(*Paper No. 2826.*)

"The Prevention of Silting in Irrigation Canals."

By Robert Greig Kennedy, Ex. Eng. P.W.D., India.

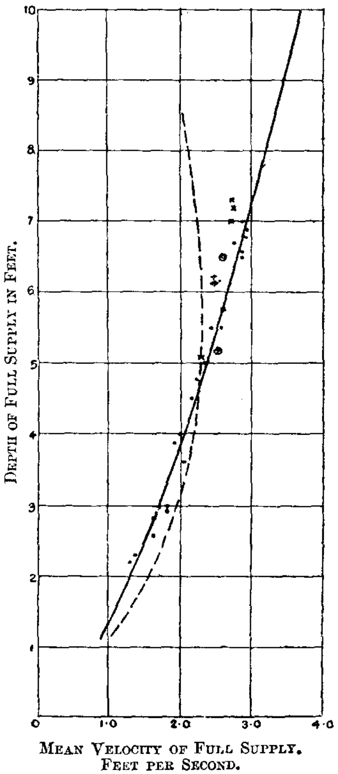
One of the most important and at the same time most difficult conditions to ensure in the construction of an irrigation canal, is that no silting shall take place in it. The observations upon which the results described in this Paper are founded were made upon a portion of the main canal of the Bari Doab canal-system and extended over about 90 miles of its lower reaches. This canal discharged at a maximum rate of about 1,700 cubic feet per second, and included about twelve distributing channels which had carrying-capacities ranging between 30 cubic feet and 250 cubic feet per second. The Bari Doab canal-system is specially valuable for observations of the kind under consideration, because its channels have assumed permanent sections by silting or scouring, and all the sites the data of which will be hereafter discussed had silted beds, often 2 feet or 3 feet deep, deposited during many years of steady flow. No silt was ever cleared away from these reaches, and therefore for a considerable time the silt-transporting power in each has been just sufficient to carry all the sediment brought down; and each reach could have carried neither more nor less than this amount under the given conditions of width of bed, depth of channel and velocity of flow. It has been invariably found that the form of the cross-section thus arrived at is nearly rectangular, the sides being vertical and of fine sediment, and the bed horizontal and of coarser sand.

Thus by observation of the various widths of bed, depths of channel and mean velocities of the stream at each site under normal conditions, means are afforded of discerning how these three factors must be varied among themselves to ensure that no bed-silting shall take place. When this is attained no trouble will arise from side-silting, which consists only of the finer particles, and which takes place, as a rule, only when weeds are allowed to grow upon the edge of the canal. During variations in supply, the thickness of the silted bed was found to vary slightly through a few inches. The mean velocity of the stream at each reach was derived from the known "full supply" discharge and the measured cross-sectional area of the channel. According to the system of distribution adopted, each distributary was as far as possible always discharged with its full supply, the total discharge of the canal being only sufficient to provide at one time for say nine out of the twelve minor channels; so that usually, neglecting loss by absorption, the sum of the discharges of the distributaries open at any time was equal to the supply in the canal.

In these lower reaches of the canal the amount of silt carried for any given discharge was more or less constant all the year round, though it was by no means so at the head. The extra amount of mud in the river-water during summer was deposited chiefly in the upper reaches in which the velocities of the stream were least; but it was again picked up and carried forward when the cold weather pure water came down; so that the farther reaches of the system were about equally turbid all the year round, except when an excessive flood in the river brought down a different kind and colour of silt. Thus the canal-system may be said to have been in a state of silt equilibrium, all the sediment being practically carried through the very numerous outlets of the distributaries into the water-courses, from which all necessary clearances were made by the cultivator. In other words, the total amount of silt carried by the usual full supply in the canal is equal to the sum of the amounts carried into and forward by the distributaries flowing at the time. All the data now to be detailed refer to these "full supplies," which are somewhat greater than the average discharge, and less than the flood discharge. In all the channels the full supply will bear a more or less constant ratio to the average. The average discharge might equally well have been taken, but the results obtained will be much more useful if they are directly applicable to the full supply required.

