The Forward Problem Study of EIT on Curved Surface

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***Abstract*—In some applications as head electrical impedance tomography (EIT), the 3 dimension (3D) electrodes array is difficult to arrange for the surface unflatness, and the 2 dimension (2D) EIT can only obtain the flat surface impedance information where the electrodes arrange. The EIT on curved surface can obtain partial 3D EIT effect based on 2D electrodes distribution and algorithm complexity. For the 2D manifold is equivalent with the curved surface, the Maxwell's equations in 2D manifold are deduced to gain the EIT model on curved surface. Then, according to the structure of the EIT model on curved surface, the finite volume method (FVM) is adopted to solve the forward problem. Lastly, we utilize the multi-grid method (MGM) to accelerate the solution procedure.The tests reveal the forward problem solution accuracy on curved surface approximates to the flat surface. The EIT on curved surface greatly improves the flexibility of EIT application.**

I. INTRODUCTION

The EIT is to gain the internal conductivity distribution of a bounded region based on the measurement of the surface voltages when injecting the currents into the target region [1]. The image reconstruction process of EIT demands for calculating forward problem and inverse problem. The definition of the forward problem is solving the potential distribution of the target region by known internal conductivity distribution and stimulation currents. The inverse problem is to calculate the conductivity distribution through the finite boundary measured voltages and stimulation currents. The inverse problem is nonlinear, ill-conditioned and underdetermined, which either does not admit an unique solution or its results does not depend continuously upon the data. The researchers in EIT solve the forward problem by utilizing the FEM mainly, which can be used to process the forward problem in 2D and 3D by triangle and tetrahedron mesh.

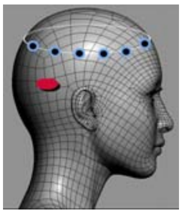

Figure 1. Sketch of the EIT for the head.

(*The red point denotes the foreign body, and the black points represent the electrodes)

EIT modeling is restricted by the arrangement of electrodes on the interface of the measured object. For example, in the EIT of the head, the electrodes are only convenient to put on the horizontal plane of the forehead. Hence, it is difficult to gain other EIT images except the 2D cross section image of the forehead. As shown in Fig.1, the location of the foreign body is under the cross section, and it is difficult to show it in EIT 2D images, and is also difficult to place several rings electrodes to reconstruct 3D images.

In this paper, we proposed an EIT curved surface model, which implement the local image reconstruction of 3D by the 2D electrodes distribution and algorithm complexity. The Maxwell's equations are generalized to curved surface to construct physics model [1, 2]. Because the FEM is not adaptive on the curved surface, the FVM is adopted, which is one of the numerical methodologies in computational fluid dynamic problems [3]. We generalize the FVM on curved surface to solve the EIT forward problem [2]. The comparisons of EIT modeling and forward problem solution between flat surface and curved surface are discussed. Finally, the analysis of FVM algorithm's accuracy and complexity for the model is also accomplished.

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II. EIT FORWARD PROBLEM ON CURVED SURFACE

Manifold is the space with local characteristic of Euclidean space [4]. The 3D curved surface is equal to 2D manifold. The Maxwell's equations and FVM are generalized on manifold [1, 2, 4, 5]. We deduce them in the 3D curved surface, equal to 2D manifold. Furthermore, the EIT curved surface model is constructed and the corresponding forward problem is solved.

A. Maxwell's Equations Generalization

In the manifold, we introduce the Hodge star operator to replace the differential operator, which calculate the dual part of the target. Furthermore, we utilize the Hodge star operator to generalize the EIT differential equation.

