \cdot v^m \cdot \log_e q} \int_v^\infty \epsilon^{-v} \cdot v^{m-1} \cdot dv$$

or

$$\frac{v}{\epsilon^{-v} \cdot v^m \cdot \log_e q} \int_v^\infty \epsilon^{-v} \cdot v^{m-1} dv = \frac{1}{\log_e q} + \frac{m-1}{\epsilon^{-v} \cdot v^{m-1} \cdot \log_e q} \int_v^\infty \epsilon^{-v} \cdot v^{m-2} dv$$

By means of these equations, having found the value of an annuity corresponding to any given value of m , we may with facility determine the annuity corresponding to $m \pm 1$. It is by virtue of this property that we are able to limit the tabulated integral to values

of n from 1 to 2,—all other values being easily deduced from those found in the table.

Comparing the expression for the integral tabulated, viz.—

$$\log \cdot \frac{1}{10^{-10^z} \cdot \epsilon^{-nz}} \int_z^\infty 10^{-10^z} \cdot \epsilon^{-nz} \cdot dz$$

with that representing the logarithmic values of annuities, viz.—

$$\log \frac{1}{g^{\alpha x} \epsilon^{-(a+\delta)x}} \int_x^\infty g^{\alpha x} \epsilon^{-(a+\delta)x} \cdot dx$$

it will be seen that the two are *identical in form*. This identity suggests a simple method of computing the former integral by the rules laid down by Mr. Woolhouse for the calculation of continuous annuities, which is the course I have found most convenient in the construction of the accompanying table.

The figures in antique type in the table are the divided differential coefficients of the function; and the process of interpolation is performed in accordance with the rules given in my paper on this subject in vol. xvi of the *Journal* (see page 111). The following examples will be sufficient to illustrate the process:—

Example I. Required the value of a continuous annuity at 5 per cent on a life aged 30; according to the values of the constants deduced by Mr. Woolhouse from the new H^{MF} experience (see *Journal*, vol. xv, p. 405.)

Here $\log \frac{1}{g} =$	·0004121	$\log^{\frac{2}{g}} =$	4·6150
$\log q =$	·04	$\log q^{30} =$	1·2
$a = \log_e \alpha =$	·00659	$z =$	3·8150
$\delta =$	·04879		
$a + \delta =$	·05538		
$\frac{a + \delta}{\log q} = n =$	1·3845		

Nearest tabular value (corresponding to $z=3\cdot8$ and $n=1\cdot3$)

	$= \bar{1}81260$
— 895 × ·150 = — 134·2	
— 53 × (·150) ² = — 1·2	
— 2360 × ·845 = — 1994·2	$-2129\cdot6 = \bar{1}7870$
+	
44 × (·845) ² = + 31·5	
+ 71 × ·150 × ·845 = + 9·0	$+40\cdot5 = 40$
	$= \bar{1}79170$
Deduct	$\log(\log q) = \bar{2}60206$
log of annuity required =	<u>$\bar{1}18964$</u>

When the required annuity is payable during the joint continuance of s lives, the value expressed in terms of Gompertz's function is

$$\frac{1}{g'q^x \epsilon^{-(sa+\delta)x}} \int_x^\infty g'q^x \epsilon^{-(sa+\delta)x} dx$$

where x is the youngest age and (d, d', d'' denoting the differences between x and the other ages respectively) g' is determined by the equation

$$\log \frac{1}{g'} = \log \frac{1}{g} \times (1 + q^d + q^{d'} + \dots)$$

or
$$\log^2 \frac{1}{g'} = \log^2 \frac{1}{g} + \log(1 + q^d + q^{d'} + \dots).$$

Example II. Required the value of a continuous annuity on two joint lives (aged 27 and 37 respectively) the rate of interest being 4 per-cent, and the constants of mortality being

$$\begin{aligned} \log^2 \frac{1}{g} &= \bar{4} \cdot 407492 \\ \bar{a} &= \cdot 0081972 \\ \log q &= \cdot 041667 \end{aligned}$$

Here $\log^2 \frac{1}{g'} = \log^2 \frac{1}{g} + \log(1 + q^{10}) = \bar{4} \cdot 4075 + \cdot 5575 = \bar{4} \cdot 9650$
 $x \log q = \cdot 041667 \times 27 = \underline{1 \cdot 1250}$
 $\bar{2} \cdot 0900 = z$

and $n = \frac{2a + \delta}{\log q} = \frac{\cdot 01639 + \cdot 03922}{\cdot 041667} = \underline{1 \cdot 3346} = n$

Nearest tabular value ($z = \bar{2} \cdot 0, n = 1 \cdot 3$) $= \bar{1} \cdot 79245$

- 1128 × 9	= - 1015·2	
- 65 × (·9)²	= - 52·7	
- 2211 × ·346	= - 765·0	- 1832·9
+ 38 × (·346)²	= + 4·5	
+ 77 × 9 × ·346	= + 24·0	+ 28·5 = - 1804
		<u>1·77441</u>

Deduct $\log(\log q) = \bar{2} \cdot 61979$

log of annuity required = 1·15462

The corresponding annuity is 14·276. The value of the same annuity deduced independently from a private mortality table (constructed from the constants above given) is 14·275.

It will be observed that the trouble involved in the process of determining the required values arises solely from the limited

extent of the table, necessitating the use of two orders of differences, and may be entirely obviated by the construction of a table on a sufficiently extended scale.

Generally speaking, the values of annuities on a single life from 3 to 8 per-cent, and on as many as six joint lives at any rate up to 4 per-cent may be deduced directly from the table. This will no doubt be found sufficient for most practical purposes; but by means of the property of the gamma-function before adverted to the application of the table, if necessary, may be extended to cases beyond the above-mentioned limits.

I believe myself entitled to claim the credit of having first pointed out the equivalence of annuities payable at various intervals, when the period of the next payment is undetermined; and the consequent error of the practice of substituting the equivalent annual payment for Half-yearly and Quarterly premiums in the Valuations of Assurance Companies by the customary method of classification according to age. Mr. Woolhouse has since shown that in valuing (in the gross) a large number of independent annuities, payable at different intervals and at different periods of time, the employment of the continuous-annuity value is the strictly accurate mode of procedure (see his paper "On an Improved Theory of Annuities and Assurances" in vol. xv). Hence it would seem to follow that the tabulation of the continuous-annuity values, in lieu of the arbitrary "annually-payable" annuities, would be more consistent both with scientific accuracy and practical convenience. When the very common question is put, how many years' purchase is a given annuity worth?—without specifying either the *interval* or the *period* of payment,—the true answer, and the *only* true answer, is, the continuous-annuity value for the given age.

