On the Flow of Electricity in a uniform plane conducting Surface - Part II

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1874 Proc. Phys. Soc. London 1 193

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on the fork is a knife-edge, C. To C is attached a vertical screw-rod, E, carrying a nut (invisible in the figure) which supports the heavy weight D, consisting of a series of leaden disks. Also fastened to C is a cardboard sector, F. The circular edge of F is split, and into the crack is pasted a paper gutte. A silk thread, G, is fastened to F, and carries a little paraffin disk, H. The pendulum-length is altered by the nut E until the disk H is isochronous in its motion with the water in V. The vessel V is shown out of proportion small. When the right length of E is obtained, the disk H moves up and down with the water without a ripple for twenty minutes or more, showing how truly pendular is the wave-motion. It follows that, for small disturbances, the rate of vertical motion at the normal level must be closely proportional to the altitude of the wave or amplitude of undulation.

XXIII. *On the Flow of Electricity in a uniform plane conducting Surface.*—Part II. By Professor G. CAREY FOSTER, F.R.S., and OLIVER J. LODGE, B.Sc.*

43. IN an unlimited uniform conducting surface, at given points of which there are sources and sinks of electricity, the electrical condition of each point within a limited area must be determined by the electrical condition of its boundaries. Moreover, if an equal similar area be taken in a *limited* conducting sheet, whose conductivity and thickness outside this area vary in any manner, the electrical state within the area will still be the same as in the previous case, if only the conditions at the boundaries remain unchanged; for evidently the electrical flow within the area will depend on the actual electrical state of the boundaries, and not upon the way in which this state has been produced. Consequently the truth or falsehood of conclusions arrived at, like those given in Part I. of this communication, on the assumption of an unlimited uniform conducting surface, may be tested by experiments on a conducting sheet of finite size, if we can ensure that, at the edges, the electrical conditions are the same as they would be at the boundaries of a portion of the same

* The substance of this paper was read to the Physical Society on the 27th of February, 1875; but it contains also experimental results obtained since that date.

size and shape taken in an infinite sheet in which there is a known distribution of sources and sinks.

44. In order to carry out such experiments, it is needful that the boundaries of the part of the conducting sheet examined should be formed either wholly by equipotential lines, or partly by equipotential lines and partly by lines of flow. It is evident that an area within which there is a flow of electricity (unless the circuit is complete within this area, as in a conducting plate moving in a magnetic field) cannot be bounded entirely by lines of flow; for in that case no electricity could enter or leave the area.

Any line in a conducting sheet can be practically made to be an equipotential line by laying along it, and in close contact with the sheet, a conductor whose resistance in directions parallel to the sheet is insensible—for instance, a thick band of copper. And any line along which the sheet is bounded by non-conductors (*e. g.* the free edges of the sheet) is necessarily a line of flow, since no electricity can pass across such a boundary in either direction. The experimental conditions can thus be chosen so that a conducting area may be bounded either by equipotential lines wholly, or partly by equipotential lines and partly by lines of flow, placed so as to correspond to any given system of poles.

45. Methods of testing experimentally the conclusions deduced from theory, founded on the general principles above indicated, have been employed by various investigators. Thus Kirchhoff, in his paper of 1845 (*Pogg. Ann.* vol. lxiv.), traced the form and distribution of the equipotential lines on thin sheets of metal bounded by four circular arcs, two of which, being free edges, were lines of flow and belonged to the same circle; while the others, of much smaller extent than the first two, were equipotential lines determined by the contact of small cylindrical electrodes (conducting wires) with the margin of the sheet. He traced the course of the equipotential lines by fixing a wire connected with one terminal of a galvanometer in succession at several points arbitrarily chosen, and in each case finding, by trial with a wire from the other terminal, a sufficient number of points having the same potential as the selected point. To ascertain the distribution required to make the difference of potential between consecutive lines constant, he determined a number of points along the straight line joining the electrodes, such that the difference of potential between each one and the next was just

sufficient to compensate the constant electromotive force of a thermoelectric element. The form of the equipotential lines in some rather more complicated cases was determined in the same manner by Quincke*, and in other cases again by students in the Edinburgh University Laboratory†. Schwedoff‡ also has investigated the direction of the flow at various points in a rectangular sheet of tinfoil.