$$\Omega : \nabla \bullet \sigma \nabla \varphi = 0 \tag{1}$$

$$\Gamma_1 : \varphi = \varphi_0 \tag{2}$$

$$\Gamma_2 : \sigma \frac{\partial \varphi}{\partial n} = -J_n \tag{3}$$

The differential equation on 3D curved surface, with the identical nature of 2D manifold, can be expressed as:

$$*d(\sigma * d\varphi) = 0 \tag{4}$$

Operating surface integration, we obtain

$$\iint_G * d(\sigma * d\varphi) = \iint_G 0 = 0 \tag{5}$$

The result of integration and dual operation to 0 is still 0. By using Green formula

$$\int_{\partial G} \sigma * d\varphi = 0 \tag{6}$$

Lastly, the EIT difference equation on curved surface is deduced as:

$$\sum_i \sigma \bullet \frac{\varphi(i) - \varphi(0)}{l_{i0}} \bullet (*l_{i0}) = 0 \tag{7}$$

where $*l_{i0}$ means get the dual part of the l_{i0}, which is the line segment vertical with l_{i0} and belonging to the edge of the dual cell. The structure of the derivation result approximates to the FVM difference equation. Therefore, it's the ultimate reason deciding the FVM but FEM adopted.

B. FVM Generalization

On the 3D curved surface, the triangles are not located in the horizon plane, as shown in Fig.2.

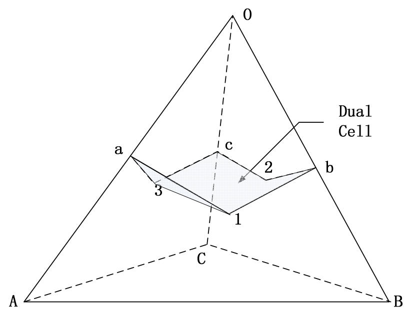

Figure 2. Illustration of internal node mesh on curved surface.

As the processing as flat surface, we cross the perpendicular bisectors of each side to find circumcentres. The distinctiveness of the curved surface exists on which circumcentres are located in the different flat surfaces. We deduce FVM directly without casting the projection of the curved surface previously.

We define

$$l_{12} = l_{1b} + l_{2b}, l_{23} = l_{2c} + l_{3c}, l_{31} = l_{3a} + l_{1a} \tag{8}$$

$$P_{123} = A_{01ab} + A_{02bc} + A_{03ac} \tag{9}$$

Considering the EIT difference equation on curved surface,

$$\sum_i \sigma \bullet \frac{\varphi(i) - \varphi(0)}{l_{i0}} \bullet (*l_{i0}) = 0 \tag{10}$$

where $*l_{i0}$, the dual part of the l_{i0}, corresponds to the l_{12}, l_{23}, l_{31}. We have

$$\sigma(\varphi_A - \varphi_0)\frac{l_{31}}{l_{A0}} + \sigma(\varphi_B - \varphi_0)\frac{l_{12}}{l_{B0}} + \sigma(\varphi_C - \varphi_0)\frac{l_{13}}{l_{C0}} + P_{123} \bullet R_{G_0}(\varphi) = 0 \tag{11}$$

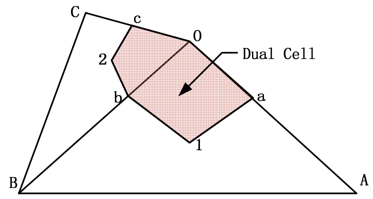

Figure 3. Illustration of boundary node mesh on curved surface.

As shown in Fig.3, if the boundary node satisfies boundary condition (2), we can gain

$$\varphi_{P_0} = \varphi(P_0) \tag{12}$$

As to the boundary condition (3), we can deduce the boundary difference equation as:

$$\int_{\Gamma_{A0}} \sigma \frac{\partial \varphi}{\partial n} + \sigma(\varphi_B - \varphi_0)\frac{l_{12}}{l_{B0}} + \int_{\Gamma_{C0}} \sigma \frac{\partial \varphi}{\partial n} = 0 \tag{13}$$

The electric potential of node *B* satisfies:

$$\sigma(\varphi_B - \varphi_0)\frac{l_{12}}{l_{B0}} = \frac{1}{2} J_n (l_{A0} + l_{C0}) \tag{14}$$

We generalize the differential equation on manifold to get the EIT difference equations on curved surface. As the above equations show, the EIT difference equations in both flat and curved surface are similar, so the good properties, such as symmetry and sparsity, of coefficient matrix are reserved.