In the continuous system the "force of discount" becomes the *nominal* rate of interest; and would therefore be quoted in integers or the aliquot parts of integers. Thus (to take the case in Example II) in lieu of $\cdot03922 \dots$ (*ad inf.*) which is the *force of discount* corresponding to 4 per-cent, we should take $\delta = \cdot04$. The value of the reversion on a life aged x at the *nominal* rate of 4 per-cent (continuous) would therefore be $1 - \cdot04 \times \bar{a}_x$, the numerical value of which expression may readily be ascertained by inspection from the table of annuity values. Many other advantages of a practical character would follow from the adoption of the continuous-annuity basis (as suggested by me in a letter published in the *Journal* for July 1861) for the valuation of life contingencies.

z	$n=1\cdot0$			$n=1\cdot1$			$n=1\cdot2$			$n=1\cdot3$			$n=1\cdot4$		
$\bar{4}\cdot0$	$\bar{1}\cdot98182$ -3795 +127 -4	$\bar{1}\cdot94510$ -3552 +116 -4	$\bar{1}\cdot91070$ -3331 +106 -3	$\bar{1}\cdot87841$ -3129 +96 -3	$\bar{1}\cdot84805$ -2946 +88 -3	$\bar{1}\cdot81719$ -2792 +86 -3	$\bar{1}\cdot78784$ -3129 +96 -3	$\bar{1}\cdot75784$ -3129 +96 -3	$\bar{1}\cdot72784$ -3129 +96 -3	$\bar{1}\cdot69784$ -3129 +96 -3	$\bar{1}\cdot66784$ -3129 +96 -3	$\bar{1}\cdot63784$ -3129 +96 -3	$\bar{1}\cdot60784$ -3129 +96 -3	$\bar{1}\cdot57784$ -3129 +96 -3	$\bar{1}\cdot54784$ -3129 +96 -3
$\bar{4}\cdot1$	$\bar{1}\cdot97989$ -3754 +122 -4	$\bar{1}\cdot94358$ -3520 +112 -3	$\bar{1}\cdot90942$ -3305 +103 -3	$\bar{1}\cdot87787$ -3108 +94 -3	$\bar{1}\cdot84719$ -2929 +86 -3	$\bar{1}\cdot81719$ -2929 +86 -3	$\bar{1}\cdot78787$ -3108 +94 -3	$\bar{1}\cdot75787$ -3108 +94 -3	$\bar{1}\cdot72787$ -3108 +94 -3	$\bar{1}\cdot69787$ -3108 +94 -3	$\bar{1}\cdot66787$ -3108 +94 -3	$\bar{1}\cdot63787$ -3108 +94 -3	$\bar{1}\cdot60787$ -3108 +94 -3	$\bar{1}\cdot57787$ -3108 +94 -3	$\bar{1}\cdot54787$ -3108 +94 -3
$\bar{4}\cdot2$	$\bar{1}\cdot97775$ -3712 +118 -4	$\bar{1}\cdot94177$ -3485 +109 -3	$\bar{1}\cdot90798$ -3276 +100 -3	$\bar{1}\cdot87618$ -3085 +92 -3	$\bar{1}\cdot84622$ -2911 +84 -2	$\bar{1}\cdot81622$ -2911 +84 -2	$\bar{1}\cdot84622$ -3085 +92 -3	$\bar{1}\cdot81622$ -3085 +92 -3	$\bar{1}\cdot78622$ -3085 +92 -3	$\bar{1}\cdot75622$ -3085 +92 -3	$\bar{1}\cdot72622$ -3085 +92 -3	$\bar{1}\cdot69622$ -3085 +92 -3	$\bar{1}\cdot66622$ -3085 +92 -3	$\bar{1}\cdot63622$ -3085 +92 -3	$\bar{1}\cdot60622$ -3085 +92 -3
$\bar{4}\cdot3$	$\bar{1}\cdot97589$ -3668 +114 -4	$\bar{1}\cdot93982$ -3448 +105 -3	$\bar{1}\cdot90686$ -3246 +97 -3	$\bar{1}\cdot87484$ -3060 +89 -3	$\bar{1}\cdot84510$ -2890 +82 -2	$\bar{1}\cdot81510$ -2890 +82 -2	$\bar{1}\cdot87484$ -3060 +89 -3	$\bar{1}\cdot84484$ -3060 +89 -3	$\bar{1}\cdot81484$ -3060 +89 -3	$\bar{1}\cdot78484$ -3060 +89 -3	$\bar{1}\cdot75484$ -3060 +89 -3	$\bar{1}\cdot72484$ -3060 +89 -3	$\bar{1}\cdot69484$ -3060 +89 -3	$\bar{1}\cdot66484$ -3060 +89 -3	$\bar{1}\cdot63484$ -3060 +89 -3
$\bar{4}\cdot4$	$\bar{1}\cdot97277$ -3622 +111 -3	$\bar{1}\cdot93763$ -3409 +103 -3	$\bar{1}\cdot90463$ -3213 +94 -3	$\bar{1}\cdot87331$ -3033 +87 -2	$\bar{1}\cdot84362$ -2867 +79 -2	$\bar{1}\cdot81362$ -2867 +79 -2	$\bar{1}\cdot87331$ -3033 +87 -2	$\bar{1}\cdot84331$ -3033 +87 -2	$\bar{1}\cdot81331$ -3033 +87 -2	$\bar{1}\cdot78331$ -3033 +87 -2	$\bar{1}\cdot75331$ -3033 +87 -2	$\bar{1}\cdot72331$ -3033 +87 -2	$\bar{1}\cdot69331$ -3033 +87 -2	$\bar{1}\cdot66331$ -3033 +87 -2	$\bar{1}\cdot63331$ -3033 +87 -2
$\bar{4}\cdot5$	$\bar{1}\cdot96987$ -3572 +107 -3	$\bar{1}\cdot93519$ -3367 +99 -3	$\bar{1}\cdot90248$ -3178 +91 -3	$\bar{1}\cdot87156$ -3004 +84 -2	$\bar{1}\cdot84236$ -2843 +77 -2	$\bar{1}\cdot81236$ -2843 +77 -2	$\bar{1}\cdot87156$ -3004 +84 -2	$\bar{1}\cdot84156$ -3004 +84 -2	$\bar{1}\cdot81156$ -3004 +84 -2	$\bar{1}\cdot78156$ -3004 +84 -2	$\bar{1}\cdot75156$ -3004 +84 -2	$\bar{1}\cdot72156$ -3004 +84 -2	$\bar{1}\cdot69156$ -3004 +84 -2	$\bar{1}\cdot66156$ -3004 +84 -2	$\bar{1}\cdot63156$ -3004 +84 -2

z	n = 1.5		n = 1.6		n = 1.7		n = 1.8		n = 1.9	
	+	-	+	-	+	-	+	-	+	-
4.0	81948	-2780	79241	-2629	76688	-2489	74258	-2363	71958	-2250
	-65	+13	-54	+11	-45	+9	-37	+7	-31	+5
4.1	81873	-2766	79188	-2617	76635	-2480	74218	-2356	71919	-2244
	-75	+14	-62	+11	-52	+10	-43	+8	-36	+6
4.2	81792	-2751	79116	-2605	76579	-2470	74171	-2347	71880	-2238
	-87	+17	-72	+13	-60	+10	-51	+9	-43	+7
4.3	81699	-2734	79088	-2591	76514	-2459	74116	-2338	71833	-2231
	-100	+18	-84	+15	-71	+12	-61	+10	-51	+8
4.4	81592	-2716	78947	-2575	76437	-2446	74050	-2338	71777	-2222
	-116	+20	-98	+16	-83	+13	-71	+11	-61	+9
4.5	81468	-2695	78842	-2558	76348	-2432	73974	-2317	71711	-2212
	-130	+21	-110	+17	-93	+14	-81	+12	-69	+10