Recently an elaborate series of experiments, by a method similar to Kirchhoff's, has been carried out by Professor W. G. Adams§, who has also extended the same method of inquiry to conductors of three dimensions by the employment of conducting liquids (mercury, sulphate of copper, sulphate of zinc).

Verifications by Kirchhoff and by Mach of the theoretical expressions for the strength of the current at different parts of a conducting sheet have already been referred to (see Part I. § 15, footnote).

Experimental measurements of resistance, in addition to the attempts of Kirchhoff already mentioned (Part I. § 1), have been made by Gaugain||, who measured the resistance of a solution of sulphate of copper contained between two circular copper cylinders placed one inside the other with their axes parallel, and thus verified Blavier's formula (see Part I. § 24, footnote)—by v. Obermayer¶, who verified Kirchhoff's formula for circular disks, and also a formula deduced by Stefan for the resistance of an oblong strip with circular electrodes placed on the central line, by experiments with disks and strips of platinum-foil—and by Domalip**, who measured the resistance of thin circular layers of a solution of sulphate of zinc contained between parallel glass disks, the electrodes being zinc rods inserted through holes in the upper piece of glass.

* Pogg. *Ann.* (1856) vol. xvii. p. 382.

† W. Robertson Smith, *Proc. Edinb. Roy. Soc.* 1869-70, p. 96.

‡ Schwedoff (Pogg. *Ann.* 1873, *Ergänzungs.* vi. p. 85) inferred the direction of the resultant flow at any point of a conducting sheet in which there are two equal opposite poles, from simple geometrical considerations, to a great extent similar to those employed by Professor W. R. Smith and by ourselves (*Phil. Mag.* vol. xlix. p. 394). We should have referred to his paper before, if we had known of its existence.

§ *Proc. Roy. Soc.* (1875) vol. xxiii. p. 280.

|| *Ann. de Chim. et de Phys.* [3] vol. lxiv. p. 200 (1862).

¶ *Wien. Akad. Ber.* (1869) vol. lx. pt. 2. p. 245.

** Carl's *Repertorium für Experimental-Physik*, 1874, vol. x. p. 23.

46. The results of all these investigations are in such close agreement with calculation that it cannot be considered that any further confirmation of the general theory is needed. It nevertheless seems worth while to point out, and to illustrate by specimens of the results obtained by it, a simple method which serves both for determining the form and distribution of the equipotential lines on a conductor of two dimensions, and for measuring the resistance of the part of it contained between any two given lines. In all except the very simplest cases, it would be much easier to determine the equipotential lines and resistance experimentally in this way than by the corresponding calculation.

47. The method referred to is an application of the principle of Wheatstone's bridge, and will be intelligible from fig. 1. The

Fig. 1..

poles of a battery (Z) are connected with the two ends of a graduated German-silver wire, EF, and also with two fixed points, A and B, of the conducting disk on which the equipotential lines are to be traced; a galvanometer, G, is connected to a sliding contact block D on the wire EF, and to a contact-pin C, with the point of which the sheet can be explored. Under these circumstances the points C and D will be at the same potential whenever the galvanometer shows no deflection; hence if the pin C be moved about on the disk, and a mark made at every position for which the galvanometer is not affected, all these marks will lie on one equipotential line. By repeating this operation, after shifting the slider D along the graduated wire, another equipotential line can be marked out on the sheet; and any number of lines can be traced, so as to correspond to equal differences of potential, by shifting the slider through successive equal distances.

48. In the actual experiments the connexions were not made exactly as shown in fig. 1, but the positions of battery and galvanometer were interchanged. This arrangement made it unnecessary to use a separate key for closing the battery-circuit, as the circuit was completed by the contact of the pin C with the

sheet ; and since these contacts were only momentary, no sensible disturbance can have arisen from unequal heating. The battery used was a single cell of Smee; the galvanometer was a reflecting one of low resistance. The disk was a circular piece of tinfoil, of 27·8 centims. diameter, pasted on the bottom of a tray or shallow box of hard wood, on the edges of which rested a wooden bar supporting the contact-pin C. This last slid in a vertical tube fixed to the wooden bar ; a spiral spring inside the tube kept the pin out of contact with the sheet except when it was pressed down. When the right position on the sheet had been found by means of the blunt point of the pin, this position was marked by pressing the pin down harder, so as to make a slight indentation in the tinfoil.

