ART. XLII.—*Determination of v , the ratio of the Electro-magnetic to the Electrostatic Unit*; by EDWARD B. ROSA, Student in Physics in the Johns Hopkins University.

THIS investigation was conducted in the physical laboratory of Johns Hopkins University during the months of March to June, 1889, under the direction of Associate Professor A. L. Kimball. The writer takes great pleasure in acknowledging his obligation to Dr. Kimball for valuable advice and encouragement throughout the progress of the work.

The method employed is essentially that given by Maxwell, vol. ii, § 776. It was used by J. J. Thomson in his determination of v , published in the Phil. Trans. for 1883. The following is substantially his description of the method. In a Wheatstone bridge ABCD (fig. 1), the circuit BD is not closed but the points B and D are joined to two poles S and R of a commutator, between which vibrates the armature P, which is connected with the inner shell of a spherical condenser. When P touches S the condenser will be charged, and there will be a momentary current through the various arms of the bridge, through the galvanometer from D to C.

When P touches R the two surfaces of the condenser are connected and the latter discharges itself through DR. If now the armature be made to vibrate continuously there will be a series of momentary currents through the galvanometer, and by adjusting the resistance a (c and d being large, fixed resistances), these interrupted currents may be exactly counterbalanced by the steady current from C to D, and the resultant deflection of the galvanometer is zero. When this is the case there is a relation between the capacity

of the condenser, the number of times the latter is charged and discharged per second and the resistances in the various arms of the bridge. Maxwell gives an approximate value of this relation. Thomson's more complete investigation gives the following equation :

$$nC = \frac{a \left\{ 1 - \frac{a^2}{(a+c+g)(a+b+d)} \right\}}{cd \left\{ 1 + \frac{ab}{c(a+b+d)} \right\} \left\{ 1 + \frac{ag}{d(a+c+g)} \right\}}$$

where n is the number of complete oscillations of the armature P per second ; C is the capacity of the condenser in electromagnetic measure ; and the other letters are the resistances of the various arms of the bridge, as shown in the figure. In the present case the values of these resistances were about as follows :

$$\begin{array}{ll} a = 40 \text{ to } 1900 \text{ ohms.} & d = 100,000 \text{ ohms.} \\ b = 0 \text{ nearly.} & g = 6,000 \text{ " } \\ c = 1,570,000 \text{ to } 2,450,000 \text{ ohms.} & \end{array}$$

Owing to the very high values of c and d as compared with a , b and g the above equation may be replaced by the approximate one $C = \frac{a}{ncd}$, which is true to within a hundredth of one per cent. The electrostatic capacity, C' , is determined by calculation from the geometrical constants of the condenser. The ratio of these values of the capacity $\frac{C'}{C}$ is v^2 , the square root of which, v , is the quantity sought.

Advantages of the method.

Thus appears at once an important advantage of the method of determining the ratio of the units from the values of a capacity, namely, that v is the square root of the ratio of the capacities, and any error in the latter enters into v by only half its amount.

There are several important advantages of this method of measuring the electromagnetic capacity. In the first place, a knowledge of the exact electromotive force and resistance of the battery is not required, and their constancy is not essential. In the second place, since it is a null method, such uncertain quantities as logarithmic decrement, torsion of the suspending fibre and period of the needle are not required ; the galvanometer can readily be made more sensitive than a ballistic galvanometer ; its "constant" need not be known ; and the field of force may be variable both in intensity and direction without prejudice to the experiment. On the other hand,

the quantities which are required are the period of the vibrator and the values of three resistances, quantities which are capable of determination to a very high degree of accuracy. In the present case the vibrator was either a tuning fork or else it was driven by a tuning fork, and by the arrangement adopted the uncertainty in its period was reduced to an extremely small quantity. The difficulties and limits of the method will appear under the head of Sources of Error.

Instruments.