For the sake of improving the computation efficiency, the Multi-grid Method (MGM) is introduced to EIT in [6]. The finite element equation solution in the finest grid is transformed to the simpler solution in the coarse grid, decreasing the algorithm computation. The algorithm includes steps of smooth, reminder restrain, solution and correction, shown in the Fig.4.

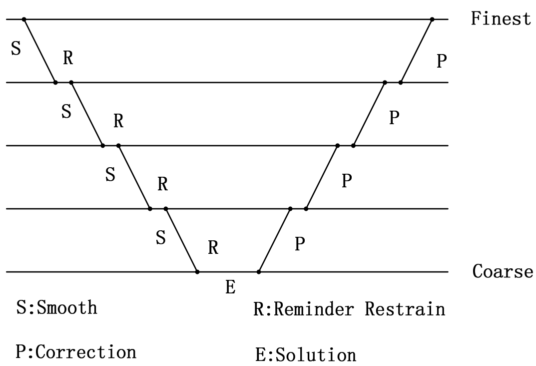

Figure 4. Illustration of MGM procedure.

III. PERFORMANCE EVALUATION

A. Accuracy

The EIT forward problem solution for hemispherical model is shown in Fig.5. Compared with the basic FEM solution for circular face in Fig.6, we can conclude that the FVM accuracy approximates to the FEM. In fact, it was proved that if the step size between the nodes is small enough, the error estimations of both FEM and FVM are unification [7]. The error estimations can be expressed as:

$$|\varphi-\varphi_h\|_1=\|e\|_1\leq Ch\,|\varphi|_2 \quad (15)$$

where h denotes the step size between the nodes, and $|\varphi|_2$ represents the L_2 modulus of the second-order partial derivative.

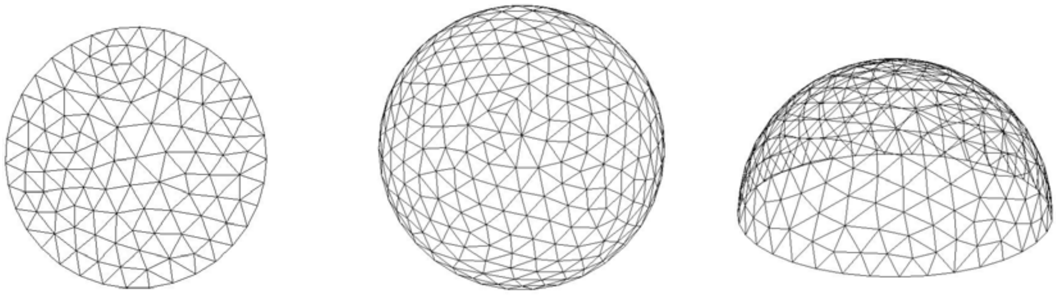

Figure 5. 2D round surface mesh, 3D hemisphere surface mesh (90 degrees view), 3D hemisphere surface mesh (45 degrees view) (from the left to right).

Since in the general EIT, the forward problem solution is calculated once previously, the advantage in calculation complexity is hard to expose. But if involving with the dynamic forward problem, which eliminates the negative effect of the body shape change, it is useful. Besides, [8] proves the FVM can gain accurate result even in coarse mesh.

IV. CONCLUSION

We study the EIT forward problem in curved surface. The EIT curved surface model is proposed, and the FVM is utilized to solve the equation with MGM acceleration.The manifold principles are used to derive the EIT model on curved surface for the 2D manifold equivalent with 3D curved surface. For the structure of derivation result is similar with the FVM difference equation, we generalize the FVM on curved surface to solve the forward problem of the EIT model. The evaluation results show that the EIT model can be structured on curved surface successfully. It's useful for EIT because EIT on curved surface can obtain the local 3D image reconstruction effect by 2D EIT cost.

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