Observations were made at thirty sites, data for which are given in Table I of the Appendix. Each was known by long local experience to have been in a state of permanent regime, the canal having been flowing for years on its self-silted bed. Such permanency had been attained only by long prior remodelling and change of slope. Of the thirty points which are plotted, *Fig. 1*, with depths as ordinates and mean velocities as abscissas, five (marked ×) are for reaches in which the alterations had been comparatively recent, and the bed had therefore not yet quite reached its limit. These points, as was to be expected, all lie somewhat to the left of the full-line curve, which may be taken as representing the average result.

Fig. 1.



Curve for sections having the same discharge and slope D = 150 c'; S = 0·25; N = 0·0225, — — — —.
Curve $V_o = 0{\cdot}84\,d^{0{\cdot}64}$, ———.

NOTE.—The points marked × are for reaches where the water-level has lately been raised and the silting has not yet reached its limit, *i.e.*, the mean velocity is less than it will ultimately be.
The points marked ⊙ are for the Upper Sutlej Canal system.

RELATION OF "CRITICAL" MEAN VELOCITY (V) TO DEPTH (d) FOR FULL SUPPLY, PLOTTED FOR SITES WHERE A PERMANENT REGIME HAS BEEN ARRIVED AT BY SILTING OF THE BED.

Two points (marked ⊙) refer to another canal, the "Katora," but neither was observed for the full-supply discharge. The true point about midway between them will, however, very nearly coincide with the corresponding one upon the curve drawn for the Bari Doab Canal.

The tabulated data in Table I, Appendix, show that the various channels are very dissimilar as regards the ratio of width to depth, which varies between about 15 and 4. Notwithstanding this, however, the results when plotted are quite as consistent as could be expected in the circumstances, so that apparently the bed-width has no place in the equation connecting the depth (d) and the mean velocity (V_o). The five points, for example, with depths between 3·9 and 5·0 inclusive, are quite consistent, although the ratio $\frac{b}{d}$ varies between 3½ and 12. This seems to show clearly that V_o is a function of d alone, and it is found that the equation is approximately expressed by the empirical expression

$$V_o = c\,d^m = 0{\cdot}84\,d^{0{\cdot}64},$$

and it is from this that the curve has been delineated.

This mean velocity V_o may be called the critical velocity, being that at which for the given depth d silting is just prevented; so that in order to avoid silting in any canal-system, the mean velocity of the stream must not be kept constant in all the channels, but must be increased with the depth according to the above law. V_o is also evidently the least mean velocity for which new channels must be designed, and at which they can be maintained. When the soil permits, a greater velocity than V_o may be given, but never a less; and in the case of very sandy soil, V_o is the only mean velocity which can permanently obtain. On different canal-systems, the values of c and m, in the equation $V_o = c\ d^m$, may vary slightly, though from what follows it may be anticipated any such variation will be but small.

The case of a channel to carry 150 cubic feet of water per second, with a slope of 1 in 4,000, and with the value of N in Kutter's formula for mean velocity = 0·0225, has been taken, and numerous possible cross-sections, each fulfilling the above conditions, have been worked out with different depths of channel, widths of bed, and mean velocities of the stream. The resulting depths and mean velocities have been plotted on the same diagram, and a dotted curve drawn through them. For all depths less than 4·7 feet, this new curve lies on the right of the critical velocity curve, showing that the proposed velocity would be here greater than that required, and scouring would result. At the depth 4·7 feet, the two curves cross, that is to say, the channel would at this point just carry its silt, and at a greater depth than 4·7 feet the velocity becomes less than V_o, and silting would take place. This particular curve, drawn for a discharge of 150 cubic feet per second, may be taken as typical of the most usual cases in its alignment as regards V_o, but each discharge would have a different curve, and with certain data it is even possible that for small depths the curve might lie to the left of the V_o curve, and then cross to the right instead of the reverse, as here delineated. In the case shown by the curve, silting would be cured by decreasing the depth; in the rarer but possible one the cure would be to increase the depth. In cases in which the slope is quite insufficient the curve would lie wholly to the left of the V_o curve, and silting could then be minimised (as afterwards shown), but could not be prevented. With very steep slopes, on the other hand, the curve would lie altogether on the right of the V_o curve, and scouring would then be unavoidable, and modified only by the nature of the soil.