1. *Condenser*.—This was made from designs by Prof. Rowland. It consists of a hollow sphere whose radius is 12·7 cm. and within which may be hung either of two balls of 10·1 and 8·9 cm. radius, respectively. The condenser has a capacity of about 50 absolute electrostatic units with the larger ball and 30 with the smaller. The spherical surfaces are accurately ground, nickel plated and polished to a mirror surface. The ball is suspended by a silk cord C, fig. 3, passing through a hole, 7 mm. in diameter, in the outer shell, and attached to the insulated end of a pivoted beam and counterpoised. By means of a rack and pinion movement and vernier the ball may be accurately set in any desired position. Maxwell* objects to this form of a condenser on account of the difficulty of working the surfaces accurately spherical, making them truly concentric and determining with sufficient accuracy their dimensions. That these difficulties have in the present case been entirely surmounted will, I think, appear from the discussion under the heads of Displacement of the Ball (p. 305), and Electrostatic Capacity (p. 305).

2. *Galvanometer*.—This was one of Elliott Bros.' Thomson, high resistance, astatic galvanometers, made very sensitive.

3. *Tuning forks*.—Two of König's forks were used, whose frequencies were approximately 32 and 130 per second. They were driven by three or four Bunsen cells, the same current in the case of the slower fork operating the vibrator P (fig. 1). Their exact periods were determined by Michelson's method.†

4. *Vibrators*.—The oscillating piece P in the case of the slower fork was a commutator, such as that used by Thomson.‡ The action of this form of a vibrator was regular and satisfactory in the case of the slower fork; but with the higher fork great difficulty was experienced in obtaining sufficient uniformity, and finally it was abandoned and the following plan devised as a substitute. T, T' (figs. 2 and 3), are two prongs of the tuning fork, driven by the electromagnet M; the interrupter, attached to the end of one of the prongs, not being

* Vol. i, p. 321.

† This Journal, Jan., 1883.

‡ Thomson, Phil. Trans., 1883, or Glazebrook, Phil. Mag., vol. xviii, p. 98.

shown. b, b' (fig. 3), are two fine brass wires, uniting at n , and tipped with platinum at p, p' , where they are bent at right

angles and fastened to the fork with an insulating cement. V, V' are two small blocks of vulcanite attached to a firm support A . Below the platinum points are two cavities in the vulcanite which are filled with mercury, and as the fork vibrates, first one and then the other of the points dips into the mercury. Thus the mercury cups, which are joined to B and D , respectively (fig. 1), answer to the posts S and R , while the wires b, b' unite and run to the ball of the condenser. When

the prongs separate p' dips into the lower cup and the condenser is charged; when they approach p dips into the other cup and the condenser is discharged. The points must be at least half a millimeter above the surfaces of the mercury when the fork is at rest, in order to avoid both dipping at once and short circuiting the condenser. With an amplitude of about three millimeters perfect contact is made at each vibration, and the regularity of action, as shown by the steadiness of the spot of light on the scale, is extremely satisfactory. The deflection of the needle when the steady current is not balanced by the intermittent current amounts in the case of the high fork to 125 scale divisions, using the one-tenth shunt; i. e., without the shunt as it was used in practice to 1250 scale divisions. With the fork's best action the resistances were adjusted until closing the key would cause a deflection of less than half a

scale division, corresponding to less than $\frac{1}{2500}$ th part of the whole current. To obtain a regularity of action which permitted such accurate observations required a very delicate adjustment of the distance between the surfaces of the mercury and the points above, as well as clean surfaces and a steady current.

5. *Battery*.—About forty cells of a storage battery with a total electromotive force of about eighty volts were used. A higher electromotive force, at first proposed, was thought to be unnecessary.

6. *Resistances*.—The resistance a was taken from a box of Elliott Bros., whose total resistance was about 12,000 ohms; the resistance d was a 100,000 box from the same firm. The first of these, box A, was carefully calibrated by comparing the several coils on a Fleming bridge with three standard coils of 10, 100 and 1000 B. A. units respectively. The first was a Warden Muirhead No. 292, 10 B. A. Units. Its value, determined by Glazebrook, Oct., 1887, is 9.99416 at 16°.5 C. The other two had been previously carefully compared with this. The values of the resistances of box A adopted were the means of three different and closely agreeing determinations, made at different temperatures. The several coils of box B were carefully compared with the known resistances of A. The temperature-coefficients of both boxes were also carefully determined.