z	$n=1.0$		$n=1.1$		$n=1.2$		$n=1.3$		$n=1.4$	
4.5	\bar{I} .96987	-3572 + 107 -3	\bar{I} .93519	-3367 + 99 -3	\bar{I} .90248	-3178 + 91 -3	\bar{I} .87156	-3004 + 84 -2	\bar{I} .84286	-2843 + 77 -2
	-305 + 51 -4 0 0 0 0	-258 + 43 -4 0 0 0 0	-218 + 36 -3 0 0 0 0	-185 + 31 -3 0 0 0 0	-157 + 26 -2 0 0 0 0	-130 + 21 -2 0 0 0 0	-105 + 16 -1 0 0 0 0	-82 + 11 -1 0 0 0 0	-60 + 6 -1 0 0 0 0	-40 + 3 -1 0 0 0 0
4.6	\bar{I} .96667	-3520 + 103 -3	\bar{I} .93247	-3323 + 95 -2	\bar{I} .90017	-3140 + 88 -3	\bar{I} .86962	-2972 + 81 -2	\bar{I} .84069	-2816 + 75 -2
	-337 + 54 -4 0 0 0 0	-286 + 46 -4 0 0 0 0	-245 + 39 -3 0 0 0 0	-209 + 33 -3 0 0 0 0	-179 + 28 -2 0 0 0 0	-152 + 23 -2 0 0 0 0	-128 + 18 -1 0 0 0 0	-106 + 14 -1 0 0 0 0	-86 + 10 -1 0 0 0 0	-68 + 7 -1 0 0 0 0
4.7	\bar{I} .96312	-3464 + 99 -3	\bar{I} .92944	-3275 + 91 -2	\bar{I} .89758	-3100 + 84 -2	\bar{I} .86740	-2938 + 78 -2	\bar{I} .83878	-2787 + 73 -2
	-373 + 58 -5 0 0 0 0	-319 + 49 -4 0 0 0 0	-275 + 42 -3 0 0 0 0	-236 + 36 -3 0 0 0 0	-203 + 30 -2 0 0 0 0	-175 + 25 -2 0 0 0 0	-151 + 21 -2 0 0 0 0	-131 + 17 -1 0 0 0 0	-113 + 13 -1 0 0 0 0	-96 + 9 -1 0 0 0 0
4.8	\bar{I} .95919	-3404 + 94 -3	\bar{I} .92606	-3225 + 87 -2	\bar{I} .89466	-3056 + 81 -2	\bar{I} .86488	-2900 + 75 -2	\bar{I} .83661	-2755 + 70 -2
	-414 + 61 -5 0 0 0 0	-357 + 52 -3 0 0 0 0	-309 + 45 -3 0 0 0 0	-267 + 39 -3 0 0 0 0	-232 + 32 -2 0 0 0 0	-199 + 26 -2 0 0 0 0	-171 + 20 -1 0 0 0 0	-147 + 15 -1 0 0 0 0	-125 + 11 -1 0 0 0 0	-107 + 8 -1 0 0 0 0
4.9	\bar{I} .95483	-3342 + 89 -2	\bar{I} .92228	-3171 + 83 -2	\bar{I} .89188	-3010 + 78 -2	\bar{I} .86203	-2860 + 72 -2	\bar{I} .83413	-2721 + 67 -2
	-457 + 64 -4 0 0 0 0	-398 + 55 -4 0 0 0 0	-347 + 48 -4 0 0 0 0	-302 + 41 -3 0 0 0 0	-267 + 35 -3 0 0 0 0	-236 + 29 -2 0 0 0 0	-209 + 23 -2 0 0 0 0	-185 + 18 -1 0 0 0 0	-167 + 14 -1 0 0 0 0	-152 + 11 -1 0 0 0 0
5.0	\bar{I} .95002	-3277 + 85 -2	\bar{I} .91807	-3114 + 79 -2	\bar{I} .88770	-2961 + 74 -2	\bar{I} .85881	-2818 + 69 -2	\bar{I} .83130	-2685 + 65 -1
	-487 + 59 -4 0 0 0 0	-447 + 51 -3 0 0 0 0	-407 + 43 -3 0 0 0 0	-367 + 35 -2 0 0 0 0	-332 + 28 -2 0 0 0 0	-301 + 22 -1 0 0 0 0	-273 + 17 -1 0 0 0 0	-249 + 13 -1 0 0 0 0	-229 + 10 -1 0 0 0 0	-213 + 7 -1 0 0 0 0

z	$n=1.5$		$n=1.6$		$n=1.7$		$n=1.8$		$n=1.9$	
$\bar{4}.5$	$\bar{1}.81468$ -134 +22 -9 +1 0 0	+71 -2 0 0 0 0	$\bar{1}.78842$ -114 +19 -8 0 0	+66 -2 0 0 0 0	$\bar{1}.76848$ -4432 +15 +1 0 0	+60 -2 0 0 0 0	$\bar{1}.78974$ -83 +12 +1 0 0	+55 -1 0 0 0 0	$\bar{1}.77111$ -72 -6 +1 0 0	+49 -1 0 0 0 0
$\bar{4}.6$	$\bar{1}.81825$ -152 +24 -10 -1 0	+60 -2 0 0 0 0	$\bar{1}.78720$ -3530 +19 -130 +1 -10 -1 0	+64 -2 0 0 0 0	$\bar{1}.76243$ -2416 +15 +1 -9 0 0	+59 -2 0 0 0 0	$\bar{1}.78884$ -98 +14 -8 0 0	+54 -1 0 0 0 0	$\bar{1}.71633$ -85 -7 +1 0 0	+49 -1 0 0 0 0
$\bar{4}.7$	$\bar{1}.81162$ -176 +26 -12 +1 -1 0	+67 -2 0 0 0 0	$\bar{1}.78579$ -3519 +22 -151 +1 -1 0	+62 -1 0 0 0 0	$\bar{1}.76121$ -2400 +19 -131 +1 -10 -1 0	+58 -1 0 0 0 0	$\bar{1}.78778$ -2289 +15 -114 +1 -9 -1 0	+54 -1 0 0 0 0	$\bar{1}.71541$ -101 +14 -8 +2 -1 0	+49 -1 0 0 0 0
$\bar{4}.8$	$\bar{1}.80978$ -201 +28 -14 +1 -1 0	+65 -1 0 0 0 0	$\bar{1}.78416$ -2496 +24 -175 +1 -13 -1 0	+60 -1 0 0 0 0	$\bar{1}.75979$ -2380 +21 -153 +1 -12 -1 0	+56 -1 0 0 0 0	$\bar{1}.78654$ -2273 +18 -134 +1 -11 -1 0	+52 -1 0 0 0 0	$\bar{1}.71432$ -117 +15 -10 +1 -1 0	+49 -1 0 0 0 0
$\bar{4}.9$	$\bar{1}.80757$ -231 +31 -16 +1 -1 0	+63 -1 0 0 0 0	$\bar{1}.78227$ -2471 +26 -203 +1 -15 -1 0	+58 -1 0 0 0 0	$\bar{1}.75813$ -2358 +23 -178 +1 -13 -1 0	+54 -1 0 0 0 0	$\bar{1}.78508$ -2254 +20 -157 +1 -12 -1 0	+51 -1 0 0 0 0	$\bar{1}.71804$ -139 +10 -12 +1 -1 0	+48 -1 0 0 0 0
$\bar{5}.0$	$\bar{1}.80509$ -2560	+60 -1	$\bar{1}.78008$ -2443	+56 -1	$\bar{1}.75621$ -2334	+52 -1	$\bar{1}.78388$ -2233	+49 -1	$\bar{1}.71153$ -2138	+46 -1