Table II in the Appendix shows for given discharges the necessary minimum velocities and slopes for three different depths of full supply, and covers the usual range of design in India.

From this it will be seen that, especially in the cases of small channels, the greater the depth the steeper must be the slope; or, in other words, when the slope is fixed and insufficient, there is an advantage in most cases in widening the bed and decreasing the depth. This is only true, however, within certain limits, as will be shown below; and as already noted there may be certain unusual cases in which the depth should be increased instead of diminished. The Author's experience shows that silting in small channels has frequently been cured by merely widening the bed.

Sediment in a flowing canal is kept in suspension solely by the vertical components of the constant eddies, which can always be observed in any stream, boiling up gently to the surface. From the sides also, some such eddies may occur to a much smaller degree, but any such must be for the greater part horizontal, and of no silt-supporting power. In order, therefore, to obtain an expression for the silt-supporting power of the stream, it may safely be assumed that the quantity of silt supported will be proportional to the width of the bed, all other conditions remaining the same. It must also vary with the velocity of the stream V, say as V^{n-1}, since the greater the velocity the greater must be the force of the eddies, which become zero when the velocity is zero. There is a third variable, the depth, but this could affect neither the number nor the force of the eddies. The amount of silt supported in a stream may therefore be expressed by AbV^{n-1}, where A is some constant unknown. But whilst thus supported, the silt is being moved forward at velocity V, so that the amount of silt transported will be equal to AbV^n. It is here presumed that all sediment is in suspension, but there is doubtless a small portion of the heavier silt simply rolling along the bed. This amount would vary as bV, instead of as bV^n; so that the value of n to include the rolling silt will be somewhat less than it would be if the suspended silt alone were considered. If it be assumed that the amount of silt supported is proportional to the upward pressure of the deflected currents of water, which varies simply as the square of the velocity, or as V^2, the expression $n - 1 = 2$, or $n = 3$, is arrived at. From these considerations, therefore, and allowing for rolling silt, a value something less than three would be derived for n. It will be shown below that its actual value, deduced from quite independent considerations, is 2·56, or say $\frac{5}{2}$.

In order to obtain an expression for the amount of silt to be carried, let p represent the ratio between the amount of silt carried and the volume of water containing it. The whole system of canal and distributaries has for years been in steady flow, and in regular intercirculation, so that the water must be of equal turbidity throughout; that is to say, the ratio p is constant. For any discharge, D, the amount of silt carried is therefore expressed by $D\,p$, or is, in other words, simply proportional to the discharge.[1]

But it has been shown above that, in the case of each of these self-adjusted channels, $V_o = c\ d^m = 0 \cdot 84\ d^{0 \cdot 64}$. This expression, therefore, not only gives the critical or non-silting velocity for any depth, but also the ratio between the mean velocity and the depth at which the silt ratio p will be constant. It does not from this consideration seem probable that the constants c and m will vary greatly, if at all on different canal-systems. On the other hand, the value of p will vary with the character of the silt, and with its specific gravity. The silt ratio will increase with the depth below the surface; but the value given to p is the average ratio from bed to surface for the whole width between the volume of silt and the volume of discharge in a permanently self-adjusted stream. In the case of a stream of clear water entering a canal and picking up its full amount of silt from previous bed deposits, p then becomes the measure of the erosion of the channel.