The resistance c was of graphite. Plate glass was ground with fine emery and lines ruled upon it. Under a magnifying power of several hundred diameters the layer of graphite appears as made up of patches which run together at numerous points. The resistance of a strip of graphite of given length and breadth depends upon how well these patches are joined together. The glass and graphite are given a heavy coat of shellac and thoroughly dried. A series of ten such resistances was prepared and mounted, connection being made at the ends by tin foil, held firmly in contact with the graphite by rubber packing, wires passing out from the tin foil. The resistances were placed in cylindrical boxes with vulcanite tops, in which were set binding screws, joined to the wire terminals. The boxes can be surrounded by water or other material to lessen the temperature fluctuations. These resistances proved quite constant and reliable. Two were used in this experiment, R_1 and R_2 , whose resistances were approximately 1,570,000 and 2,440,000 ohms. During the six weeks preceding May 9 their alteration, aside from temperature fluctuations, was inappreciable. But between May 9 and May 13, when not in use, from some as yet unknown cause, both increased about one-half of one per cent, and up to June 8, when last used, remained

nearly constant at the new value. Inasmuch as glass and shellac are poor conductors, the temperature of the graphite resistances cannot safely be assumed to be the same as that of the air within the box, unless the latter has been kept constant for some time. In order, therefore, to avoid all uncertainty as to their values these resistances were determined anew whenever used, and, if their temperature changed materially, both just before and just after using. They were compared with the resistances of boxes A and B, two arms of a Wheatstone bridge, with a ratio of 99.89, being taken from A. Here is a specimen observation and calculation.

May 22. Bridge reading 24,430. Temperatures $\left\{ \begin{array}{l} \text{Graphite } 19^{\circ}\cdot 8 \\ \text{A} = 20^{\circ}\cdot 3 \\ \text{B} = 20^{\circ}\cdot 6 \end{array} \right.$
 $24,430 = \left\{ \begin{array}{l} 20000 \text{ from B} = 20,012 \text{ at } 20^{\circ} = 20,015 \text{ at } 20^{\circ}\cdot 6 \\ 4430 \text{ " A } 4443 \text{ " } 4444 \text{ " } 20^{\circ}\cdot 3 \end{array} \right.$
 $24,459 \times 99\cdot 89 = 2,443,200 \text{ ohms at } 19^{\circ}\cdot 8 = \text{temp. at which used.}$
 This value is reliable to within one part in five thousand.

It is proper to add that if these graphite resistances are put into a circuit where there is a large difference of potential between their terminals, their resistance is immediately diminished by heating. With three Bunsen cells used in measuring their resistance no heating was perceptible. In the determination of capacity there was a difference of potential between the terminals always less than two volts and usually less than one. When the temperatures were maintained constant the resistance after use was always precisely the same as before. While, therefore, the use of high graphite resistance is somewhat restricted where great accuracy is desired, they still may serve a very useful purpose in many cases, and are the most convenient and reliable of any high resistance, aside from metal wires, that I know anything about.

Arrangement of the apparatus.

The vibrators were fixed as near as possible to the condenser to reduce the capacity of the charging wires to a minimum. The condenser, galvanometer and other parts of the apparatus were insulated with great care; and yet in spite of all precautions leakage made its appearance on rainy days, and a slight trace of leakage could usually be detected. Observations were consequently confined to fair weather. The apparatus for determining the frequency of the forks was kept always ready for use.

Sources of Error.

1. *Resistances.*—The constant errors in the resistances must have been very small, and corrections for temperature fluctuations were made with great care.