z	n=1.0			n=1.1			n=1.2			n=1.3			n=1.4				
3.0	1.95002 -507 -26	-3277 +67 +1	+85 -2 0	1.91807 -3114 +58	-444 +2 0	+79 -4 0	-2 0 0	1.88770 -2961 +51	-359 +1 0	+74 -4 0	-2 0 0	1.85881 -2818 +44	-342 +1 0	+69 -2 0	1.83130 -2685 +38	-301 +2 0	+65 -1 0
3.1	1.94468 -560 -29	-3209 +70 +2	+80 -4 0	1.91388 -3054 +61	-496 +2 0	+75 -4 0	-2 0 0	1.88337 -2909 +54	-438 +2 0	+70 -3 0	-1 0 0	1.85517 -2773 +47	-387 +2 0	+66 -1 0	1.82809 -2645 +41	-343 +2 0	+62 -1 0
3.2	1.93878 -621 -32	-3137 +73 +1	+76 -4 0	1.90814 -2991 +64	-552 +2 0	+71 -3 0	-1 0 0	1.87883 -2853 +57	-492 +2 0	+67 -3 0	-1 0 0	1.85105 -2724 +50	-437 +2 0	+63 -1 0	1.82443 -2602 +44	-390 +2 0	+59 -1 0
3.3	1.93224 -687 -35	-3063 +75 +1	+72 -4 0	1.90231 -2925 +67	-616 +1 0	+68 -3 0	-1 0 0	1.87372 -2794 +60	-551 +2 0	+64 -3 0	-1 0 0	1.84640 -2672 +54	-494 +2 0	+60 -1 0	1.82027 -2556 +47	-444 +2 0	+56 -1 0
3.4	1.92501 -761 -39	-2986 +79 +2	+68 -4 0	1.89581 -2856 +71	-685 +2 0	+64 -3 0	-1 0 0	1.86788 -2732 +64	-618 +1 0	+60 -3 0	-1 0 0	1.84115 -2616 +58	-557 +2 0	+56 -1 0	1.81554 -2507 +51	-593 +2 0	+53 -1 0
3.5	1.91700 -2905	-2905	+64 -1	1.88858 -2783	+60 -1	+60 -1	-1	1.86134 -2667	+56 -1	+56 -1	-1	1.83523 -2557	+53 -1	+53 -1	1.81018 -2454	+50 -1	-1

z	$n=1.5$		$n=1.6$		$n=1.7$		$n=1.8$		$n=1.9$	
$\bar{3}0$	$\bar{1}80509$ -265 +34 -18 +1 -1 0	+60 -1 -2 0 0 0	$\bar{1}78008$ -2443 +29 -17 +1 -1 0	+56 -1 -2 0 0 0	$\bar{1}75621$ -2334 +25 -16 +1 -1 0	+52 -1 -1 0 0 0	$\bar{1}78388$ -2233 +22 -14 +1 -1 0	+49 -1 -2 0 0 0	$\bar{1}77153$ -2138 +19 -13 +1 -1 0	+46 -1 -2 0 0 0
$\bar{3}1$	$\bar{1}80225$ -304 +36 -21 +1 -1 0	+58 -1 -2 0 0 0	$\bar{1}77757$ -2413 +32 -19 +1 -1 0	+54 -1 -2 0 0 0	$\bar{1}75897$ -2308 +28 -24 +1 -1 0	+51 -1 -2 0 0 0	$\bar{1}78139$ -2210 +24 -17 +2 -1 0	+47 -1 -1 0 0 0	$\bar{1}70975$ -2118 +21 -19 +1 -1 0	+44 -1 -2 0 0 0
$\bar{3}2$	$\bar{1}79899$ -348 +39 -24 +2 -1 0	+56 -1 -2 0 0 0	$\bar{1}77466$ -2380 +35 -31 +1 -22 -1 0	+52 -1 -2 0 0 0	$\bar{1}75137$ -2279 +31 -29 +1 -1 0	+49 -1 -2 0 0 0	$\bar{1}72906$ -2184 +27 -25 +1 -19 -1 0	+46 -1 -2 0 0 0	$\bar{1}70767$ -2096 +23 -18 +1 -1 0	+42 -1 -1 0 0 0
$\bar{3}3$	$\bar{1}79526$ -398 +42 -27 +2 -1 0	+54 -1 -3 0 0 0	$\bar{1}77192$ -2344 +37 -38 +1 -25 -1 0	+50 -1 -2 0 0 0	$\bar{1}74837$ -2247 +33 -24 +1 -1 0	+47 -1 -2 0 0 0	$\bar{1}72636$ -2156 +29 -29 +1 -22 -1 0	+44 -1 -2 0 0 0	$\bar{1}70522$ -2072 +26 -26 +1 -20 -1 0	+41 -1 -2 0 0 0
$\bar{3}4$	$\bar{1}79100$ -455 +45 -30 +2 -1 0	+51 -1 -3 0 0 0	$\bar{1}76747$ -2305 +41 -42 +1 -28 -1 0	+48 -1 -3 0 0 0	$\bar{1}74489$ -2213 +36 -37 +2 -27 -1 0	+45 -2 -2 0 0 0	$\bar{1}72321$ -2126 +33 -34 +1 -25 -1 0	+42 -2 -2 0 0 0	$\bar{1}70237$ -2045 +30 -29 +1 -24 -1 0	+39 -1 -1 0 0 0
$\bar{3}5$	$\bar{1}78614$ -2356	+48	$\bar{1}76306$ -2263	+45 -1	$\bar{1}74087$ -2175	+43	$\bar{1}71955$ -2092	+40	$\bar{1}69903$ -2014	+38

