Optimal geometry toward uniform current density electrodes

Yizhuang Song[1], Eunjung Lee[1], Eung Je Woo[2] and Jin Keun Seo[1]

[1]Department of Computational Science and Engineering, Yonsei University, 134 Sinchon-dong, seodaemun-gu, Seoul, 120-749, South Korea
(Tel : +82-2-2123-6128, Fax : +82-2-2123-8194) [2]Department of Biomedical Engineering, Kyung Hee University

E-mail: `yzsong@yonsei.ac.kr`

Abstract. Electrodes are commonly used to inject current into the human body in various biomedical applications such as functional electrical stimulation, defibrillation, electrosurgery, RF ablation, impedance imaging, and so on. When a highly conducting electrode makes direct contact with biological tissues, the induced current density has strong singularity along the periphery of the electrode, which may cause painful sensation or burn. Especially in impedance imaging methods such as the Magnetic Resonance Electrical Impedance Tomography (MREIT), we should avoid such singularity since more uniform current density underneath a current-injection electrode is desirable. In this paper, we study an optimal geometry of a recessed electrode to produce a well-distributed current density on the contact area under the electrode. We investigate the geometry of the electrode surface to minimize the edge singularity and produce nearly uniform current density on the contact area. We propose a mathematical framework for the uniform current density electrode and its optimal geometry. The theoretical results are supported by numerical simulations.

1. Introduction

In biomedical applications such as functional electrical stimulation, defibrillation, electrosurgery, RF ablation, and impedance imaging, we inject current into the human body through a pair of electrodes or more. Electrodes are usually made of highly conductive materials and biological tissues have moderate conductivity values of less than a few S/m. For a typical electrode configuration, a pair of electrodes are attached on the surface of the human body.

As analyzed by Wiley and Webster [9], there exists a singularity of the current density along the edge of the surface electrode. This may cause painful sensation and burn under the dispersive electrode in electrosurgery, for example. Concentration of current density along the periphery of the electrode should be avoided in defibrillation and RF ablation where more uniform energy transfer over a wide local region is desirable.

Previous studies on the uniform current density electrode Geuze [1], Langberg [4], Ksienski [5], Rubinstein *et al* [6], Suesserman *et al* [7] and Tungjitkusolmun *et al* [8] suggested empirical designs without a rigorous mathematical analysis; they improved the uniformity of current density by altering either the electrode geometry or material property based primarily on numerical simulations. Providing a rigorous mathematical analysis of the uniform current density electrode, in this paper, we describe a novel design of such an electrode which is quite different from any of the previous empirical designs. Analyzing the singularity of the current density along the electrode perimeter, we will propose a design using the layer potential technique.

2. Mathematical model of uniform current density electrode

To simplify the electrode design process, we consider the following half space model. Let $\Omega = \mathbb{R}^3_-$ and $\mathcal{D}$ be the hydrogel layer. We denote by $\Gamma \subset \partial\Omega$ the skin contact surface. Then we can express the geometry $\mathcal{D}$ with a function of two variables x and y. Assume that σ is homogenous, say $\sigma = 1$ in both Ω and $\mathcal{D}$. We also assume that the boundary $\partial\Gamma$ is a simply closed smooth curve, Γ is symmetric with respect to x- and y-axis and its gravitational center is the origin. Define $\mathcal{D}$ as $\mathcal{D} = \{\mathbf{r} = (x, y, z) : 0 \le z < \phi(x, y) \text{ for } (x, y, 0) \in \Gamma\}$, where ϕ is a continuous function defined on Γ describing the electrode contact surface $\mathcal{E} = \{\mathbf{r} = (x, y, z) : z = \phi(x, y) \text{ for } (x, y, 0) \in \Gamma\}$. The hydrogel layer $\mathcal{D}$ is located vertically on the flat surface of Ω.

In the half space model, the governing equation is

$$\begin{cases} \Delta u = 0 \qquad \text{in } \widetilde{\Omega} \\ \int_{\mathcal{E}} \frac{\partial u}{\partial \mathbf{n}} dS = I, \quad \mathbf{n} \times \nabla u|_{\mathcal{E}} = 0 \\ \frac{\partial u}{\partial \mathbf{n}} = 0 \quad \text{on } \partial\widetilde{\Omega} \backslash \mathcal{E}, \end{cases} \tag{1}$$

where $\widetilde{\Omega} = \Omega \cup \mathcal{D}$ and $\lim\limits_{|\mathbf{r}|\to\infty} u(\mathbf{r}) = 0$ (see [2] and [3] or [10]).