2. *Tuning forks.*—Michelson's method furnishes a very exact determination of the period of an electric tuning fork, but unfortunately the period does not remain constant. This is especially the case with the higher fork, the charging wires and spring contact having a varying effect upon the rate in different adjustments. But the slower fork with mercury contact was not, even after making proper temperature-corrections, perfectly constant. To avoid all uncertainty and obviate the necessity of applying a temperature-correction the rates of the fork were determined each time anew, usually before and after, or in the midst of a series of observations on capacity. As stated, the apparatus for the purpose was always ready for use, and without stopping the fork or changing its circumstances in any way whatever, by simply closing the clock circuit and the primary circuit of the induction coil, I could in three to five minutes count a sufficient number of flashes to give me the period of the fork true to within less than one part in ten thousand. Occasionally a slight change in the sound emitted by the fork, due to variation in contact or current, suggested a possible change in the period; a moment's glance in the microscope would answer the question. This method of dealing with the rates of the forks avoids the introduction of small constant and large accidental errors, which may happen when the rates are determined once for all.

3. *Charging wires.*—The vibrating armature P (fig. 1), the wires b , b' (fig. 3) as well as the joining wire e have a certain capacity which adds itself to that of the condenser when they are connected, but which may be determined separately by disconnecting the charging wire at m . Thus, on April 15, with R_6 and fast fork, the resistance a was 1874.5 and 153.0 respectively in the two cases mentioned, which gives 1721.5 ohms as the resistance corresponding to the condenser alone. This assumes that the capacity of the charging wire is the same when joined to the ball as when separated. The capacity of the 2.5 cm. of fine brass wire between the ball and the shell, (fig. 3) is nearly one per cent of the capacity of the condenser, determined experimentally. It would seem that this capacity might be slightly greater when the wire was disconnected from the ball and at a different potential; but being lifted one or two millimeters in disconnecting its capacity would be thereby reduced. The effects of these two modifying circumstances were separately very carefully studied. With the rapid fork running very smoothly a change of half an ohm could be easily detected; this would be equal to a change of 1-3500th of the capacity of the condenser. No difference could, however, be observed, although the trial was several times repeated. The two effects have opposite signs, and if each is inappreciable

much more would their sum be so. I therefore conclude that the difference of the observed capacities of condenser and charging system together and of charging system alone is a true measure of the capacity of the condenser.

4. *Displacement of the ball.*—The upper half of the spherical shell was lifted and the lower half adjusted upon its supports until the distance of the ball from the shell was the same at all points on the equatorial circumference. The upper half of the shell was then replaced, and by means of the rack and pinion the ball was first lowered and then raised until it touched the shell, the exact moment of touching being indicated by an electrical contact, and several readings taken on the vernier in each position. The mean of the readings in the two positions gave the central position. In this manner the ball was adjusted vertically to within 0.1 mm. and equatorially within 0.2 mm. Thus the ball is centered to within less than one per cent of the distance between the ball and shell, which is 25 mm. Thomson has investigated a formula for the capacity of eccentric cylinders. The formula shows that for a displacement of one per cent the capacity is increased $\frac{1}{200}$ th of one per cent. Evidently the capacity of spherical shells is less affected by slight eccentricity than cylinders. Therefore we may safely conclude that no error is due to eccentricity. This conclusion was verified experimentally, a displacement of four per cent causing an inappreciable change.

5. *Adjusting resistances.*—The accidental errors occurring in adjusting the resistances a so as to produce zero deflection will be eliminated by a large number of observations. Their magnitude depends on the strength of the current, delicacy of the galvanometer, regularity of the vibrator, etc., and are larger with the slow fork than with the fast. The stronger the current and more sensitive the galvanometer the greater the deflection due to a certain error in a , but on the other hand the greater the unsteadiness of the spot, so there is a practical limit in that direction. That these accidental errors are small is, I think, attested by the uniformity of the results obtained.

Electrostatic capacity.

The electrostatic capacity of the condenser was calculated from the formula $C = \frac{rr'}{r-r'}$, where r, r' are the radii of the shell and ball respectively. The radii are determined by finding the volume of water which fills the shell and which is displaced by the ball. These results are confirmed by direct measurement upon the dividing engine.