z	n=1·0			n=1·1			n=1·2			n=1·3			n=1·4							
3·5	I·91700	-2905	+64	-1	-1	-783	+60	-1	I·86184	-2667	+56	-1	I·88523	-2557	+53	-1	I·81018	-2454	+50	-1
	-842	+83	-4	0	0	+75	-4	0	-602	+68	-3	0	-628	+61	-3	0	-571	+54	-3	0
	-43	+1	0	0	0	+2	0	0	-39	+2	0	0	-38	+1	0	0	-36	+2	0	0
	-1	0	0	0	0	-2	0	0	-2	0	0	0	-1	0	0	0	-1	0	0	0
3·6	I·90814	-2821	+60	-1	-1	-2706	+56	-1	I·85401	-2597	+53	-1	I·82856	-2495	+50	-1	I·80410	-2398	+47	-1
	-931	+86	-4	0	0	+78	-4	0	-775	+71	-4	0	-708	+64	-3	0	-647	+59	-2	0
	-47	+2	0	0	0	+2	0	0	-44	+1	0	0	-42	+2	0	0	-40	+2	0	0
	-2	0	0	0	0	-1	0	0	-1	0	0	-2	0	0	0	-2	0	0	0	0
3·7	I·89884	-2733	+56	-1	-1	-2626	+52	-1	I·84581	-2525	+49	-1	I·82104	-2429	+47	-1	I·79721	-2337	+45	-1
	-1029	+89	-4	0	0	+81	-3	0	-867	+74	-3	0	-796	+67	-3	0	-732	+62	-3	0
	-52	+2	0	0	0	+2	0	0	-49	+2	0	0	-47	+2	0	0	-45	+1	0	0
	-2	0	0	0	0	-2	0	0	-1	0	0	-1	0	0	0	-1	0	0	0	0
3·8	I·88751	-2642	+52	-1	-1	-2543	+49	-1	I·83664	-2449	+46	-1	I·81260	-2360	+44	-1	I·78943	-2274	+42	-1
	-1139	+92	-4	0	0	+84	-4	0	-909	+77	-4	0	-895	+71	-3	0	-827	+65	-3	0
	-57	+2	0	0	0	+2	0	0	-54	+2	0	0	-53	+2	0	0	-51	+2	0	0
	-2	0	0	0	0	-1	0	-2	0	0	0	-2	0	0	0	-2	0	0	0	
3·9	I·87563	-2548	+48	-1	-1	-2456	+45	-1	I·82639	-2370	+42	-1	I·80310	-2287	+41	-1	I·78063	-2207	+39	-1
	-1258	+95	-5	0	0	+88	-4	0	-1033	+81	-3	0	-1005	+74	-3	0	-934	+68	-3	0
	-63	+1	0	0	0	+1	0	0	-60	+2	0	0	-58	+2	0	0	-56	+2	0	0
	-2	0	0	0	0	-2	0	-2	0	0	0	-2	0	0	0	-2	0	0	0	
2·0	I·86230	-2452	+43	-1	-1	-2367	+41	-1	I·81494	-2287	+39	-1	I·79246	-2211	+38	-1	I·77071	-2137	+36	-1

z	n=1.5			n=1.6			n=1.7			n=1.8			n=1.9			
3.5	I.78614 -520 -34 -1	-2356 +49 +2 0	+48 -3 0 0	I.76906 -474 -32 -1	-2263 +45 +1 0	+45 -2 0 0	I.74087 -431 -31 -1	-2175 +39 +2 0	+43 -2 0 0	I.71955 -395 -29 -1	-2092 +36 +1 0	+40 -1 0 0	I.69908 -361 -28 -1	-2014 +33 +1 0	+38 -1 0 0	
3.6	I.78069 -501 -38 -2	-2305 +53 +1 0	+45 -3 0 0	I.75799 -541 -37 -2	-2217 +47 +2 0	+43 -3 0 0	I.73624 -496 -35 -2	-2134 +42 +2 0	+41 -3 0 0	I.71380 -456 -33 -2	-2055 +39 +1 0	+39 -2 0 0	I.69518 -419 -32 -2	-1980 +36 +1 0	+37 -1 0 0	
3.7	I.77428 -674 -43 -2	-2251 +57 +1 0	+42 -2 0 0	I.75219 -619 -42 -2	-2168 +50 +2 0	+40 -2 0 0	I.73091 -572 -40 -2	-2090 +46 +2 0	+38 -2 0 0	I.71089 -528 -38 -2	-2015 +43 +1 0	+37 -2 0 0	I.69060 -487 -37 -2	-1943 +39 +1 0	+36 -2 0 0	
3.8	I.76709 -765 -48 -2	-2193 +60 +1 0	+40 -3 0 0	I.74556 -708 -48 -2	-2116 +53 +3 0	+38 -2 0 0	I.72477 -656 -45 -2	-2042 +59 +2 0	+36 -2 0 0	I.70471 -609 -43 -2	-1971 +46 +1 0	+35 -2 0 0	I.68534 -565 -42 -2	-1903 +42 +1 0	+34 -2 0 0	
3.9	I.75894 -868 -55 -2	-2132 +63 +1 0	+37 -3 0 0	I.73798 -808 -53 -2	-2060 +58 +2 0	+36 -3 0 0	I.71774 -753 -51 -2	-1990 +53 +2 0	+34 -2 0 0	I.69817 -702 -49 -2	-1924 +49 +1 0	+33 -2 0 0	I.67925 -655 -48 -2	-1860 +45 +1 0	+32 -2 0 0	
3.0	I.74969	-2068	+34	I.72935	-2000	+33	I.70968	-1935	+32	-1	I.69064	-1874	+31	I.67220	-1814	+30