To make the hydrogel layer as short as possible, we introduce an admissible set $\mathcal{A}$ defined as $\mathcal{A} = \{\phi \in C(\Gamma) \mid \min_\Gamma \phi = 0\}$. Our goal is to get a uniform current density along the contact surface Γ, that is,

$$\text{to find an optimal } \phi \in \mathcal{A} \text{ which minimizes } \Phi(\phi) = \int_\Gamma \left| \frac{\partial u^\phi}{\partial \mathbf{n}} - \alpha^\phi \right|^2 dS, \tag{2}$$

where u^ϕ satisfies (1) and $\alpha^\phi = \frac{1}{|\Gamma|}\int_\Gamma \frac{\partial u^\phi}{\partial \mathbf{n}} dS$. Here the superscript ϕ in u^ϕ represents the dependence of the solution u on the geometry of the domain $\widetilde{\Omega}$, that is ϕ.

The key idea to solve the optimization problem (2) is to use the following special potential: $w(x,y,z) = \int_\Gamma \frac{1}{4\pi\sqrt{(x-x')^2+(y-y')^2+z^2}} dx' dy'$. We now introduce a new minimization problem which is closely related to the minimization problem (2):

$$\begin{aligned} &\text{to find an optimal } \phi \in \mathcal{A} \text{ which minimizes} \\ &\Psi(\phi) = \frac{1}{2}\int_\Gamma \left|\phi(x,y) + w\left(x,y,\phi\left(x,y\right)\right) - w\left(0,0,0\right)\right|^2 dxdy. \end{aligned} \tag{3}$$

The above problem (3) is tractable because the objective functional depends on ϕ explicitly. Indeed, the minimizer is obtained by solving the simple equation of $\phi(x,y) + w\left(x,y,\phi\left(x,y\right)\right) = w(0,0,0)$ for each $(x,y) \in \Gamma$.

By introducing $\tilde{u}$ in Theorem 2.1 below, we can transform the highly nonlinear problem (2) into a much simpler problem of finding a level surface of $\tilde{u}$; a solution of (3) is simply the level surface $\{(x,y,\phi(x,y)) \mid \tilde{u}(x,y,\phi(x,y)) = \tilde{u}(0,0,0),\ (x,y) \in \Gamma\}$. With this ϕ, $\tilde{u}$ will satisfy the boundary conditions in (5). We should note that $\tilde{u}$ is independent of ϕ and $\frac{\partial \tilde{u}}{\partial \mathbf{n}}$ is constant over Γ.

Theorem 2.1. *Suppose $\phi \in \mathcal{A}$ is a minimizer of (3). Define*

$$\tilde{u}(\mathbf{r}) := \begin{cases} w(\mathbf{r}) & \text{in } \Omega \\ w(\mathbf{r}) + z & \text{in } \mathcal{D}, \end{cases} \tag{4}$$

where $\mathcal{D}$ is set vertically on Ω. Then $\tilde{u}$ satisfies

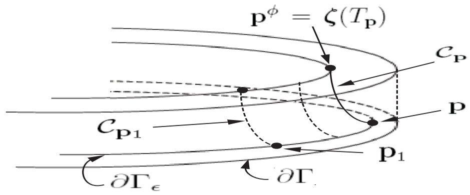
$$\begin{cases} \Delta \tilde{u} = 0 & \text{in } \widetilde{\Omega} \\ \nabla \tilde{u} \times \mathbf{n} = 0 & \text{on } \mathcal{E} \\ \frac{\partial \tilde{u}}{\partial \mathbf{n}} = 0 & \text{on } \partial\Omega \setminus \Gamma \end{cases} \tag{5}$$

and $\frac{\partial \tilde{u}}{\partial \mathbf{n}} = \frac{1}{2}$ on Γ. Moreover, $\mathcal{E}$ lies on the surface $\{\mathbf{r} \in \mathbb{R}^3_+ \ : \ w(\mathbf{r}) + z = w(0)\}$.

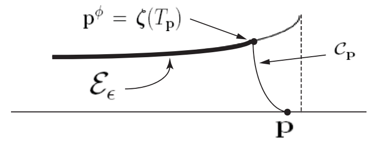
It is crucial to observe that we can trim the hydrogel layer $\mathcal{D}$ to produce a perfectly uniform current density on Γ. For a positive $\epsilon \approx 0$, we denote $\Gamma_\epsilon = \{\mathbf{r} \in \Gamma |\ \mathrm{dist}(\mathbf{r}, \partial\Gamma) > \epsilon\}$. To properly trim the lateral side V_ϕ of $\mathcal{D}$ so that $\partial\tilde{u}/\partial\mathbf{n} = 0$ along V_ϕ, we chop V_ϕ using the trajectory of

