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tification detached itself from the anode and joined the stratifications in the narrow portion of the tube. This process could be repeated until there appeared to be formed a species of stationary wave, due apparently to the setting free of the gases from the anode meeting a cathode liberation. When the strength of the current is diminished, the stratifications disappear in the terminals, more noticeably in the anode than in the cathode. This process can be traced to the x -ray stage. Since there are no electrical oscillations in the circuit, for I had several megohms in circuit, the molecular theory of bombardment, together with the phenomenon of occlusion, seem to be most important. The end-on tube which was employed was filled with dry hydrogen obtained by electrolysis from phosphoric acid and distilled water. The gas was passed through caustic potash and phosphoric pentoxide. Hittorf's resistance of iodide of cadmium in amyl alcohol will undoubtedly be better than distilled water in quantitative experiments.

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VII. On Approximately Simple Waves.

By LORD RAYLEIGH, F.R.S.*

THE first question that arises is as to the character of *absolutely* simple waves; and here "it may be well to emphasize that a simple vibration implies *infinite* continuance, and does not admit of variations of phase or amplitude. To suppose, as is sometimes done in optical speculations, that a train of simple waves may begin at a given epoch, continue for a certain time involving it may be a large number of periods, and ultimately cease, is a contradiction in terms"†. A like contradiction is involved if we speak of unpolarized light as homogeneous, really homogeneous light being necessarily polarized.

This much being understood, approximately simple waves might be defined as waves which *for a considerable succession* deviate but little from a simple train. Under this definition large changes of amplitude and frequency would not be excluded, provided only that they entered slowly enough. More frequently further limitation would be imposed, and approximately simple waves would be understood to mean waves which for a considerable succession can be approximately identified with a simple train of given frequency, if not of

* Communicated by the Author.

† 'Theory of Sound,' 2nd ed. § 65 *a*, 1894.

given amplitude. But the *phase** of the simple train approximately representing the given waves would vary from place to place, slowly indeed but to any extent.

Thus if we take, as analytically expressing the dependence of the displacement upon time,

$$H \cos pt + K \sin pt, \quad (1)$$

where H and K are slowly varying functions of t , the frequency may be regarded as constant, while the amplitude $\sqrt{H^2 + K^2}$ and the phase $\tan^{-1}(K/H)$ vary slowly but without limit. It scarcely needs to be pointed out that a *slow* uniform progression of phase is equivalent to a *small* change of frequency.

In one important class of cases the phase remains constant and then, since a constant addition to t need not be regarded, (1) is sufficiently represented by

$$H \cos pt \quad (2)$$

simply. If the changes of amplitude are periodic, we may write

$$H = H_0 + H_1 \cos qt + H_1' \sin qt \\ + H_2 \cos 2qt + H_2' \sin 2qt + \dots, \quad (3)$$

in which q is supposed to be small. The vibration (2) is then always equivalent to a combination of simple vibrations of frequencies represented by

$$p, \quad p+q, \quad p-q, \quad p+2q, \quad p-2q, \quad \&c.$$

Under this head may be mentioned the case of ordinary beats, so familiar in Acoustics. Here

$$H = H_1 \cos qt, \quad (4)$$

$$\text{and} \quad H \cos pt = \frac{1}{2} H_1 \cos (p+q)t + \frac{1}{2} H_1 \cos (p-q)t. \quad . . . (5)$$

It may be observed that although the phase is regarded as constant, the change of sign in the amplitude has the same effect as an alteration of phase of 180° .

Another important example is that of intermittent vibrations. If we put

$$H = 2(1 + \cos qt), \quad (6)$$

the amplitude is always of one sign, and

$$H \cos pt = 2 \cos pt + \cos (p+q)t + \cos (p-q)t. \quad . . . (7)$$

Three simple vibrations are here required to represent the effect.

* What is here called for brevity the phase is more properly the *derivation* of phase from that of an absolutely simple train of waves.

Again ('Theory of Sound,' § 65 *a*), if

$$H = 4 \cos^4 qt, \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

we have

$$H \cos pt = \frac{3}{2} \cos pt + \cos (p+q)t + \cos (p-q)t \\ + \frac{1}{4} \cos (p+2q)t + \frac{1}{4} \cos (p-2q)t. \quad . \quad . \quad (9)$$

If K also be variable and periodic in the same period as H , so that

$$K = K_0 + K_1 \cos qt + K_1' \sin qt + K_2 \cos 2qt + K_2' \sin 2qt + \dots, \quad (10)$$

we have the most general periodicity expressed when we substitute these values in (1) ; and the general conclusion as to the periods of the simple vibrations required to represent the effect remains undisturbed.

If K and H vary together in such a manner that the amplitude $\sqrt{(H^2 + K^2)}$ remains constant, the sole variation is one of phase. My object at present is to call attention to this class of cases, so far as I know hitherto neglected, unless an example (Phil. Mag. xxiv. p. 409, 1892) in which an otherwise constant amplitude is periodically and suddenly reversed be considered an exception.