z	$n=1.0$			$n=1.1$			$n=1.2$			$n=1.3$			$n=1.4$		
$\bar{2}.0$	$\bar{1}.86280$ -2452 +98 -1300 -69 -2	+43 -4 0 0 0	-1 0 0 0 0	$\bar{1}.88821$ -2367 +91 -1206 -68 -1	+41 -4 0 0 0	-1 0 0 0 0	$\bar{1}.81494$ -2287 +84 -1200 -66 -2	+39 -3 +1 0 0	-1 0 0 0 0	$\bar{1}.79245$ -2211 +77 -1128 -65 -2	+38 -3 +2 0 0	-1 0 0 0 0	$\bar{1}.77071$ -2137 +71 -1053 +1 -2	+36 -3 +1 0 0	-1 0 0 0 0
$\bar{2}.1$	$\bar{1}.84769$ -3352 +101 -1534 -76 -2	+39 -4 +1 0 0	0 0 0 0 0	$\bar{1}.82456$ -2276 +94 -1440 -73 -2	+37 -3 +1 0 0	0 0 0 0 0	$\bar{1}.80217$ -2203 +87 -1348 -71 -2	+36 -3 +2 0 0	0 0 0 0 0	$\bar{1}.78050$ -2132 +81 -1263 +1 -3	+35 -3 +1 0 0	0 0 0 0 0	$\bar{1}.75953$ -2065 +75 -1185 -70 -3	+33 -2 +2 0 0	0 0 0 0 0
$\bar{2}.2$	$\bar{1}.83157$ -2250 +102 -1692 -83 -3	+35 -3 +1 0 0	0 0 0 0 0	$\bar{1}.80941$ -2181 +96 -1593 -81 -3	+34 -3 +1 0 0	0 0 0 0 0	$\bar{1}.78794$ -2114 +90 -1500 -80 -3	+33 -3 +1 0 0	0 0 0 0 0	$\bar{1}.76713$ -2050 +84 -1413 +2 -3	+32 -3 +2 0 0	0 0 0 0 0	$\bar{1}.74695$ -1988 +79 -1331 -78 -3	+31 -3 +2 0 0	0 0 0 0 0
$\bar{2}.3$	$\bar{1}.81879$ -2147 +105 -1865 -91 -3	+32 -3 +1 0 0	0 0 0 0 0	$\bar{1}.79264$ -2084 +99 -1764 -89 -3	+31 -3 +1 0 0	0 0 0 0 0	$\bar{1}.77211$ -2023 +92 -1668 -88 -3	+30 -3 +1 0 0	0 0 0 0 0	$\bar{1}.75218$ -1964 +86 -1579 -87 -3	+29 -2 +1 0 0	0 0 0 0 0	$\bar{1}.73283$ -1907 +81 -1495 -86 -3	+28 -2 +1 0 0	0 0 0 0 0
$\bar{2}.4$	$\bar{1}.79420$ -2041 +107 -2053 -99 -3	+29 -3 +1 0 0	0 0 0 0 0	$\bar{1}.77408$ -1984 +100 -1950 -98 -3	+28 -3 +1 0 0	0 0 0 0 0	$\bar{1}.75462$ -1930 +94 -1853 -97 -3	+27 -2 +2 0 0	0 0 0 0 0	$\bar{1}.73549$ -1877 +89 -1761 +2 -3	+27 -3 +2 0 0	0 0 0 0 0	$\bar{1}.71699$ -1835 +84 -1674 -94 -3	+26 -3 +2 0 0	0 0 0 0 0
$\bar{2}.5$	$\bar{1}.77265$ -1933	+26	0	$\bar{1}.75327$ -1883	+25	0	$\bar{1}.73499$ -1834	+25	0	$\bar{1}.71690$ -1786	+24	0	$\bar{1}.69928$ -1739	+23	0

z	n=1:5			n=1:6			n=1:7			n=1:8			n=1:9		
2̄0	1̄74969 -985 +67 -61 +2 -2	0 -2 0 0 0	+34 +268 +2	1̄72985 -2000 +62 -3 +1 -2	0 +33 0 -3 0 0	0 +32 -1 -3 0 0	1̄70968 -861 +57 -38 +1 -2	0 +31 0 0 0	1̄69064 -1874 +53 +2 -2	0 +31 0 0 0	1̄67220 -1814 +49 +2 -2	0 +30 0 0 0			
2̄1	1̄78921 -1113 +69 -68 +2 -2	0 -3 0 0 0	+32 +1999 -3 +2 0	1̄71954 -1937 +65 -2 +2 -2	0 +30 0 -2 0 0	0 +29 0 -2 0 0	1̄70047 -1877 +60 -984 +2 -65 -2	0 +28 0 0 0	1̄68199 -1819 +55 +2 -63 -2	0 +28 0 0 0	1̄66408 -1763 +51 +2 -61 -2	0 +28 0 0 0			
2̄2	1̄72788 -1256 +74 -76 -3	0 -2 0 0 0	+29 +1928 -2 +2 0	1̄70889 -1870 +69 -3 +1 -3	0 +28 0 -3 0 0	0 +27 0 -2 0 0	1̄68997 -1815 +64 -1119 +2 -73 -3	0 +26 0 0 0	1̄67209 -1762 +59 +2 -1037 -3	0 +26 0 0 0	1̄65478 -1711 +55 +2 -69 -3	0 +26 0 0 0			
2̄3	1̄71403 -1416 +76 -84 -3	0 -3 0 0 0	+27 +1852 -3 +2 0	1̄69577 -1800 +71 -2 +2 -3	0 +25 0 -2 0 0	0 +25 0 -2 0 0	1̄67802 -1749 +67 -1272 +1 -81 -3	0 +24 0 0 0	1̄66078 -1701 +64 +1 -1207 -3	0 +24 0 0 0	1̄64401 -1653 +60 +1 -1146 -3	9 +24 0 0 0			
2̄4	1̄69900 -1592 +78 -93 -3	0 -2 0 0 0	+24 +1774 -2 +2 0	1̄68150 -1727 +74 -2 +2 -3	0 +23 0 -2 0 0	0 +23 0 -2 0 0	1̄66446 -1681 +70 -1443 +2 -90 -3	0 +22 0 0 0	1̄64788 -1636 +67 +1 -1375 -3	0 +22 0 0 0	1̄63174 -1592 +63 +1 -1310 -3	0 +22 0 0 0			
2̄5	1̄68212 -1094	0	+22	1̄66540 -1651	0	+21	1̄64910 -1609	0	1̄63322 -1568	0	+20	1̄61774 -1528	0	+20	

z	$n=1.0$			$n=1.1$			$n=1.2$			$n=1.3$			$n=1.4$						
$\bar{2}.5$	$\bar{1}.7265$	+26	0	$\bar{1}.76357$	+35	0	$\bar{1}.78499$	-1834	+25	0	$\bar{1}.71690$	-1786	+24	0	$\bar{1}.69928$	-1739	+23	0	
	-2260	+109	-3	-2155	+103	-3	-2055	+97	-3	-1961	+92	-3	-1872	+87	-2	-1872	+87	-2	0
	-107	+1	0	-106	+1	0	-105	+1	0	-104	+1	0	-103	+1	0	-103	+1	0	0
	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0	0
$\bar{2}.6$	$\bar{1}.74895$	+23	0	$\bar{1}.78093$	+32	0	$\bar{1}.71386$	-1736	+22	0	$\bar{1}.69622$	-1693	+21	0	$\bar{1}.67950$	-1651	+21	0	0
	-2481	+109	-3	-2376	+105	-3	-2274	+99	-3	-2178	+93	-2	-2114	+88	-2	-2087	+88	-2	0
	-116	0	0	-115	0	0	-115	+1	0	-114	+1	0	-113	+1	0	-113	+1	0	0
	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0	0
$\bar{2}.7$	$\bar{1}.72395$	+20	0	$\bar{1}.70599$	+19	0	$\bar{1}.68944$	-1636	+19	0	$\bar{1}.67327$	-1599	+19	0	$\bar{1}.65747$	-1562	+19	0	0
	-2723	+108	-2	-2616	+104	-2	-2514	+100	-2	-2416	+95	-2	-2323	+91	-3	-2323	+91	-3	0
	-125	0	0	-124	0	0	-124	0	0	-124	+1	0	-123	+1	0	-123	+1	0	0
	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0	0
$\bar{2}.8$	$\bar{1}.69444$	+18	0	$\bar{1}.67856$	+17	0	$\bar{1}.66303$	-1536	+17	0	$\bar{1}.64784$	-1503	+17	0	$\bar{1}.63296$	-1470	+16	0	0
	-2981	+109	-2	-2874	+105	-2	-2771	+100	-2	-2673	+97	-2	-2578	+93	-2	-2578	+93	-2	0
	-134	0	0	-134	0	0	-134	+1	0	-133	0	0	-133	0	0	-133	0	0	0
	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0	0
$\bar{2}.9$	$\bar{1}.66326$	+16	0	$\bar{1}.64845$	+15	0	$\bar{1}.63395$	-1435	+15	0	$\bar{1}.61975$	-1406	+15	0	$\bar{1}.60854$	-1377	+14	0	0
	-3258	+109	-2	-3151	+104	-2	-3048	+100	-2	-2950	+97	-2	-2855	+94	-1	-2855	+94	-1	0
	-143	0	0	-143	0	0	-143	0	0	-143	0	0	-143	0	0	-143	0	0	0
	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0	0
$\bar{1}.0$	$\bar{1}.62922$	+14	0	$\bar{1}.61548$	+13	0	$\bar{1}.60201$	-1335	+13	0	$\bar{1}.58879$	-1309	+13	0	$\bar{1}.57688$	-1283	+13	0	0