The quantity of silt which a self-adjusted channel is able to carry and that which it carries under certain conditions, can now be equated thus:—$A\,b\,V_o^n = p\,D$. But since the channels are nearly rectangular, $D = b\,d\,V_o$, by substituting which, $A\,b\,V^n = p\,b\,d\,V_o$, or $A\,V_o^{n-1} = p\,d$, or $V_o = \left(\frac{p}{A}d\right)^{\frac{1}{n-1}}$, is obtained. This equation is of the same form as that experimentally deduced above, so that the theory advanced is shown to be in accordance

---

[1] This can be shown algebraically as follows:—Let D represent the full-supply discharge of the main canal, and p its silt ratio, and D_1 and p_1, D_2 and p_2, D_3 and p_3, &c., those of the smaller channels. Since all the silt and supply is carried forward by the distributaries—

$$D\,p = D_1\,p_1 + D_2\,p_2 + D_3\,p_3 + \text{ \&c.} \quad . \quad . \quad . \quad . \quad (1)$$

and

$$D = D_1 + D_2 + D_3 + \text{ \&c.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

With this value substituted for D, (1) becomes—

$$D_1\,(p - p_1) + D_2\,(p - p_2) + D_3\,(p - p_3) + \text{ \&c.} = 0 \quad . \quad . \quad (3)$$

If p, p_1, p_2, &c., are not all equal, some of the factors $(p - p_1)$, $(p - p_2)$, &c., must, therefore, be negative, and some positive; that is to say, some of the smaller channels carry a greater, and some a smaller, quantity of silt than the main canal, the depth, breadth and velocity in which are all greater than in the minor channels. This is manifestly untenable, and hence the silt ratio cannot but be constant throughout the system, and the amount of silt carried cannot but vary directly with the discharge.

with observed facts. The equation $V_o = c\,d^m = 0 \cdot 84\,d^{0 \cdot 64}$ is therefore identical with $V_o = \left(\frac{p}{A} d\right)^{\frac{1}{n-1}}$, so that $\frac{1}{n-1} = m = 0 \cdot 64$, or $n = 2 \cdot 56$, that is to say, the silt-transporting power of a stream varies as its mean velocity raised to the 2·56th power, or approximately as $V^{\frac{5}{2}}$. Both p and A must be very small fractions, and vary together with the character of the silt carried in different canal-systems.

The rate at which silt is deposited, when the mean velocity falls from any cause below that necessary to prevent settlement and to carry the full quanity p, can be found as follows:—Calling x the quantity of sediment which the stream can carry at the mean velocity V, the quantity carried at the "critical velocity" V_o for the given depth of channel being p, and the carrying power varying as $V^{\frac{5}{2}}$, it follows that $p : V_o^{\frac{5}{2}} :: x : V^{\frac{5}{2}}$, or $x = p\left(\frac{V}{V_o}\right)^{\frac{5}{2}}$. As, however, the supply channel is presumably n permanent regime, and carries its proper quantity p, the fraction of sediment deposited will be—

$$p - x, \text{ or } p\left(1 - \left(\frac{V}{V_o}\right)^{\frac{5}{2}}\right).$$

The values of $\frac{p-x}{p}$, corresponding with several values of $\frac{V}{V_o}$, are given in the Table below, which shows how rapidly silt settles as soon as V falls below V_o:—

$\frac{V}{V_o}$	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
$\frac{p-x}{p}$	1·00	0·996	0·983	0·951	0·899	0·823	0·722	0·592	0·429	0·233	0·00

Another case, curious in its result, is that presented by a channel in designing which sufficient slope to prevent silting in some degree is unobtainable. The cross-section at which the stream will silt least, and therefore cost least in maintenance when sufficient slope is unobtainable, is determined as follows:—The feeder canal or river being assumed to carry its proper quantity p, the portion of the silt deposited as above for any velocity V and depth d, is—

$$p - x = p\left(1 - \left(\frac{V}{V_o}\right)^{\frac{5}{2}}\right) = p\left(1 - \left(\frac{V}{c\,d^m}\right)^{\frac{5}{2}}\right).$$

This expression is a minimum when the expression $\frac{V}{c\,d^{m}}$ is a maximum. In the case of a rectangular channel, where $D = b\,d\,V$, $\frac{V}{c\,d^{m}}$ becomes $\frac{D}{c\,b\,d^{m+1}}$, which again for any given discharge D is a maximum when $b\,d^{m\quad 1}$ is a minimum, that is to say, $b\,d^{1\cdot 64}$. From all the possible cross-sections, therefore, giving the required discharge with the available slope, that one which gives the least value for $b\,d^{1\cdot 64}$ must be selected. In general this will give a comparatively small depth; but in the case of larger channels, at any rate, this will not be less than is quite usual in canal design.

