# Minimizing computational costs in large scale 3D EIT by using sparse Jacobian matrix and parallel CGLS reconstruction

*C Yang, M Soleimani*

*Department of Electronic and Electrical Engineering, University of Bath, UK*

**Abstract:** Electrical impedance tomography (EIT) is an imaging technique for detecting the conductivity distribution inside an electrodes surrounding system by using exterior voltage measurements. This paper describes the basic content of three dimensional EIT and introduces a newly both time and memory efficient method on solving large scale 3D EIT problems. The block-wise conjugate gradient method can make a contribution on reducing the computational time and memory storage for 3D EIT problem. This will effectively be a matrix free approach to the EIT imaging. The block-wise implementation allows an easy parallelisation for multiple CPU reconstruction which has been used in this study. Furthermore, sparse matrix reduction technique is an interesting method of minimizing the storage size of the Jacobian matrix. By making the jacobian matrix into sparse format, elements with zeros would be eliminated by sparse storage method, which results in saving memory space. Matlab simulations have been done to demonstrate the improvement.

**Keywords:** Electrical impedance tomography, large scale inverse problem, conjugate gradient method

## 1. Introduction

Electrical impedance tomography (EIT) is one of the emerging imaging techniques applied for both medical and industrial imaging. The aim of EIT is that to reconstruct an object image by producing the electrical conductivity distribution inside a conductive region. A typical EIT system includes a voltage measurement unit and an EIT sensor surrounding by conducting electrodes. In EIT measurement, set of small signal currents are applied to pair of electrodes, then the resulting voltages are measured. The control computer is then used to calculate the conductivity distribution from the voltage measurement which is called the inverse problem of EIT. Two dimensional EIT has been well developed as a fast and simple reconstruction method in the field. However, three dimensional EIT appeared to be more attractive and challenging since it can be more approached to the clinical applications. In this paper, only three dimensional EIT is taken into consideration. More researchers are considering large number of electrodes, which results in large number of measured data. Large number of data could be generated with multiple frequency or time series EIT data acquisition, which is gaining momentum in the field. Limitations appear to the traditional back projection or Tikhnonov regularised reconstruction algorithm when facing large scale 3D EIT problem as huge information needed to be stored and calculation of Jacobian matrix becomes unfeasible. In this paper, a reconstruction technique combining Block-wise conjugate gradient method and sparse matrix simplification is developed to overcome these problems. The new combined reconstruction technique has significant advantages on the computational time of large scale EIT reconstruction without losing too much image quality. An image, or solution, is obtained by repeatedly running a forward model and an inverse solver until convergence is attained. The forward problem of EIT is to determine the voltage distribution arising from the current pattern injection onto the object. To generate EIT result images, the forward problem needs to be solved and this can be performed by using a finite element method [5], the forward problem forms a sparse matrix as well that can be solved through either direct or iterative methods effectively [1,3]. Efficient calculation of the forward problem, although important, is beyond scope of this study. One of the most typical regularization schemes would be Tikhonov regularization. When dealing with problems that include large number of pixels but small amount of measurements, the Weiner filter can be used. However, when the 3D

EIT problem comes to both large number of pixels and measurements, the Weiner filter technique cannot be applied due to memory problem. Block-wise CGLS is suitable and convenient to overcome the problems at the stage.

## 2. Sparse Jacobian

For this study we are assuming that a large scale EIT system consists of total 128 electrodes with 4 planes 32 channels. The number of measurements taken would then be 128X125=16,000 or 8,000 independent measurements. In order to generate mesh data for accurate forward modelling, the element number could be as large as 100,000. This implies that the Jacobian matrix would be in a size of approximately 16,000 X 100,000. This will require huge memory space for storing and the inversion of the Jacobian has become unfeasible. Therefore, any work can be done to reduce the size of the Jacobian without losing accuracy of the image will be vital. Several space reduction methods can be used to reduce the size of inverse matrix, such as spherical harmonic methods, level set method and etc. In this paper we present an efficient Matrix modification technique called sparse matrix. Due to the nature of EIT, only the regions near the object would have obvious conductivity difference $\Delta\sigma$. From the sensitivity map, one can be observed that most regions far away from the source have small change in conductivity. Measurements from those regions are appeared to be small or zero in the sensitivity matrix. These low sensitivity values will be removed with the act of regularisation effect. However, the near zero elements are still remaining in the matrix which consumes memory storage and being involved in the inverse calculation. Hence sparse matrix reduction method indicates that values which are relatively small, typically below a certain threshold can be located and transformed to zeros [2]. These zero elements are then eliminated from the Jacobian matrix. This effect would decrease the total number of non-zeros of the Jacobian matrix hence reduce the memory storage. As there is no significant effect on the sensitive region, the image quality would remain.