z	n = 1.5			n = 1.6			n = 1.7			n = 1.8			n = 1.9		
2.5	1.68212	-1694	+22	1.66540	-1651	+21	1.64910	-1609	+21	1.63322	-1568	+20	1.61774	-1528	+20
	-1787	+83	-2	-1707	+78	-2	-1622	+74	-2	-1360	+69	-2	-1493	+65	-2
	-102	+1	0	-101	+1	0	-99	+1	0	-98	+2	0	-96	+2	0
	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0
2.6	1.66820	-1610	+20	1.64720	-1572	+19	1.63176	-1534	+19	1.61661	-1497	+18	1.60182	-1461	+18
	-2000	+84	-2	-1918	+80	-1	-1840	+76	-2	-1766	+72	-1	-1695	+69	-2
	-112	+1	0	-111	+1	0	-109	+1	0	-108	+1	0	-107	+1	0
	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0
2.7	1.64204	-1525	+18	1.62697	-1491	+18	1.61224	-1457	+17	1.59784	-1424	+17	1.58377	-1391	+16
	-2234	+86	-2	-2180	+83	-2	-2069	+79	-2	-1992	+75	-2	-1919	+71	-2
	-122	+1	0	-121	+1	0	-120	+1	0	-119	+1	0	-118	+1	0
	-3	0	0	-3	0	0	-3	0	0	-3	0	0	-3	0	0
2.8	1.61845	-1438	+16	1.60423	-1407	+16	1.59032	-1377	+15	1.57670	-1348	+15	1.56337	-1319	+14
	-2488	+89	-2	-2401	+84	-2	-2319	+81	-2	-2240	+78	-2	-2164	+75	-1
	-132	0	0	-132	+1	0	-131	+1	0	-129	+1	0	-129	+1	0
	-4	0	0	-4	0	0	-4	0	0	-4	0	0	-4	0	0
2.9	1.59221	-1349	+14	1.57886	-1323	+14	1.56578	-1295	+13	1.55296	-1269	+13	1.54040	-1243	+13
	-2762	+89	-2	-2675	+86	-2	-2591	+83	-2	-2510	+79	-2	-2433	+75	-2
	-143	+1	0	-142	+1	0	-141	+1	0	-141	+1	0	-140	+1	0
	-4	0	0	-4	0	0	-4	0	0	-4	0	0	-4	0	0
1.0	1.56312	-1259	+12	1.55065	-1235	+12	1.53842	-1212	+11	1.52641	-1189	+11	1.51463	-1167	+11

z	n=1.0		n=1.1		n=1.2		n=1.3		n=1.4		
1.0	1.52922 -3554 -152 -3	+1388 +108 0 0	1.61548 -3447 -153 -3	+1361 +105 0 0	1.60201 -1335 +101 -2	+13 -2 0 0	1.58879 -3246 -153 -3	+1309 +97 0 0	+13 -2 0 0	1.57588 -1283 +93 0 0	
1.1	1.59213 -3866 -161 -3	+1280 +106 -1 0	1.57945 -3762 -161 -3	+1256 +103 -1 0	1.56700 -1234 +100 -1 -3	+11 -1 0 0	1.55477 -3561 -103 -3	+1212 +96 0 0	+11 -2 0 0	1.54276 -1100 +92 -103 -3	
1.2	1.55188 -4197 -169 -2	+1175 +105 -1 0	1.54019 -4094 -169 -3	+1154 +102 -1 0	1.52875 -3994 -171 -2	+10 -2 0 0	1.51750 -3897 -171 -3	+1116 +96 -1 0	+9 -1 0 0	1.50643 -1098 +92 -172 -3	
1.3	1.50815 -4542 -175 -2	+1071 +103 -2 0	1.49753 -4440 -177 -3	+1053 +99 -2 0	1.48708 -4343 -178 -2	+8 -1 0 0	1.47679 -4248 -179 -2	+1021 +93 -1 0	+8 -1 0 0	1.46666 -1006 +91 -180 -3	
1.4	1.46096 -4899 -181 -1	+970 +99 -2 0	1.45134 -4801 -183 -2	+956 +97 -1 0	1.44185 -4706 -184 -2	+7 -2 0 0	1.43250 -4613 -186 -2	+929 +92 -1 0	+7 -2 0 0	1.42328 -916 +90 -187 -3	
1.5	1.41015	+873	1.40148	+860	1.39293	+849	1.38449	+838	+5	1.37615	+827