Ball A.

May 1. Weight in air 2903.83 g. Temperature $18^{\circ}.9$; Bar. 76 cm. Volume of ball 4339 cc. Correction for displaced air is consequently $+4.83$ g. $\therefore$ Weight of ball in vacuo = 2908.66 g. A second determination gave 2908.64 g. I therefore take for the true weight in vacuo 2908.65. The ball being lighter than water a sinker was attached and the following weighings made :

May 3. W't in distilled water, ball and sinker, 210.62 g. at $17^{\circ}.05$ C.
 " " " sinker alone, 1635.59 " at $17^{\circ}.10$ C.

difference,	1424.97 "
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Correction for $\frac{1425}{8.4}$ cc. air displaced by weights .21. "	

	1424.76 "
Weight of ball in vacuo,	2908.65 "

Loss of weight in water at $17^{\circ}.05$ C. =	4338.41 "
" " " 4° . "	4338.68 " =

volume in cubic centimeters. Another determination gave 4338.87. I take as a mean 4338.8, which makes the mean radius $r_A' = 10.1180$ cm. An error of 0.1 in the number 4338.8 would cause an error of less than a thousandth of a millimeter in r_A' .

Ball B.

May 1. Weight in air, 2321.40 g.
 Correction for displaced air, 3.20 " = 2324.60 g. in vacuo.
 May 3. W't in distilled water, ball and sinker, 208.96 g. at $16^{\circ}.45$ C.
 " " " sinker alone, 807.86 g. " $16^{\circ}.70$ C.

difference,	598.90 "
Correction for air displaced by weights	.09 "

	598.81 "
Weight of ball in vacuo,	2324.60 "
Loss of weight in water at $16^{\circ}.45$ C. =	2923.41 "
" " " 4° . "	2926.65 " =

volume in cubic centimeters; this gives $r_B' = 8.8735$. A second determination gave a closely agreeing result.

In these weighings the bodies were lifted completely out of water, replaced and air bubbles carefully removed at least three times in each weighing. The mean of the several values, which differed in the centigrams, was each time taken. These differences were usually due to slight changes in the temperature of the water, the balances being far more sensitive than the thermometer. As, however, the temperature was read to $\frac{1}{2}^{\circ}$ th of a degree several times during a weighing and the

mean taken, it is thought the temperature is true to within $0^{\circ}.1$ and this corresponds in the case of the larger ball to about $.07$ g. I think the values of the radii given above are true to within two or three thousandths of a millimeter.

Shell.

The weighings of water contained by the shell were made by replacement. The shell was sealed about the junction of its two halves with white paint, filled with distilled water and allowed to stand to absorb any air bubbles which might have escaped the brushing with a wire which was given the inner surface after filling. The condenser was placed on the platform of the scales, approximately counterpoised and then accurately balanced by adding weights to the platform; about 100 cc. of water was then withdrawn, temperature taken, shell refilled, (the space around the opening being thoroughly raked with a wire to prevent error from small air bubbles which tended to lodge there), and weights again added to balance. The following weighings were thus made:

130.7 g.	130.5	131.2	131.3	
132.0	132.0	131.7	131.4	Mean = 131.4 g.
		131.6	131.4	Mean temp. $18^{\circ}.4$

The condenser being emptied and carefully dried required the following weights to balance the counterpoise.

8650.3,	8650.8,	8650.9,	8650.9;	mean 8650.7
8650.7 - 131.4 = 8519.3 = W't of water at $18^{\circ}.4$ in air.				
	8531.8 =	"	"	$4^{\circ}.0$ "
Correction for displaced air	9.1			
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	8540.9 g. =	"	"	" in vacuo

= volume of the shell in cubic centimeters. This makes the radius $r = 12.6805$ cm. It seems reasonable to suppose that the number 8540.9 is true to within less than a gram. This would make the error in r less than $.0005$ cm.