$$J_k = \begin{cases} J_{ij} & if \quad \sum_{i=1}^{nm} J_{ij} \geq threshold \\ 0 & if \quad \sum_{i=1}^{nm} J_{ij} < threshold \end{cases}$$

Where i,*j* corresponded to measurement and the element number within the domain. We can estimate the threshold to be a small percentage of an averaging of top values. The Jacobian matrix $J_k$ is now having large amount of zero values. Where the conductivity of one element is equal to zero, the entire column corresponded to that node is removed from the Jacobian, thus reducing the size of Jacobian matrix [2]. In this study, we are making the threshold that J=0 if abs(J)<0.002 x max(abs(J)), which gives number of non-zero elements in Jacobian to approximately 35% of the total number of entries. If we set threshold to higher than that the image quality may go down. Further analysis needs to be done for suitable thresholding.

The theresholding here means that we keep all the voxels in the image reconstruction process; they may set to zero for one particular combination of the measurements and then become useful with another set of measured data. This could be particular more relevant in 3D imaging when dealing with multiple layer of electrodes. One way to have a more uniform sensitivity would be to have multiple excitation patterns, which could significantly increase the complexity of the EIT hardware, especially when large number of electrodes is involved.

## 3. Block-wise matrix vector multiplication for CGLS

Conjugate gradient (CG) methods and in particular regularised vision of that has been proposed as a suitable method for large scale EIT imaging problem [6]. CG methods are well suited for sparse matrix calculation, which is the case for our proposed matrix reduction scheme presented in section 2. Tikhonov regularisation can be applied to the Jacobian by adding regularisation term $\begin{bmatrix} J \\ \alpha I \end{bmatrix} \mathrm{x} = \begin{bmatrix} b \\ 0 \end{bmatrix}$ Where $\alpha$ is the regularisation parameter and I is identity matrix. The current computer storage capacity and in particular in Matlab environment makes it very difficult to store Jacobian matrix *J* in large scale 3D EIT problem. For large data set and large number of pixels even the sparse Jacobian cannot be stored. Block-wise matrix method suggests that we separate the jacobian *J* into blocks, and solve the forward problem individually as CGLS does not require access to the full matrix *J*. Therefore all that is needed would be a simple matrix vector multiplication with each of *J* and *J* per iteration. The matrix J can be divided in to $l+1$ blocks

$$J_k = \begin{bmatrix} J_1 \\ J_2 \\ \ldots \\ J_l \\ J_{l+1} \end{bmatrix}$$

Where $k$ = 1 , 2 , 3 … l , l+1 , each block $J_k$ has a dimension of m/l by n and $J_{l+1}$ is the *n* x *n* regularised identity matrix used for Tikhonov regularisation. It is well known in the CGLS, two matrix multiplications are mainly considered which are $Jp$ multiplication and $J^T r$ multiplication [4]. Therefore, the $Jp$ multiplication can be done sequential, where *P* is a vector with size *n* x 1. Similarly, in the $J^T r$ multiplication, r has a size of 1x (*m* + *n*) which can be partitioned into *l* + 1 blocks, where each block has a dimension of $(m+n)/l$. As a result, $J^T r$ can be expressed as the sum of all $J_k^T r_k$ blocks, $J^T r = \sum_{k=1}^{l+1} J_k^T r_k$ . Each block-wise sensitivity matrix $J_k$ can be loaded at the same time and matrix multiplication steps can be done in parallel by using computers with multiple cores, this would improve the reconstruction speed more significantly.

