A METHOD FOR THE DETERMINATION OF COEFFICIENTS OF MUTUAL INDUCTION.

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THE principal methods for the determination of coefficients of mutual induction fall into three groups :

1. By comparison with self induction (which may be separately determined in absolute measure).¹

2. By comparison with electrostatic capacity.²

3. By comparison with ohmic resistance.³

The first two are zero methods — the third is not, as it involves the use of a ballistic galvanometer.

For some time past I have been using a method in the laboratory which falls into group 1 and which has proved so satisfactory both as regards accuracy and simplicity that I publish it, thinking it may, for these reasons, commend itself. The very simplicity of the method made me at first suspect that it must have been adopted by others but on going over the literature of the subject as thoroughly as I am able, I find no description of the method.

Two coils of mutual inductance M and of self-inductances L_1 and L_2 when joined in series with the windings in the same direction are equivalent to a single coil of self-inductance equal to the sum of the coefficients of self-induction increased by twice the coefficient of mutual induction.

When the coils are joined in series so that the lines of force from one oppose the lines from the others, the system is equivalent to a single self-inductance equal to the sum of the coefficients of self-induction diminished by twice the coefficient of mutual induction.

The two coils (provided with a commutator for reversing the direction of the current through one of them) may be put in one branch of a Wheatstone bridge, the other branches of which consist of two noninductive resistances and a variable standard of self-induction.

¹ See Maxwell, "Electricity and Magnetism," Vol. II., 365, or Carhart and Patterson, "Electrical Measurements," p. 272.

² See Carey-Foster, *Phil. Mag.*, 23, 121.

³ See Glazebrook, *Phil. Trans.*, 1883.

A balance for steady currents and for variable currents being obtained when the currents are in the same direction through the two coils gives $L' = L_1 + 2M + L_2$.

Repeating the balancing when the currents are in opposite directions through the two coils gives $L'' = L_1 - 2M + L_2$.

Combining these expressions gives $L' - L'' = 4M$ or $M = \frac{1}{4}(L' - L'')$. Thus M is got simply from the difference in the settings of the variable standard of self-induction in the two cases.

If such a standard is not available the determination of M becomes more tedious and may be effected as follows: A coil of self-inductance L may be prepared and used as a substitute for the variable standard mentioned above. It is well to choose the value of L about intermediate between L' and L'' .

By the bridge method the ratios of L' to L and of L'' to L may be found — calling these p and q , respectively, the expression $M = \frac{1}{4}(L' - L'')$ becomes $M = L/4(p - q)$.

If L be determined by balancing against electrostatic capacity then M may be calculated since p and q are numerical expressions obtained by the use of the bridge.

In the following table are given the results of a series of observations to determine the coefficients of mutual induction of a certain pair of coils. Those obtained with the aid of a variable standard are given in column I. Those obtained in the manner just described are given in the second column.

As a basis of comparison of the results obtainable by the method of reversing the coils with those obtainable by the same observer using some of the standard methods the coefficient of the coils was determined by the Maxwell method (group 1), by the Carey-Foster Method (group 2) and by the Glazebrook method (group 3).

The results are given in the third, fourth and fifth columns of the tables. The departures of the individual observations from the mean are also shown in each case.

The mutual induction consisted two similar circular brass frames 93.3 mm. in diameter, the annular region in which the wire was wound being 56.7 mm. in the plane of the windings and 22 mm. at right angles to it.

Each coil consisted of 23 turns per layer and 20 layers of No. 20 copper wire.

The resistance and self-inductance of each coil were respectively, 6.40 ohms and 0.0330 Henry.

All necessary precautions were taken to prevent the disturbing effects of thermal electromotive forces and of direct magnetic action of the coils on the galvanometer. The results given in the first four columns of the table were obtained by the use of a Thomson astatic galvanometer of 1070 ohms resistance and of sensibility of the order of 10^{-9} amperes.

The results shown in the last column were obtained by the use of a Nalder pattern ballistic galvanometer of 2400 ohms resistance of high sensibility adjusted to a period of about 5 seconds.

Table of determinations of M in Henrys by the various methods.

I.	II.	III.	IV.	V.
0.01535	0.01560	0.01567	0.01547	0.01528
0.01545	0.01530	0.01564	0.01568	0.01504
0.01540	0.01530	0.01542	0.01590	0.01499
0.01535	0.01507	0.01510	0.01545	0.01528
0.01548	0.01570	0.01564	0.01490	0.01470
Average, 0.01540	0.01539	0.01549	0.01548	0.01506

Table of departures of individual values from the mean.

I.	II.	III.	IV.	V.
-0.00005	+0.00021	+0.00018	-0.00001	+0.00022
+0.00005	-0.00009	+0.00015	+0.00020	-0.00002
± 0.00000	-0.00009	-0.00007	+0.00042	-0.00007
-0.00005	-0.00013	-0.00039	-0.00003	+0.00022
+0.00008	+0.00031	+0.00015	-0.00050	-0.00036
Average, 0.00005	0.00021	0.00019	0.00023	0.00018

I am indebted to Miss Georgiana M. Whitcomb for her very conscientious work in obtaining the data contained in the table.

It appears that with the ordinary appliances of a laboratory equipped for advanced electrical measurements the first method is the most accurate, while the others show about an equal accuracy.

As regards the time required to perform the experiment, the first method has a very decided advantage.

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