The case is a common one in which, at the head of a canal, the question arises as to the relative quantities of silt deposited, with very turbid water in the river, at high and low discharges; and in which there exists the option of discharging at a greater rate than is needed for the irrigation requirements, the surplus being disposed of by escapes farther down the canal. For the low discharge let the data be D_1, d_1 and V_1; and for the higher D_2, d_2 and V_2; and the silt ratios in the river at both p_1, presumably greater than p which the canal can carry when the channel has finally adjusted itself. The quantity which can be carried at the discharge D_1 is $p\left(\frac{V_1}{c\,d_1{}^{m}}\right)^{\frac{5}{2}}$, and therefore the quantity which will be deposited will be $p_1 - p\left(\frac{V_1}{c\,d_1{}^{m}}\right)^{\frac{5}{2}}$.

The total amount of sediment deposited at the discharges D_1 and D_2 respectively will then be—

$$D_1\left(p_1 - p\left(\frac{V_1}{c\,d_1{}^{m}}\right)^{\frac{5}{2}}\right);\text{ and } D_2\left(p_1 - p\left(\frac{V_2}{c\,d_2{}^{m}}\right)^{\frac{5}{2}}\right).$$

As a case in point, it can be shown that if $p_1 = 2\,p$, and the width of the canal bed is 200 feet, the amount of silt deposited in the canal with a depth of 7 feet is 1·69 time that deposited with a depth of 5 feet; and in general, by actual trial it is seen that except for very low discharges, the second of these expressions is greater than the first. It is therefore erroneous to increase the discharge in a canal-head during heavy floods, on the assumption that at its increased velocity the stream more than carries off the excess of silt brought in.

The Paper is accompanied by a tracing from which the *Fig.* has been prepared.

[APPENDIX.

APPENDIX.

TABLE I.—DETAILS OF SELECTED SITES.

Channel.	Full Supply in Cubic Feet per Second. (D).	Bed-width in Feet. (*b*).	Depth of Water for Full Supply. (*d*).	Mean Velocity for Full Supply. (V_o).
Bari Doab Canal—				
M.B.L. 2 miles above Bhuchar [1] . .	1,250	70	6·5	2·81
Just above Bhuchar Fall [1] .	940	66	5·7	2·55
At Jaman [1]	940	66	5·5	2·55
Above Lulliani Bridge [1] . .	700	61	5·0	2·33
,, Bhambha [1] ,, . .	650	48	5·5	2·40
,, Gandian [1] ,, . .	390	36	4·8	2·25
Bari Doab Canal—				
Raja. heads, Chabhal Rajbaha [1] . .	85	16	3·0	1·70
Amritsar [1] ,, . . .	120	14	4·0	2·00
Doda [1] ,, . . .	70	12	3·0	1·80
Bhuchar [1] ,, . . .	220	22	4·5	2·15
Gilpan [1] ,, . . .	75	14	3·0	1·70
Kanha [1] ,, . . .	65	15	2·6	1·60
Lulliani [1] ,, . . .	65	14	2·8	1·60
Minor heads, Athilpur Minor [1]. . .	33	11	2·2	1·30
Chunian [1] ,, . . .	26	8	2·3	1·40
Bari Doab Canal observed in 1894.				
M.B.L. above Ralliali Rapid [2] . .	1,700	86	6·8	2·90
,, Thriawal [2] ,, . . .	1,700	85	7·0	2·86
,, Doburji Raja. head [2] .	1,700	84	6·9	2·91
,, Fatteghar Rapid [2] . .	1,500	80	6·6	2·83
,, Doda Rapid [2] . . .	1,250	68	6·7	2·75
Raja. heads, Turkwind Raja. [2] . .	142	18	3·9	1·90
Raiwind Raja. [2] . . .	138	18	3·6	2·04
Channels in which the silting of the bed is not complete.				
Bari Doab Canal—				
M.B.L. above Kathunangal Rapid [3] .	1,700	86	7·3	2·71
,, Jethowal Rapid [4] . .	1,700	91	7·0	2·70
,, Tarn Taran Rapid [5] .	1,500	76	7·2	2·74
,, Bhuchar Rajbaha [5] . .	1,250	83	6·1	2·47
,, Bhuchar Fall [5] . . .	940	81	5·1	2·28
At Sohalfoot Bridge [5] . . .	1,250	82	6·2	2·46
On Upper Sutlej Canals.				
Katora Canal near Head [6]	633	50	5·2	2·52
,, ,, ,, ,, [7]	924	55	6·5	2·59