These values of the radii are confirmed by the following direct measurements made on a dividing engine, using calipers and a standard meter bar by Bartels and Diederichs, Göttingen, whose length is accurately known. Three mutually perpendicular diameters of the shell were found to be 25.357, 25.360, 25.358 cm. Mean = 25.358, giving $r = 12.679$, a very close agreement in view of the difficulty of setting the calipers. More accurate measurements on the balls were obtained.

Ball A. Following are twelve diameters.

20.2399	20.2372	20.2170
20.2358	20.2336	20.2348
20.2350	20.2382	20.2250
20.2250	20.2315	20.2401

Mean = 20.2328 + correction for the bar, .0038 = 20.2366 $\therefore r'_A = 10.1183$ cm.

Ball B. Following are six diameters.

17.7468	17.7408	17.7429
17.7465	17.7452	17.7407

Mean = 17.7438 + correction for the bar, .0034 = 17.7472 $\therefore r'_B = 8.8736$.

It is perhaps somewhat accidental that these values coincide so closely with the values of the radii found by the first method. Their importance is not insisted upon further than as furnishing satisfactory confirmation of the results of the other and more accurate method.

It will be seen that in ball A no diameter differs from the mean by as much as a tenth of a millimeter, and in B the variation is still smaller. This deviation from perfect sphericity has no appreciable effect upon the value of the capacity calculated from the ordinary formula. We now have :

$$C'_A = \frac{12.6805 \times 10.1180}{12.6805 - 10.1180} = 50.069.$$

$$C'_B = \frac{12.6805 \times 8.8735}{12.6805 - 8.8735} = 29.556.$$

The radius of the hole in the shell through which the suspending cord passes is 0.35 cm. and its area $\frac{1}{82.00}$ th part of the area of the shell. The capacity is diminished in a less ratio than the area; therefore the capacity is diminished probably not more than a hundredth of one per cent, a quantity wholly negligible.

Electromagnetic capacity.

A series of observations on the electromagnetic capacity by the method described was made, extending from March 28 to June 8, under a variety of circumstances as to weather and external surroundings. The two graphite resistances, the two tuning forks, and different resistances from box A were variously combined, and at temperatures ranging from 17° to 25° C. The shell and ball were occasionally re-adjusted, and between April 16 and May 4 the condenser was taken apart and its electrostatic capacity determined. Further, in order to measure the graphite resistances the apparatus, as shown in fig. 1, was each time disconnected and put together again. All these variations must have had the effect of eliminating to a large degree constant errors, while of course the single observations do not agree as well among themselves as

they otherwise would. Following is the last observation made, given as a specimen :

Resistances a :	1930.0 — 194.9 = 1736.0	Temperature :
	1932.0 — 195.5 1736.5	A = 22°·3
	1932.0 — 197.0 1735.0	B 23°·0
	1932.5 — 196.5 1736.0	Graphite 23°·0

The wire was first in contact at m (fig. 3), and the resistance a corresponding to joint capacity of condenser and charging system was 1930.0 ohms. The wire e (which slides with some friction in the small glass tube G , the latter being fastened at S to the ebonite R) was now lifted very slightly, and 194.0 ohms found to give no deflection on closing the key in the galvanometer circuit. The wire was then again lowered to contact and the subsequent observations made in a similar manner. Any leakage increases the numbers alike in the first two columns, and if constant does not affect the differences, which give the capacity of the condenser. But the leakages are not constant, so that small differences are thereby introduced; this accounts in part for the differences above, though of course small differences are inevitable if there be no leakage. As illustrating how a large leak if constant eliminates itself the following item from my notes of June 6 is of interest. The wooden base of the condenser was thoroughly wet with a cloth, and the leakage thereby introduced changed the readings from 1924–186 to 1933–195, the difference, 1738, remaining unaltered.