n	n=1.5			n=1.6			n=1.7			n=1.8			n=1.9			
I.0	I.56812 -3058 -153 -4	+12 -2 0 0	I.55065 -2970 -153 -4	+12 -2 0 0	I.53842 -2885 -153 -4	+11 +8 +1 0	I.52641 -2803 -152 -4	+11 +8 +1 0	I.51468 -2725 -151 -4	+11 -2 0 0	I.50892 -2855 -153 -4	+11 +8 +1 0	I.49882 -3118 -163 -4	+11 -2 0 0	I.48588 -3039 -162 -4	+11 +76 +1 0
I.1	I.53097 -3376 -163 -3	+10 -1 0 0	I.51988 -3287 -163 -3	+10 -1 0 0	I.50800 -3251 -163 -3	+10 +8 0 0	I.49822 -3118 -163 -4	+10 +8 +1 0	I.48588 -3039 -162 -4	+10 -2 0 0	I.47488 -3251 -163 -3	+10 +8 +1 0	I.46897 -3455 -173 -3	+10 -2 0 0	I.45878 -3375 -173 -3	+9 +78 +1 0
I.2	I.49555 -3712 -172 -3	+9 -1 0 0	I.48485 -3623 -173 -3	+9 -1 0 0	I.47488 -3538 -173 -3	+8 +8 0 0	I.46897 -3455 -173 -3	+8 +8 0 0	I.45878 -3375 -173 -3	+8 -1 0 0	I.44686 -3978 -182 -3	+8 +8 -1 0	I.43719 -3804 -182 -3	+7 +8 -1 0	I.42766 -3812 -183 -3	+7 +79 0 0
I.3	I.45668 -4065 -181 -3	+8 -1 0 0	I.44686 -3978 -182 -3	+8 -1 0 0	I.43719 -3804 -182 -3	+7 +8 -1 0	I.42766 -3812 -183 -3	+7 +8 -1 0	I.41827 -3732 -183 -3	+7 -1 0 0	I.40528 -4351 -189 -2	+6 +8 -1 0	I.39640 -4268 -190 -2	+6 +8 -1 0	I.38709 -4108 -191 -2	+6 +78 -1 0
I.4	I.41419 -4436 -188 -2	+7 -2 -1 0	I.40528 -4351 -189 -2	+7 -2 0 0	I.39640 -4268 -190 -2	+6 +8 -1 0	I.38768 -4187 -190 -2	+6 +8 -1 0	I.37909 -4108 -191 -2	+6 -1 0 0	I.36808 -4108 -191 -2	+6 +8 -1 0	I.35981 -4066 -190 -2	+5 +8 -1 0	I.34889 -3978 -182 -3	+5 +78 -1 0
I.5	I.36793 -817	+5	I.35981 -806	+5	I.35180 -796	+5	I.34389 -786	+5	I.33608 -776	+5	I.32817 -766	+5	I.32026 -756	+5	I.31235 -746	+5

z	n=1.0			n=1.1			n=1.2			n=1.3			n=1.4		
1.5	I-41015	-873	+6	I-40148	-860	+5	I-39293	-849	+5	I-38449	-838	+5	I-37615	-827	+5
	-5266	+99	-1	-5174	+93	-1	-5081	+90	-1	-4992	+88	-1	-4905	+86	-1
	-185	-3	0	-186	-2	0	-189	-2	0	-191	-2	0	-192	-2	0
	-1	0	-1	0	0	-1	0	0	0	-1	0	0	-1	0	0
1.6	I-35563	-781	+5	I-34787	-769	+4	I-34032	-761	+4	I-33265	-752	+4	I-32517	-743	+4
	-5641	+92	-1	-5552	+88	-1	-5464	+86	-1	-5379	+84	-1	-5294	+83	-1
	-188	-3	0	-189	-3	0	-192	-2	0	-194	-2	0	-196	-2	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.7	I-29734	-692	+4	I-29046	-684	+3	I-28366	-677	+3	I-27692	-670	+4	I-27026	-662	+4
	-6016	+86	-1	-5932	+83	-1	-5849	+82	-1	-5768	+80	-1	-5688	+78	-1
	-188	-3	0	-190	-2	0	-193	-2	0	-195	-2	0	-197	-2	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.8	I-23580	-609	+3	I-23924	-603	+2	I-23293	-597	+3	I-21729	-592	+3	I-21140	-586	+3
	-6392	+80	-1	-6314	+78	-0	-6235	+77	-1	-6157	+76	-1	-6082	+74	-1
	-186	-3	0	-188	-2	0	-191	-3	0	-194	-2	0	-196	-2	0
	+1	0	+1	0	0	+1	0	0	+1	0	0	+1	0	0	0
1.9	I-16953	-532	+2	I-16423	-527	+2	I-15898	-523	+2	I-15377	-518	+2	I-14861	-514	+2
	-6760	+74	-1	-6686	+72	-1	-6614	+71	-1	-6542	+71	-1	-6472	+69	-1
	-182	-3	0	-185	-3	0	-189	-2	0	-192	-3	0	-194	-2	0
	+2	0	+1	0	0	+1	0	0	+1	0	0	+1	0	0	0
0	I-10013	-461	+1	I-09559	-458	+1	I-09096	-454	+2	I-08644	-450	+1	I-08195	-447	+1

z	$n=1.5$			$n=1.6$			$n=1.7$			$n=1.8$			$n=1.9$		
$\bar{1}.5$	$\bar{1}.86798$ -4819 -194 -2	$+5$ -1 0 0	-817 $+84$ 0 0	$\bar{1}.85981$ -4736 -195 -2	$+5$ -1 0 0	-806 $+82$ -1 0	-796 $+80$ -1 0	$\bar{1}.84889$ -4876 -197 -2	$+5$ -1 0 0	-786 $+78$ -1 0	$\bar{1}.83608$ -4499 -198 -2	-776 $+76$ -1 0	$+5$ -1 0 0	0 0 0 0	
$\bar{1}.6$	$\bar{1}.31778$ -5212 -197 -1	$+4$ -1 0 0	-734 $+81$ -2 0	$\bar{1}.30387$ -5084 -201 -1	$+4$ -1 0 0	-725 $+79$ -2 0	-717 $+77$ -1 0	$\bar{1}.29614$ -4978 -202 -1	$+4$ -1 0 0	-709 $+76$ -1 0	$\bar{1}.28909$ -4903 -203 -1	-701 $+75$ -1 0	$+4$ 0 0 0	0 0 0 0	
$\bar{1}.7$	$\bar{1}.26368$ -5611 -199 -0	$+3$ 0 0 0	-655 $+77$ -2 0	$\bar{1}.25071$ -5459 -204 -0	$+3$ -1 0 0	-648 $+76$ -2 0	-642 $+74$ -1 0	$\bar{1}.24433$ -5386 -204 -0	$+3$ 0 0 0	-634 $+73$ -2 0	$\bar{1}.23802$ -5313 -206 -0	-627 $+72$ -2 0	$+4$ -1 0 0	0 0 0 0	
$\bar{1}.8$	$\bar{1}.20557$ -6009 -199 0	$+3$ -1 0 0	-580 $+73$ -2 0	$\bar{1}.19408$ -5866 -203 0	$+2$ 0 0 0	-574 $+71$ -2 0	-569 $+70$ -2 0	$\bar{1}.18842$ -5796 -205 0	$+3$ -1 0 0	-563 $+69$ -2 0	$\bar{1}.18282$ -5728 -207 -0	-558 $+68$ -2 0	$+3$ -1 0 0	0 0 0 0	
$\bar{1}.9$	$\bar{1}.14349$ -6404 -198 $+1$	$+2$ 0 0 0	-509 $+67$ -2 0	$\bar{1}.13389$ -6269 -202 $+1$	$+2$ 0 0 0	-505 $+67$ -2 0	-501 $+66$ -2 0	$\bar{1}.12841$ -6205 -204 $+1$	$+2$ 0 0 0	-497 $+65$ -2 0	$\bar{1}.12346$ -6140 -206 $+1$	-493 $+64$ -2 0	$+2$ -1 0 0	0 0 0 0	
$.0$	$\bar{1}.07749$	$+2$	-444	$\bar{1}.06889$	$+2$	-440	-437	$\bar{1}.06433$	$+2$	-434	$\bar{1}.06001$	-431	$+1$	0	