---

[1] Selected channels in which silting has come to its limit.

[2] No alterations made in water-levels or sections for many years, silting complete.

[3] Crest raised in 1891.

[4] Rebuilt in 1889.

[5] Crest and water-levels raised in 1893 spring.

[6] Taken July, 1892.

[7] Taken July, 1893.

TABLE II.—MINIMUM LONGITUDINAL SLOPES OF, AND VELOCITIES OF FLOW IN, CHANNELS, REQUIRED TO PREVENT SILTING FOR GIVEN DISCHARGES AND FOR VARIOUS DEPTHS OF FULL SUPPLY.

Calculated for N = *0·02375*. (Kutter's Formula.)

Discharge in Cubic feet per Second.	For Maximum Probable Depths.			For Moderate Depths.			For Minimum Probable Depths.		
	Depth in feet.	Minimum mean Velocity, V_o.	Minimum fall per 1,000.	Depth in feet.	Minimum mean Velocity, V_o.	Minimum fall per 1,000.	Depth in feet.	Minimum mean Velocity, V_o.	Minimum fall per 1,000.
10	2·1	1·30	0·50	2·0	1·30	0·48	1·8	1·21	0·43
25	2·5	1·51	0·43	2·4	1·46	0·37	2·2	1·39	0·34
35	2·8	1·63	0·39	2·6	1·55	0·34	2·4	1·46	0·31
50	3·1	1·74	0·34	2·9	1·66	0·31	2·6	1·55	0·29
75	3·4	1·85	0·32	3·1	1·74	0·29	2·9	1·66	0·27
100	3·7	1·94	0·31	3·3	1·82	0·28	3·0	1·70	0·26
125	4·0	2·04	0·30	3·6	1·91	0·27	3·2	1·77	0·26
150	4·2	2·10	0·29	3·7	1·95	0·26	3·4	1·85	0·25
175	4·3	2·13	0·28	3·9	2·01	0·25	3·5	1·88	0·24
200	4·5	2·20	0·27	4·0	2·04	0·24	3·6	1·91	0·23
250	4·8	2·26	0·26	4·2	2·10	0·24	3·8	1·98	0·23
300	5·0	2·35	0·26	4·5	2·20	0·23	4·0	2·04	0·22
350	5·3	2·41	0·25	4·6	2·23	0·23	4·2	2·10	0·22
400	5·5	2·50	0·25	4·8	2·29	0·22	4·3	2·13	0·22
450	5·7	2·56	0·25	5·0	2·35	0·22	4·4	2·17	0·21
500	5·8	2·62	0·24	5·1	2·38	0·22	4·6	2·23	0·21
600	6·1	2·66	0·24	5·3	2·44	0·21	4·8	2·29	0·21
700	6·3	2·73	0·23	5·5	2·50	0·21	5·0	2·35	0·20
800	6·6	2·80	0·23	5·8	2·62	0·21	5·2	2·41	0·20
900	6·8	2·86	0·22	5·9	2·61	0·20	5·3	2·44	0·19
1,000	7·0	2·92	0·22	6·0	2·64	0·20	5·4	2·47	0·19
1,500	7·8	3·12	0·21	6·8	2·85	0·20	6·1	2·66	0·19
2,000	8·5	3·31	0·20	7·3	3·00	0·19	6·6	2·80	0·18