The mean of the above differences is 1735.9

Correction : excess at 20° C. = 5.4; temp. corr. 1.3; = 6.7

$$\therefore a = 1742.6$$

Box B = 100, 120 at 23°·0 = d

R_s (calculated as already explained) 2,435,800 = c

Frequency of the fork 130.075 = n

1 B. A. unit = .98664 ohm.

$C = \frac{a}{ncd} \times \frac{1}{.98664 \times 10^9}$, C being the capacity of the condenser in absolute electromagnetic units.

log c	= 2,435,800	= 6.386642
" d	100,120	5.000521
" n	130.075	2.114194
"	$.98664 \times 10^9$	8.994159
		22.495516
" a	= 1742.6	3.241198
" C		20.745682 n .
" C'_A	= 50.069	1.699568
" v^2		20.953886
" v		10.476943
$\therefore v$		2.9988×10^{10} cm. per sec.

Table of Results.

[The numbers in the columns headed v when multiplied by 10^{10} give the values of v in centimeters per second.]

Group.	Date.	No.	v , slow fork	Weight.	Weighted mean of group.	v , fast fork.	Weight.	Weighted mean of group.
Group I.	March 28.	1	3.0040	2	3.0012 (24)	2.9994 2.9990 2.9975 2.9953 2.9977 2.9998 2.9986 2.9996	4 2 2 2 3 4 4 4	2.9987 (25)
	April 2.	2	3.0031	3				
		3	2.9993	3				
	4.	4	2.9980	3				
		5	3.0009	3				
	5.	6	3.0010	2				
		7	3.0036	2				
	10.	8						
	12.	9						
	13.	10						
		11						
		12						
	15.	13	2.9993	3				
		14	3.0031	3				
	16.	15						
		16						
		17						
Group II.	May 4.	18			3.0045 (16)	3.0048	3	3.0029 (33)
		19				3.0041	4	
	6.	20				3.0053	4	
		21				3.0063	2	
		22				3.0029	4	
		23				3.0024	4	
	7.	24				3.0006	4	
		25				3.0011	4	
		26				3.0009	4	
	8.	27	3.0058	3				
		28	3.0007	3				
		29	3.0039	2				
		30	3.0069	2				
	9.	31	3.0033	3				
		32	3.0073	3				
Group III.		33	3.0012	3	3.0043 (16)			
		34	3.0105	1				
	14.	35	3.0090	3				
		36	3.0059	3				
	15.	37	3.0021	2				
		38	3.0036	2				
		39	2.9990	2				
Group IV.	21.	40			3.0017 (15)	2.9947	1	2.9977 (25)
	22.	41				2.9950	2	
		42				2.9966	2	
		43	2.9996	4				
	23.	44	3.0025	4				
		45	3.0022	4				
		46	3.0028	3				
		47				2.9988	4	
	June 6.	48				2.9978	4	
		49				2.9980	4	
	8.	50				2.9980	4	
		51				2.9988	4	

The results exhibited in the preceding table have been divided into four groups. The first group consists of seventeen values found before the condenser was taken apart to measure its electrostatic capacity. During this time the upper half of the shell was lifted and the ball adjusted two or three times. The values found by the fast fork are more uniform than the others and average somewhat lower. The second group extends from May 4 to 9, inclusive, when the condenser had been set up again. There were two small glass tubes about 5 mm in diameter and drawn out to about 2 mm, where they projected (between 1 cm and 2 cm) through the shell into the space within. They had once been used to pass charging wires through. The wires had been withdrawn, and it was supposed that the glass tubes had no appreciable effect. The holes were together only $\frac{1}{3000}$ th of the area of the shell, and the tendency of the glass to slightly increase the capacity would tend to counterbalance the decreasing effect of the holes. When the condenser was set up the second time the tubes were intentionally left out, and the values of group II were noticed to be larger than those of group I. No cause could be discovered for this increase (which indicates a *less* electromagnetic capacity), but the tubes were replaced and group III taken. The mean of this group is as large as that of the preceding group. The tubes were now again withdrawn and the holes covered with gold foil, making the inner surface of the shell continuous. Group IV gave values averaging almost exactly the same as group I. The circumstances were alike in other respects so far as is known; the usual variations in the conditions of the observations, as already explained, occurring in all the groups. I do not think the presence or absence of the tubes could affect the capacity appreciably; they were altogether too small, probably not filling over a thirty-thousandth of the space between the ball and shell. But that there was a difference in the actual capacity of the condenser when groups I and IV were taken from its value when II and III were obtained seems almost certain. As yet I have not become satisfied as to the cause of this difference, but it seems probable that in putting the condenser together some obstruction lodged between the two halves of the shell and prevented them from coming completely together. Had they been separated a few hundredths of a millimeter only, the difference in question would be fully accounted for. The surfaces of contact are very accurately ground and polished, and loosening the screws does not cause them to separate, as proved by the capacity remaining constant. That the low fork should give higher values for v than the high one (which means a *lower* value for the capacity), is rather unexpected and not fully understood. The low fork gave only a quarter the current given by the