The mode of making contact at A and B which we found to answer best is shown in fig. 2. AS is a brass spring with a projecting pin A, and a screw S with nut and washer. The screw passes through the wood

Fig. 2.

of the tray and is fixed by screwing up the nut, the conducting wire being clamped between

the nut and washer. The rounded end of the pin A rests on the tinfoil surface, pressing it firmly and making a good circular contact. It was found necessary to lubricate the bar carrying C with blacklead to prevent its causing vibrations when sliding on the edges of the tray, and so deranging the contact of the springs A and B.

49. After a series of points have been marked on the tinfoil in the manner described, a line may be drawn through each set by hand. A set of lines obtained in this way for the case of two equal opposite poles on the edge of a circular disk correspond to a portion of the diagram in Plate IV. bounded by a complete line of flow. P being any one of the dots marking such a line, if they are circles with the poles as inverse points,

the ratio $\frac{AP}{BP}$ will be constant for any one line (Part I. § 16) ;

and if they are at equal differences of potential, the values of this ratio will vary from one line to the next by a constant factor μ (Part I. § 18). The numbers given in the following Table show to what extent these requirements are fulfilled by the ex-

perimental lines. Nine or ten points on an average were determined for each line; but as a specimen of the results it seems sufficient to give the ratios found for four points only, viz. (in the case of each line) for a point near either end, a point on or near the straight line joining the poles, and a point about halfway between this and the more distant end of the equipotential line.

Values of $\frac{AP}{BP}$ ($=\mu^n$) for four different points on each of eight different equipotential lines.

	μ^{-3} .	μ^{-2} .	μ^{-1} .	μ^0 .	μ .	μ^2 .	μ^3 .	μ^4 .
	·200	·335	·568	·996	1·71	2·94	5·05	8·75
	·200	·327	·568	1·00	1·70	2·95	5·13	9·01
	·190	·323	·573	1·00	1·71	2·98	5·18	8·97
	·196	·333	·573	1·02	1·73	2·98	5·53	8·91
Means.	·1965	·3295	·5705	1·005	1·71	2·96	5·22	8·66.
	$= (1·72)^{-3}$	$= (1·74)^{-2}$	$= (1·76)^{-1}$		$= (1·71)$	$= (1·72)^2$	$= (1·73)^3$	$= (1·7)$

In the experiments by which the above values were obtained the distance between the poles was 19·45 centims. Taking this as the value of $2a$, and 1·724 for μ , a system of coaxial circles was drawn by the method of § 21, Part I. A tracing of the experimental curves was then superposed on these theoretical circles; and the two sets were found to coincide almost exactly.

Another case tried was that of a circular disk with four equal poles on its edge, two positive and two negative; the equipotential lines obtained experimentally agreed in all particulars with those drawn by the superposition method of § 35.

50. The arrangement of fig. 1 serves not only to lay out the equipotential lines on a disk, but also to determine its resistance; for if the pin C be kept in contact with any point of the disk (with the electrode A for instance), while D is adjusted so that the galvanometer stands at zero, and if C is then moved to another point (say the electrode B) and D again adjusted, the resistance of the part of the disk between the equipotential lines which respectively pass through the two positions of C is the same fraction of the resistance of the branch E A B F, as the resistance of the part of the wire between the two positions of D is of the resistance of the whole wire E F. Accordingly the resistance of the disk is given, in terms of that of unit length of the wire E F,

as soon as the ratio of the resistances $EABF$ and EDF is known. In order to determine this ratio, the connexions shown in fig. 1 were somewhat modified, the arrangement finally adopted being that shown in fig. 3.

