A Bayesian approach to Cole – Cole parameter estimation with multi-frequency EIT

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In this work, reconstruction methods for multi-frequency electrical impedance tomography (MF-EIT) imaging are developed. In MF-EIT, the aim is to image a target based on a set of complex valued boundary voltage measurements corresponding to current injections with multiple frequencies. We model the frequency dependence of the electrical admittivity with the Cole-Cole equation [3]

$$\gamma(\omega, r) = \sigma_\infty(r) + \frac{\sigma_0(r) - \sigma_\infty(r)}{1 + (\mathbf{j}\,\frac{\omega}{\omega_c(r)})^{\alpha(r)}}, \quad r \in \Omega \tag{1}$$

where $\sigma_0(r)$, $\sigma_\infty(r)$, $\omega_c(r)$ and $\alpha(r)$ are spatially distributed material specific parameters and ω is the angular frequency of the time-harmonic electric field. We adopt the observation model of MF-EIT written by Brandstätter et. al. [1, 2]

$$V_i = U(\theta; \omega_i) + n_i, \quad i = 1, \ldots, N \tag{2}$$

where V_i and n_i are vectors of voltage measurements and measurement noise corresponding to frequency ω_i, and N is the number of frequencies. The parameter vector θ is defined as $\theta = (\sigma_\infty, \sigma_s, \alpha, \omega_c)^{\mathrm{T}}$ where vectors $\sigma_\infty, \sigma_s, \alpha$ and ω_c denote the finite dimensional representations of the corresponding distributed parameters. The mapping $U = U(\theta; \omega)$ is a combined function $U(\theta; \omega) = R(\gamma) \circ \gamma(\theta, \omega)$ where $R(\gamma)$ results from the finite element (FE) approximation of the electrode model of EIT and $\gamma = \gamma(\theta, \omega)$ is the spatially discretized counterpart of the Cole – Cole model (1).

We write the inverse problem of MF-EIT in the Bayesian framework, and compute the *maximum a posteriori* (MAP) estimates based on the observation model (2) and statistical *a priori* models of the distributed parameters $\sigma_\infty, \sigma_s, \alpha$ and ω_c. In cases of Gaussian models, $n_i \sim \mathcal{N}(0, \Gamma_{n_i})$, $\theta \sim \mathcal{N}(\theta_*, \Gamma_\theta)$, the MAP estimate can be written in the form

$$\theta_{\mathrm{MAP}} = \arg\min_\theta \Big\{ \sum_{i=1}^{N} \| L_{n_i} (V_i - U(\theta; \omega_i)) \|^2 + \| L_\theta (\theta - \theta_*) \|^2 \Big\}. \tag{3}$$

where the matrices L_{n_i} and L_θ are defined as the Cholesky factors of the precision matrices, i.e. $L_{n_i}^{\mathrm{T}} L_{n_i} = \Gamma_{n_i}^{-1}$, $L_\theta^{\mathrm{T}} L_\theta = \Gamma_\theta^{-1}$, and θ_* is the expectation of θ. We test the feasibility of the approach with numerical simulations. Results for one example case are shown in Figure 1.

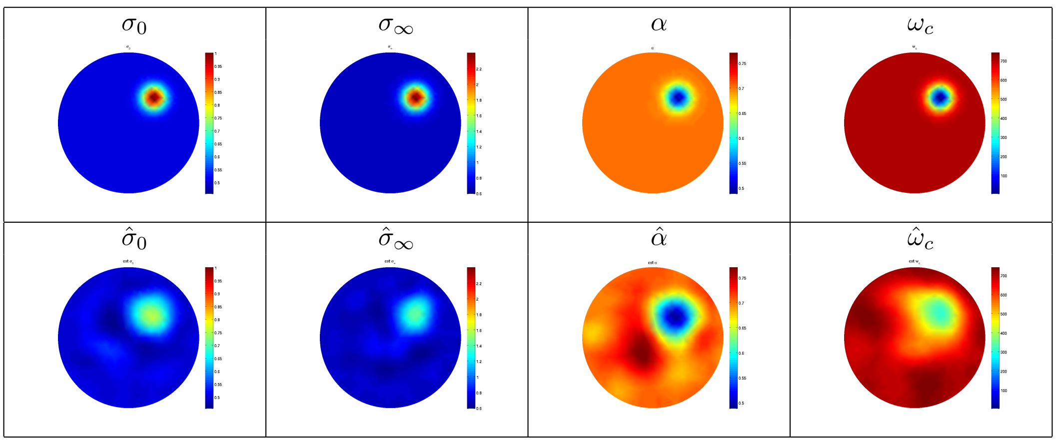

Figure 1: Estimation of Cole – Cole parameters using MF-EIT data with 15 frequencies. The target Cole – Cole parameters are shown in the first row. The second row shows the MAP estimates of the parameters.

References

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[3] Cole, K.S.; Cole, R.H. *Dispersion and Absorption in Dielectrics - I Alternating Current Characteristics*. J. Chem. Phys. **9**, 1941, pp. 341 – 352.