On the Calculation of Jacobian Matrix for Inverse Boundary Problem in 2D EIT

XU Yaoyuan, XU Guizhi, LI Ying
Province-Ministry Joint Key Laboratory of Electromagnetic Field and Electrical Apparatus Reliability
Department of Biomedical Engineering, Hebei University of Technology, Tianjin 300130, China
xu.yaoyuan@yahoo.com

Abstracts

The inverse reconstruction of EIT is severely nonlinear and ill-posed, which makes the resulting images unstable with a low resolution. The introduction of inverse boundary reconstruction makes possible to utilize prior information on the conductivity of inclusions, and enhance the computational performance. The calculation of Jacobian matrix is a key issue for iterative algorithms, as it bridges the map from boundary movements to changes on measurements. This paper presents a fast algorithm to calculate the Jacobian matrix for the inverse boundary reconstruction in EIT using the reciprocity theorem. Results are compared with those obtained by finite difference method, which show the performance of the algorithm.

Keywords: EIT; Jacobian matrix; Reciprocity theorem

1. Introduction

Electrical Impedance Tomography (EIT) is a technique to obtain the electrical impedance distribution by applying different patterns of electric currents at the periphery and measuring the resulted voltages. It enables the production of images of biological tissues in body, as they display different electrical properties. It also possesses a non-intrusive nature, and advantages in cost, speed and safety. For these reasons, EIT is a powerful tool in clinical diagnosis and monitoring, such as lung diseases, heart function, beast cancer, blood accumulation and stomach emptying. There is also a simplified technique in industrial process imaging and geophysics prospecting, called Electrical Resistance tomography (ERT), which only used the real part of measurements. The two techniques are same in mathematical theory. The forward evaluation are both governed by Laplacian equation and driven by boundary conditions of Complete Electrode Model (CEM).

The inverse reconstruction of EIT, referred as Calderon Inverse Conductivity Problem [1], is severely nonlinear and ill-posed, which makes the resulting images unstable with a low resolution. In recent years, a class of algorithm has received wide attention, which aims to determine the locations and shapes of inclusions assuming a piece-wise conductivity distribution. Thus, the inverse problem is called Inverse Boundary Reconstruction. This variation has advantages as follows.

1) Usage of prior information on conductivity of inclusions
The usage of prior information is an important measure to cope with the ill-posed nature arisen in EIT reconstruction. In many applications, the conductivity distribution is piece-wise and the values can be measured in advance. This prior information will be integrated in modeling the

inverse boundary reconstruction schedule. However, the traditional Inverse Conductivity Reconstruction will not consider it, since the conductivity value is what the inverse solvers are expected to output.

2) Enhancement on computational performance
Since the inverse problem concerns only on boundary location of inclusions, the choice of Boundary Element Method (BEM) to solve the forward evaluation is reasonable, The BEM only requires discretization on boundaries, which leads to a much smaller size of algebraic equations comparing to domain-based methods.

This paper focuses on calculating the Jacobian matrix, which bridge the map from boundary movements to changes on measurements, for the inverse boundary reconstruction in 2D EIT. This issue is of much importance, since it is a basic prerequisite for most iterative algorithms. Although there exist methods directly determine the boundaries, such as Monotonicity [2], Factorization [3] and Enclosure [4], they are only suitable for simple cases under rigid assumptions. Duraiswami [5] applied BEM to EIT boundary reconstruction using Finite Difference Method (FDM) to compute the Jacobian matrix, which requires an additional solve of the forward problem for each independent movement direction, and thus the calculative speed is limited. Otten [6] developed the direct linearization method with numerical integrations on boundary elements; this idea was improved greatly by Xu [7] using analytical quadratic BEM and strategies on the computational aspect. In this paper, we present a fast algorithm to calculate the Jacobian matrix using the reciprocity theorem.

2. Jacobian matrix calculation using the reciprocity theorem

An EIT sensor consists of an array of electrodes (typically 16 electrodes) that are distributed along the inner periphery of a non-conducting pipe at equal intervals. The data acquiring system adopts a special strategy to generate electric field and collect boundary voltage measurements. A constant current of low frequency is applied to an adjacent pair of electrodes, and the voltages measured successively from each pair of adjacent electrodes. The current is then switched to the next pair and the voltage measurements are repeated. This procedure is repeated until a full rotation of electric field around the cross-section is completed.

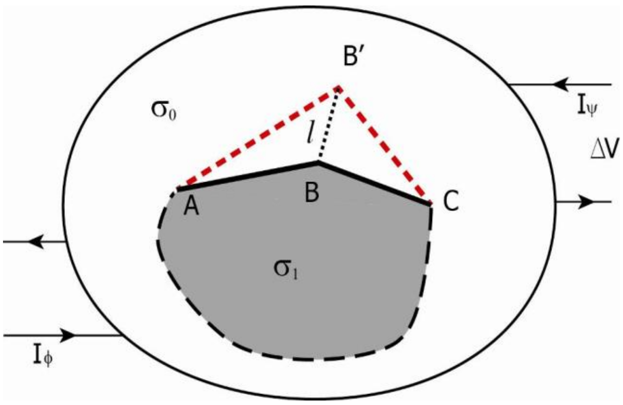

Fig. 1 Electric current driven patterns and conductivity distribution

Consider a region Ω with its conductivity distribution σ=: {σ_0 *in background sub-region;* σ_1 *in sub-region covered by an inclusion*}, see Fig. 1. The inclusion can be insulating or perfectly conducting, however, the conductivity of homogeneous background should be generally conducting due to the principle of EIT. Let Φ (Ψ) the potential distribution and $\boldsymbol{J_\Phi}$ ($\boldsymbol{J_\Psi}$) the current density throughout the region driven by current pattern I_Φ (I_Ψ), we get their relationship from Green's theorem as follows.