One pole of the battery is connected by means of a slider D with some point of the wire EF , while the other can be connected at pleasure, on the one hand, with either of the electrodes A and B upon the disk by pressing one of the contact-keys 1 and 2, or, on the other hand, with another slider H on the wire EF by pressing the key 3. The galvanometer is permanently connected with one end E of the wire, and with a fixed point K between the sliders D and H . The part of the graduated wire between the galvanometer-terminals E and K thus takes the place of the whole length EF in the arrangement shown in fig. 1. To measure the resistance between A and B in terms of that of the graduated wire, it is then only necessary to

Fig. 3.

press successively the keys 1 and 2, and each time to adjust the slider D so as to give no deflection on the galvanometer, and then to connect the battery with H , by the key 3, and to find the positions of D required to keep the galvanometer at zero for two positions of H separated by the known amount h . If we denote by s the distance through which D has to be shifted on the wire EK when the battery-contact on the disk is transferred from A to B , and by d the shift of D corresponding to the shift of H over the distance h , we have

$$\frac{\text{resistance between } A \text{ and } B}{\text{resistance of } s} = \frac{\text{resistance of } h}{\text{resistance of } d};$$

and since s , h , and d are all of them lengths measured on the same wire, their resistances are simply as their lengths. Hence, taking as unit of resistance the resistance of unit length of the graduated wire, we have

$$\text{resistance A to B} = s \frac{h}{d}.$$

It will be observed that the ratio $\frac{h}{d}$, which we will hereafter denote by $\frac{1}{\epsilon}$, is the same as that of the resistances E A B K and E D K.

51. In all measurements of resistance it is necessary, not only that the contact of the electrodes A and B with the disk should be constant and easily reproduced, but also that the area of contact should be measurable; the difficulty of realizing these conditions with accuracy is by far the most important source of error to which the determinations of resistance are liable. In order to obtain a measurable and constant surface of contact, the spring contact-pieces employed in the experiments for plotting the equipotential lines were replaced by the following arrangement. A thin cylindrical rod of brass, to which two wires could be attached, slid vertically in a V-groove down the side of a rectangular wooden block. The lower end of the rod was filed accurately flat and perpendicular to its length, so as to touch the tin-foil with its entire surface. Constant pressure against the tin-foil was maintained by placing a weight on a cap fixed to its upper end. Two such rods constituted the electrodes A and B; and tolerably constant contacts could be made with them after a little practice. Nevertheless individual readings are apt to vary a good deal, and it is necessary to take the mean of several distinct experiments in order to get a single trustworthy result.

The numbers given in the annexed Table will serve to illustrate the variations occurring among single readings. They refer to a set of experiments in which the electrodes were placed approximately at the ends of a diameter of a circular disk.

	$2a.$	$s.$
	26.30	69.4
	26.22	69.7
	26.30	68.3
	26.30	68.3
	26.00	67.5
	26.31	68.7
	25.95	69.0
	26.25	68.6
	26.30	69.8
	26.00	69.4
	26.25	67.9
	26.31	68.7
	26.05	69.2
Means	26.20	68.80

52. The principal results of the measurements of resistance made by the method explained above are given in the Tables which follow, in such a way as to show how far they agree with the values indicated by theory. Every entry in the Tables is a mean value deduced from the number of experiments stated in the first column; the last column in each Table contains a quotient which theoretically should be constant. Common logarithms are employed, except when it is otherwise stated. The symbols used in the headings of the columns have the following significations:—

R , resistance in terms of an arbitrary unit ($=.00155$ ohm; being the resistance of one centimetre of the wire EF).

x , resistance of the contacts between the electrodes and the sheet (found by special experiments).

ϵ , ratio of the resistances EDF and $EABF$ (figs. 1 or 3), or $\frac{d}{h}$ (§ 50).

s , shift of the slider D .

$2a$, or AB , distance between the poles.

ρ , radius of the circle of contact of the electrodes, or half the diameter of the brass rods.

r , radius of the tinfoil disk.

δ , thickness of the disk.

κ , specific conductivity of the material of disk.