## 4. Results and discussions

The simulations are done by using a laptop with Intel i7-2720QM CPU and 4GB ram. For 128 electrodes model, the Jacobian matrix has been divided into 80 blocks, and additional block for the identity matrix (used for regularisation here). Four parallel CPU was used for the calculation in this study. All reconstructions are done in parallel execution. The execution time of 128 electrodes system using sparse BW-CGLS is 522 seconds for 100 iterations, while it shows "out of memory" with our Weiner filter algorithm and standard Tiknonov regularised methods. The result shows when dealing with 128 electrodes EIT system, the sparse Jacobian CGLS method has significantly reduced the programming time comparing to the traditional methods. The number of voxels in inverse problem is 22,000 elements. To avoid the inverse crime the forward model is generated with a mesh with 170,000 elements. If more CPU blocks are available or in a GPU environment this calculations can be speed up more significantly.

Currently we are setting up a 32 EIT system to validate these algorithms experimentally. The block-wise CGLS has already been successfully tested with real magnetic induction tomography data in [4], which gives the confidence that it will work for the EIT. We have not yet tested the sparse Jacobian approach experimentally, so we cannot comment, how it will work with real data. Image analysis measures will be used to investigate what level of the theresholding can be used and how it will affect the image qualities. The excitation pattern will affect our ability to increase the reduction of the Jacobian matrix. In this study an adjacent pattern has been used.

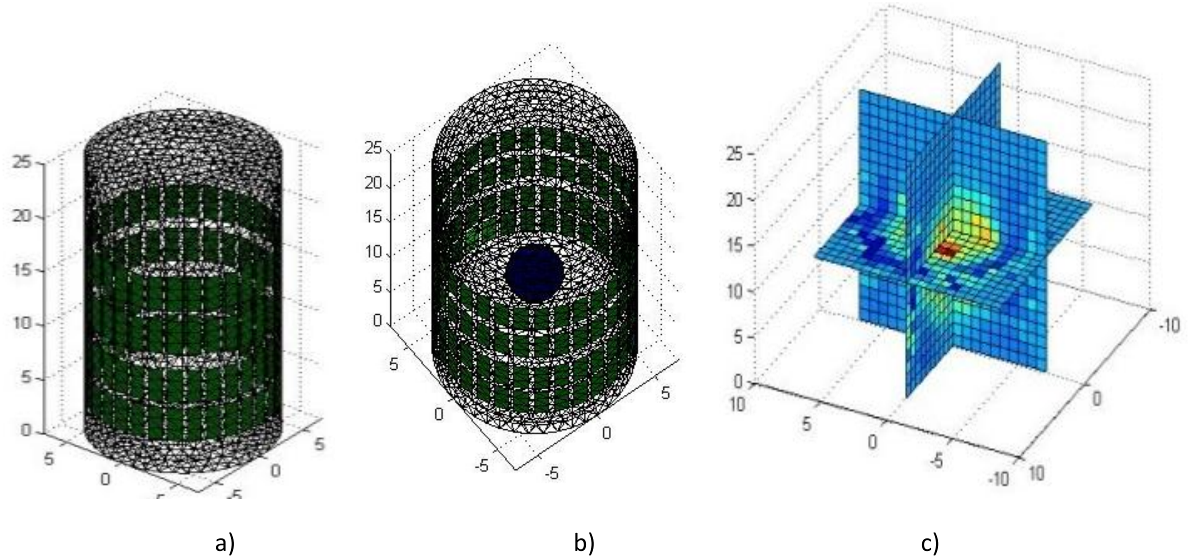


Figure a)Finite element mesh of 128 electrodes EIT system b) FEM of 128 electrodes EIT system with a sphere in the centre . c) Reconstruction of 128 electrodes EIT system with CGLS reconstruction algorithm (100 iterations).

## 5. Conclusion

The main challenges of 3D EIT are the computational aspects including memory issues and execution time. Simple thresholding of the Jacobian matrix here works on the basis that we keep all voxels in the image reconstruction process, but we set them to zero when a particular measurement is not sensitive enough on that region. Some traditional reconstruction methods are suitable when dealing with typical systems with small amount of measurements data or small number of voxels in inverse problem. While by making good use of sparse Jacobian technique, combining the method of partitioning the Jacobian into blocks, a personal computer is now efficient enough to produce satisfying results for large scale EIT system. This could be a suitable alternative for large scale EIT problem. We are developing 3D EIT system and the experimental validation of the proposed algorithms is underway.

### References

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