high one, and was less steady in its action in proportion to the current; consequently the single observations were less reliable, but this alone does not account for the nearly uniform difference.

In view of the uncertainty as to the cause of the variations it is difficult to determine how best to combine the results. The weight of each single value of v in the table is determined by considering the number of observations from which it is calculated, the uniformity of the separate observations, the steadiness of the spot, etc. If we give to groups II and III one-half the weight of I and IV in proportion to the sum of the weights of the separate values we have as the mean for the fast fork 2.9994, and for the slow fork 3.0023. Giving now double weight to the results of the fast fork on account of their greater accuracy and uniformity, we have as a mean of all $v = 3.0004 \times 10^{10}$ *cm. per sec.*

Again, if it be found that the cause suggested is the true cause of the excess of groups II and III then those groups should be thrown out and we should have 2.9982 and 3.0014 as the means, which combined as before would give for the mean of all $v = 2.9993 \times 10^{10}$ *cm. per sec.* These values are based upon the value .98664 for the British Association Unit.

It is proposed to resume this investigation next winter, when more perfect insulation can be obtained, and several improvements in the details of the apparatus will be made. The smaller ball of the condenser will then be used also, and the cause of the difference in the values given by the two forks will be studied. Although we cannot yet say whether v is greater or less than 300,000,000 meters per second, it seems certain that it is within a tenth per cent of this number, and it is hoped in the continuation of this investigation to narrow considerably further the range of uncertainty.

For convenience of reference the following values of v and of the velocity of light as found by different observers are added, the values of v being corrected to the value .98664 for the B. A. Unit:

v , ratio of the units.	Velocity of light.
¹ 1856. Weber & Kohlrausch 3.107×10^{10}	1879. Michelson 2.9991×10^{10}
² 1869. W. Thomson & King 2.808	1882. Michelson 2.9985 "
³ 1868. Maxwell 2.842	1882. Newcomb 2.9986 "
⁴ 1872. M'Kichan 2.896	and 2.9981 "
⁵ 1879. Ayrton & Perry 2.960	1874. Cornu 2.9850 "
⁶ 1880. Shida 2.955	1878. Cornu 3.0040 "
⁷ 1883. J. J. Thomson 2.963	1880-81. Young & Forbes. 3.0138 "
⁸ 1884. Klemencic 3.019	
⁹ 1888. Himstedt 3.009	
1889. W. Thomson 3.004	

Johns Hopkins University, Baltimore, June 15, 1889.

¹ Weber & Kohlrausch, *Electrodyn. Maasbestim.*, Abh. der Königl. Sächs. Gesellschaft der Wissensch., vol. v, p. 219, 1856.

² King, Report of the Committee on Electrical Standards, 1869.

³ Maxwell, *Phil. Trans.*, 1868, p. 643. ⁴ Dugald M'Kichan, *Phil. Trans.*, 1879.

⁵ Ayrton & Perry, *Jour. Soc. Tel. Engineers*, 1879, p. 126.

⁶ Shida, *Phil. Mag.*, V, x, p. 431. ⁷ J. J. Thomson, *Phil. Trans.*, 1883, p. 707.

⁸ Klemencic, *Wien. Ber.* lxxxiii, 88. ⁹ Himstedt, *Wied. Ann.*, No. 9, 1888.