L , the expression $\log_{10} \frac{AG \cdot BF \cdot A'G \cdot B'F}{\rho^2 \cdot A'F \cdot B'G}$ (see equation (20a), § 39, Part I.).

C , the constant $\frac{\log_e 10}{2\pi\kappa\delta}$.

The following relations hold good among the above quantities

$$R + x = \frac{s}{\epsilon} \quad (\S 50)$$

and

$$R = CL \quad (\S\S 38 \text{ and } 39, \text{ Part I.}),$$

whence

$$s - \epsilon x = \epsilon CL.$$

First set of Resistance experiments, April 1874.

ϵ was practically constant and $= 5.147$, $\rho = .163$, $\epsilon x = 2.42$.

Case 1. Two equal electrodes on the edge of a circular disk (Plate IV.).

Number of experiments.	r .	$2a$.	s .	$\frac{s - \epsilon x}{\log \frac{2a}{\rho}}$
13	13.16	26.20	68.80	30.1
13		18.83	65.42	30.5
9		15.465	62.08	30.0
12		13.466	59.47	29.8
16		10.185	57.05	30.6
17	7.62	15.224	61.66	30.1
6		11.675	58.02	30.0
10		10.35	57.20	30.4
5		8.78	56.60	31.3
Weighted mean.....				30.29

In this case $L = 4 \log \frac{2a}{\rho}$ by formula (13), $\S\S 25 \text{ \& } 39$, Part I., and therefore $\frac{s - \epsilon x}{\log \frac{2a}{\rho}} = 4C\epsilon = \text{const.}$: the mean value 30.29 gives $2C = 2.943$.

Case 2. Two equal electrodes, one at the centre and one on the edge of a circular disk.

Number of experiments.	r .	s .	$\frac{s - \epsilon x}{\log \frac{r}{\rho}}$
8	13.16	46.47	23.10
7	7.62	41.81	23.60
Mean.....			23.33

Here $L = 3 \log \frac{r}{\rho}$ by formula (21), § 39, Part I., and therefore $\frac{s - \epsilon x}{\log \frac{r}{\rho}} = 3C\epsilon = 23.33$, whence $2C = 3.022$.

The lines of flow and of equal potential for this case are shown in fig. 6, Plate V., by the part of the figure inside the circle. This figure represents the case of two opposite poles in an unlimited sheet, one of them being twice as strong as the other. One of the flow-lines is then a circle passing through the pole of double strength and having the other pole at its centre. The remaining flow-lines lie half of them within this circle and half of them outside. Consequently, if the sheet be cut along the circumference of this circle (which in the figure is shown as complete), a disk is obtained in which the electrical conditions are the same as in this experiment.

Case 3. Both electrodes in the disk, one on each side the centre and at equal distances from it.

Number of experiments.	r.	AB.	s.	$\frac{s - \epsilon x}{\log \frac{5 \cdot AB}{3 \cdot \rho}}$
4	13.16	13.16	36.92	16.20
5	7.62	7.62	32.78	16.05
				Mean.....16.12

Here L becomes $2 \log \frac{5 \cdot AB}{3 \cdot \rho}$, whence $\frac{s - \epsilon x}{\log \frac{5 \cdot AB}{3 \cdot \rho}} = 2C\epsilon$, and $2C = 3.132$.

Second set of Resistance experiments, April 1875.

Case 1. Both poles on the edge of the circular disk.

$r = 13.43$ centims., $\rho = .163$ centim., $x = .471$.

Number of experiments.	2a.	s.	ϵ .	$R = \frac{s}{\epsilon} - x$.	$\frac{R}{\log \frac{2a}{\rho}}$
8	26.764	13.03	.9705	12.95	5.85
6	26.773	13.23	.9700	13.07	5.90
				Mean ...	5.87

$L = 4 \log \frac{2a}{\rho}$; $\frac{R}{\log \frac{2a}{\rho}} = 4C$; $2C = 2.935$.