$$\begin{cases} \boldsymbol{\zeta}'(t) = \nabla\tilde{u}(\boldsymbol{\zeta}(t))/|\nabla\tilde{u}(\boldsymbol{\zeta}(t))| \\ \boldsymbol{\zeta}(0) = \mathbf{p} \qquad \mathbf{p} \in \Gamma_\epsilon, \end{cases} \tag{6}$$

as depicted in Figure 1 (that is, the method of characteristics). Then, we have $\frac{\partial \tilde{u}}{\partial \mathbf{n_r}} = \mathbf{n_r} \cdot \nabla\tilde{u} = 0$, where $\mathbf{n_r}$ is the normal direction of the trimmed lateral surface of $\mathcal{D}$.



(a) 3D configuration



(b) Cross section of (a)

Figure 1. Trimming process to produce a perfectly uniform current density.

3. Design examples

For practical reasons, we chose two kinds of contact surfaces including a rounded rectangular surface and a circular surface. For the rounded rectangular surface, we set the size of the rectangle as 1.1 and the radius of the 1/4 circle at the corner as 0.1. To trim the edges of the electrodes, we chose $\epsilon = 0.01$ and used the Euler method to solve (6) to get Γ_ϵ and $\mathcal{E}_\epsilon$. After this trimming process, we obtained the optimal electrode shapes in Figure 2.

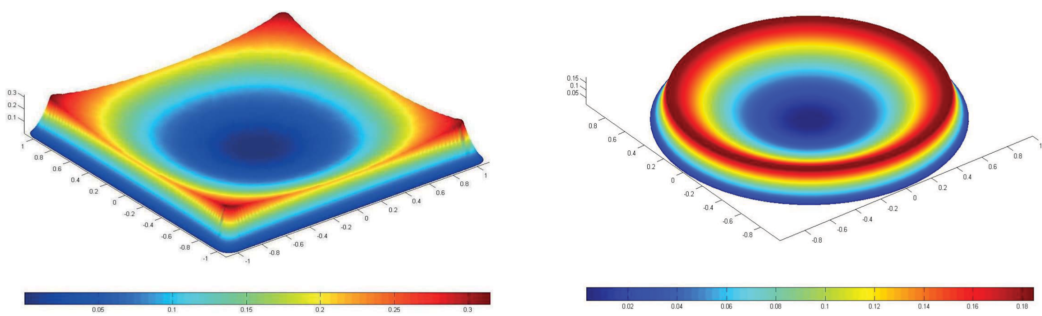


Figure 2. Final designs of two uniform current density electrodes after applying trimming process.

We present numerical simulations of current density distributions obtained using two different electrodes including the circular electrode directly attached on the skin and the circular uniform current density electrode shown in Figure 2. Figure 3 shows vector plots of $\frac{\partial u}{\partial \mathbf{n}}$ along Γ for the two cases of (a) and (b). Table 1 summarizes computed values of the ratio $\frac{\max_\Gamma \partial u/\partial \mathbf{n}}{\min_\Gamma \partial u/\partial \mathbf{n}}$ over Γ.

Mesh size	1/20	1/30	1/40
Ratio for (a)	2.35	2.83	3.71
Ratio for (b)	1.22	1.30	1.31

Table 1. Values of the ratio $\frac{\max_\Gamma \partial u/\partial \mathbf{n}}{\min_\Gamma \partial u/\partial \mathbf{n}}$ over Γ for three difference mesh sizes. (a) and (b) indicate the two cases using the simple and uniform current density electrodes, respectively.

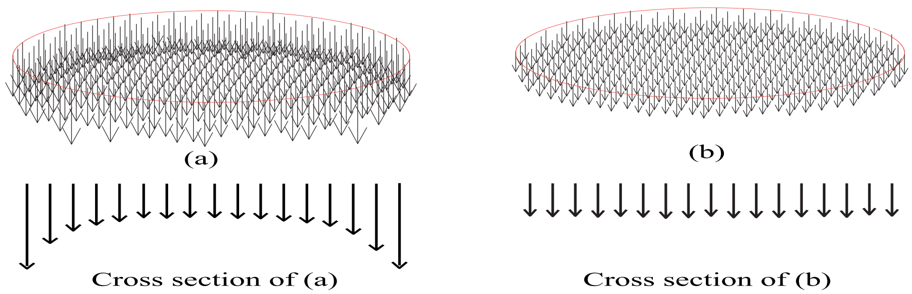


Figure 3. Plots of current density distributions $\partial u/\partial \mathbf{n}$ along Γ computed by using meshes with 1/40 size. The left and right plots correspond to the simple and uniform current density electrodes, respectively.

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