$$\oint_{\partial\Omega}\left(\Psi J_\Phi - \Phi J_\psi\right)\cdot ds = \int_{\Omega}\left(\sigma-\sigma\right)\cdot\left(\nabla\Psi\cdot\nabla\Phi\right)\cdot dv = 0 \tag{1}$$

If the conductivity distribution changes from σ to σ' due to a movement on a mesh point B, one gets

$$\int_{\mathrm{V}}\left(\sigma-\sigma'\right)\cdot\left(\nabla\Phi'\cdot\nabla\Psi\right)\cdot dv = \oint_{\partial\Omega}\left(\Psi J'_\Phi - \Phi' J_\psi\right)\cdot ds \tag{2}$$

where Φ' and J'_Φ are changed potential distribution and current density corresponding to σ'. However, the driven current is the same as the previous ($I'_\Phi = I_\Phi$). Then, considering the voltages on electrodes are constant, one gets

$$\begin{aligned}\int_{\mathrm{V}}\left(\sigma-\sigma'\right)\cdot\left(\nabla\Phi'\cdot\nabla\Psi\right)\cdot dv &= \oint_{\partial\Omega}\left(\Psi J_\Phi - \Phi' J_\psi\right)\cdot ds \\ &= \oint_{\partial\Omega}\left(\Phi J_\psi - \Phi' J_\psi\right)\cdot ds \\ &= I_\psi\left(V-V'\right) = I_\psi \Delta V\end{aligned} \tag{3}$$

In order to obtain the Jacobian, one need calculate the limit of

$$J = \lim_{l\to 0}\frac{\Delta V}{l} = \frac{\left(\sigma_0-\sigma_1\right)}{I_\psi}\cdot\lim_{l\to 0}\frac{\int_{\mathrm{ABCB'}}\left(\nabla\Phi'\cdot\nabla\psi\right)\cdot dv}{l} \tag{4}$$

where J is the Jacobian coefficient. We assume the current density distribution throughout the region does not vary considering the slight movement, and then gets

$$J = \frac{\left(\sigma_0-\sigma_1\right)}{I_\psi}\cdot\left(\begin{aligned}&\sin\alpha\frac{|AB|}{2}\int_{-1}^{1}\frac{1+t}{2}\left(\frac{\partial\Phi}{\partial\tau}\frac{\partial\Psi}{\partial\tau}+\frac{\sigma_0}{\sigma_1}\frac{\partial\Phi}{\partial n}\frac{\partial\Psi}{\partial n}\right)_{AB}dt + \\ &\sin\beta\frac{|BC|}{2}\int_{-1}^{1}\frac{1-t}{2}\left(\frac{\partial\Phi}{\partial\tau}\frac{\partial\Psi}{\partial\tau}+\frac{\sigma_0}{\sigma_1}\frac{\partial\Phi}{\partial n}\frac{\partial\Psi}{\partial n}\right)_{BC}dt\end{aligned}\right) \tag{5}$$

where α and β are the angles ABB' and $B'BC$, respectively. Operations $\partial/\partial n$ and $\partial/\partial\tau$ are derivatives in tangential and normal directions.

3. Results

In order to test the performance of the algorithm, consider a simple case with a circular inclusion. The sensor is 12.5 cm by diameter, and sixteen 1 cm-width electrodes are planted along the periphery. The plate and electrodes are equal in height (3 cm) thus the problem can be addressed in two dimensions. The amplitude of driven currents is 5 mA. The conductivity of homogeneous background is set to 662 mS/cm, as that of the tap water. Contact resistivity is set to 30 Ω cm^2. The circular inclusion is 3 cm by radius, with its conductivity half of that of background. The outer periphery is meshed by 160 elements for the computation, while the inner is meshed by 90 elements. Quadratic analytical BEM is used to solve the forward problems, see Ref. [7] for details. Consider a mesh point in the inclusion which is supposed to move outward in the normal direction (with a tiny distance), computer the Jacobian coefficients with the algorithm above. The results are also compared with those returned by finite difference method which is time-consuming and precious, see Fig. 2. The difference between the results is less than 0.7%, which tells the precision of presented algorithm.

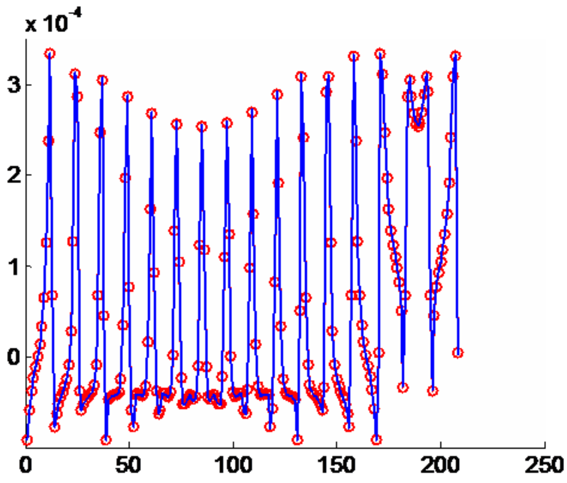

Fig. 2 Comparison on Jacobian coefficients

(Solid line: finite difference method; Circle markers: reciprocity theorem)

4. Conclusions

The introduction of inverse boundary reconstruction makes possible to utilize prior information on conductivity of inclusions, and enhance the computational performance. An effective algorithm to calculate the Jacobian matrix is of much importance, since it is a basic prerequisite for most iterative algorithms. A fast algorithm is presented, which is easy to apply, as the calculation is restricted to integrations on local elements. A test demonstrates its precision, the difference between the results is less than 0.7% compared with finite difference method.

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