Case 2. One pole at the centre, the other on the edge (as at *a*, fig. 4).

$$r = 13.43 \text{ centims.}, \rho = .163 \text{ centim.}, x = .425.$$

Number of experiments.	AB = r .	s .	ϵ .	$R = \frac{s}{\epsilon} - x$.	$\frac{R}{\log \frac{r}{\rho}}$.
3	13.43	9.37	1.035	8.63	4.504

$$L = 3 \log \frac{r}{\rho}; \quad \frac{R}{\log \frac{r}{\rho}} = 3C; \quad 2C = 3.003.$$

Case 3. Both poles inside the disk at distances CA, CB respectively from its centre.

$$r = 13.43 \text{ centims.}, \rho = .163 \text{ centim.}, x = .38.$$

AB.	CA.	CB.	s .	ϵ .	$R = \frac{s}{\epsilon} - x$.	$\frac{R}{L}$.
13.85	5.55	9.1	7.97	1.0595	7.14	1.63
20.00	10.9	9.1	8.94	1.0395	8.22	1.55
12.20	10.9	4.45	7.64	1.0625	6.81	1.58
7.25	4.5	3.62	6.45	1.0845	5.57	1.63
7.00	7.2	9.0	6.50	1.0850	5.62	1.61
9.00	0.0	9.0	7.17	1.0735	6.30	1.68
12.08	7.7	4.45	7.59	1.0650	6.75	1.64
15.12	7.7	7.7	8.04	1.0615	7.20	1.59
					Mean	1.615

The flow-lines for this case are indicated in Plate V. fig. 9, where the circle with centre C corresponds with the edge of the disk. In order to get the theoretical resistance for this arrangement, additional poles, of the same signs as A and B respectively, must be assumed at the points A' and B'; so that, if the conducting sheet were unlimited, the circle with centre C would coincide with a line of flow: that is to say, A' and B' must be on the prolongations of the radii through A and B respectively, and at the following distances from the centre, $CA' = \frac{r^2}{CA}$, $CB' = \frac{r^2}{CB}$ (see Part I. §§ 36 and 38). Taking

A and B as the middle points of the circles of contact made by the two electrodes, we have (*loc. cit.*)

$$L = \log \frac{(AB)^2 \cdot AB' \cdot A'B}{\rho^2 \cdot AA' \cdot BB'}; \quad \frac{R}{L} = C; \quad 2C = 3.23.$$

The values of AB' and $A'B$ in the above expression were found by laying down on paper, to any convenient scale, the actual positions of the poles from the data AB , CA , CB , CA' , CB' , and then measuring the distances AB' and $A'B$. This is a quicker process than the corresponding calculation.

Case 4. One electrode at the centre, the other *touching* Fig. 4. the edge (as at *b*, fig. 4).

$$r = 13.43 \text{ centims.}, \quad \rho = .163 \text{ centim.}, \quad x = .38.$$

Number of experiments.	$AB = r - \rho$	s .	ϵ .	$R = \frac{s}{\epsilon} - x$.	$\frac{R}{\log \frac{r - \rho}{\sqrt[3]{3} \cdot \rho}}$
7	13.27	9.13	1.038	8.41	4.80

The poles being assumed to be the centres of the circles of contact made by the electrodes,

$$L \text{ becomes } 3 \log \frac{r - \rho}{\sqrt[3]{3} \cdot \rho}; \quad \frac{R}{\log \frac{r - \rho}{\sqrt[3]{3} \cdot \rho}} = 3C; \quad 2C = 3.20.$$

Case 5. Both electrodes *touching* the edge.

$$r = 13.43 \text{ centims.}, \quad \rho = .163 \text{ centim.}, \quad x = .38.$$

Number of experiments.	AB.	s .	ϵ .	$R = \frac{s}{\epsilon} - x$.	$\frac{R}{\log \frac{AB}{\sqrt[3]{3} \cdot \rho}}$
3	24.9	12.42	.983	12.25	6.30
1	16.8	11.58	.996	11.25	6.34
2	8.88	9.985	1.0065	9.54	6.36
2	10.68	10.59	1.016	10.043	6.36
2	22.0	12.34	.988	12.11	6.40
5	26.56	13.35	.969	13.40	6.79
Mean...					6.49

In this case, A and B being again taken as the middle points of the circular electrodes,

$$L \text{ becomes } 4 \log \frac{AB}{\sqrt{3} \cdot \rho}; \quad \frac{R}{\log \frac{AB}{\sqrt{3} \cdot \rho}} = 4C; \quad 2C = 3.245.$$

$$53. \text{ Summary of mean values of } 2C = \frac{\log_e 10}{\pi \kappa \delta}.$$

Date	1874.		1875.		Mean.
	No. of experiments.	2C.	No. of experiments.	2C.	
Case 1. Both poles on the edge	101	2.943	14	2.935	2.94
Case 2. One in centre and one on the edge					
Case 3. Both inside the disk					
Case 4. One in centre and one touching the edge			7	3.20	3.20
Case 5. Both touching the edge			15	3.245	3.245

It will be seen by this Table that the values of C deduced from the several sets of experiments are not quite constant. In explanation of the discrepancies, it is to be observed that the equipotential lines determined by the contact of the electrodes with the tinfoil did not agree quite strictly with equipotential lines due to the arrangement of poles assumed in calculating the values of L, except in the experiments (distinguished as Case 1) where both the electrodes were on the edge of the disk. The differences between the results of these experiments and those with other arrangements of the electrodes are in the direction which would be caused by the actual departure from the theoretical conditions.

The discrepancy is greatest in Case 5, where both the electrodes *touch* the edge of the disk. In this case the additional poles A' and B', required in an imaginary unlimited continuation of the conducting sheet in order to make the circumference of the disk a line of flow, would be points outside the circumference, but very near to it and to A and B respectively. If, for

an instant, we regard the electrodes as mere points whose distance from the circumference of the disk is very small compared with its radius or with their distance from each other, the equipotential lines for a short distance from A and B will be very nearly indeed a system of lemniscates (see Part I. § 36, at the beginning)—the node of the self-cutting curve, or lemniscate proper, being sensibly on the circumference. The circle of contact formed by each electrode may be regarded as approximately representing one branch of such a lemniscate, if, instead of taking its centre for the actual pole (A or B), we take a point nearer the centre of the disk by $(\sqrt{2}-1)\rho$, where ρ is the radius of the electrode. A recalculation of the experiments of Case 5 in this way reduces the mean value of $2C$ from 3.245 to 3.11; and a similar correction applied to Case 4 gives $2C=3.10$ instead of 3.20. Thus corrected, the results of these experiments agree fairly with those of Case 1.

54. Taking the mean value of $2C$ obtained with electrodes on the edge of the disk and multiplying it by $\frac{\pi}{\log_e 10}$, we get

$$4.011,$$

which ought to be the resistance of a square centimetre of the tinfoil used. For comparison with the value so found, the same quantity was determined by measuring the resistance of an oblong strip of tinfoil, the electrodes being flat pieces of brass, firmly clamped down on the ends of the strip, and making contact with its entire width. Five measurements, made with different lengths of a strip 3 centims. broad, gave as the resistance of a square centimetre

$$4.032,$$

the unit being, as before (see § 52), 0.00155 ohm; and five measurements, with a strip 1.5 centim. broad, gave

$$4.029.$$

The mean of these determinations is

$$4.031,$$

which agrees closely with the value calculated from the experiments with the disks.

55. It seems proper to point out in conclusion, that Professor J. Clerk Maxwell indicated so long ago as 1856 (Brit.-Assoc. Report, 1856, Trans. of Sect. p. 12; see also, for fuller developments, Camb. Phil. Trans. vol. x. p. 27) the fundamental idea of the method of treating questions relating to the flow of electricity in conductors of more than one dimension which was used in the first Part of this paper,—and also to mention that problems presented by finite plane conductors, in cases which become unmanageable by the simple geometrical processes employed by us (as, for instance, the case of rectangular conducting sheets), have been solved by Jochmann (Schlömilch's *Zeitschr. f. Mathemat.* 1865, p. 48) by a method founded on the superposition of the effects of separate poles, and by Heine (Crelle's *Journal*, 1874, vol. lxxix. p. 1) by an application of the geometrical principle of point-point correspondence (*Abbildung*) to Kirchhoff's results.