If we take

$$H = \cos(\alpha \sin qt), \quad K = \sin(\alpha \sin qt), \quad . \quad . \quad (11)$$

H and K are of the required periodicity, and the condition of a constant amplitude is satisfied. In fact (1) becomes

$$\cos (pt-\alpha \sin qt). \quad . \quad . \quad . \quad . \quad (12)$$

Now, since

$$e^{i\alpha \cos \theta} = J_0(\alpha) + 2iJ_1(\alpha) \cos \theta + 2i^2J_2(\alpha) \cos 2\theta + \dots \\ + 2i^nJ_n(\alpha) \cos n\theta + \dots,$$

we have

$$e^{i\alpha \sin qt} = J_0 + 2iJ_1 \sin qt + 2J_2 \cos 2qt + 2iJ_3 \sin 3qt + 2J_4 \cos 4qt + \dots, \quad (13)$$

and thus

$$\cos(\alpha \sin qt) = J_0(\alpha) + 2J_2(\alpha) \cos 2qt + 2J_4(\alpha) \cos 4qt + \dots, \quad (14)$$

$$\sin(\alpha \sin qt) = 2J_1(\alpha) \sin qt + 2J_3(\alpha) \sin 3qt + \dots, \quad (15)$$

where J_0, J_1 , &c. denote (as usual) the Bessel's functions of the various orders. In the notation of (3) and (10)

$$\begin{aligned} H_1 &= H_3 = \dots = 0, & H_1' &= H_2' = H_3' = \dots = 0, \\ H_0 &= J_0(a), & H_2 &= 2J_2(a), & H_4 &= 2J_4(a), & \&c., \\ K_0 &= K_1 = K_2 = \dots = 0, & K_2' &= K_4' = \dots = 0, \\ K_1' &= 2J_1(a), & K_3' &= 2J_3(a), & K_5' &= 2J_5(a), & \&c. \end{aligned}$$

Accordingly (12), expressed as a combination of simple waves, is

$$\begin{aligned} & J_0(\alpha) \cos pt + J_2(\alpha) \{ \cos (p-2q)t + \cos (p+2q)t \} \\ & \quad + J_4(\alpha) \{ \cos (p+4q)t + \cos (p-4q)t \} + \dots \\ & \quad + J_1(\alpha) \{ \cos (p-q)t - \cos (p+q)t \} \\ & \quad + J_3(\alpha) \{ \cos (p-3q)t - \cos (p+3q)t \} + \dots \quad (16) \end{aligned}$$

If α , representing the maximum disturbance of phase, be small, we may write approximately

$$J_0 = 1 - \frac{1}{4}\alpha^2, \quad J_1 = \frac{1}{2}\alpha, \quad J_2 = \frac{1}{8}\alpha^2,$$

n .	J_n (3).	J_n (6).	J_n (12).	J_n (18).	J_n (24).
0	-.26005	+.15065	+.04769	-.01336	-.05623
1	+.33906	-.27668	-.22345	-.18799	-.15404
2	+.48609	-.24287	-.08493	-.00753	+.04339
3	+.20906	+.11477	+.19514	+.18632	+.16127
4	+.13203	+.35764	+.18250	+.06964	-.00308
5	+.04303	+.36209	-.07347	-.15537	-.16230
6	+.01139	+.24584	-.24372	-.15596	-.06455
7	+.00255	+.12959	-.17025	+.05140	+.13002
8	+.00049	+.05653	+.04510	+.19593	+.14039
9	+.00008	+.02117	+.23038	+.12276	-.03643
10	+.00001	+.00696	+.30048	+.07317	-.16771
11		+.00205	+.27041	-.20406	-.10333
12		+.00055	+.19528	-.17624	+.07299
13		+.00013	+.12015	-.03092	+.17632
14		+.00003	+.06504	+.13157	+.11803
15		+.00001	+.03161	+.23559	-.03863
16			+.01399	+.26108	-.16631
17			+.00570	+.22855	-.18312
18			+.00215	+.17063	-.09311
19			+.00076	+.11271	+.04345
20			+.00025	+.06731	+.16191
21			+.00008	+.03686	+.22640
22			+.00002	+.01871	+.23429
23			+.00001	+.00886	+.20313
24				+.00395	+.15504
25				+.00166	+.10695
26				+.00066	+.06778
27				+.00025	+.03990
28				+.00009	+.02200
29				+.00003	+.01143
30				+.00001	+.00563
31					+.00263
32					+.00118
33					+.00010
34					+.00021
35					+.00008
36					+.00003
37					+.00001

while J_3 &c. are of higher powers in α than α^2 . Thus if we stop at the first power of α , we are concerned only with the multiples of t represented by

$$p, \quad p-q, \quad p+q;$$

while if we include α^2 we have

$$p, \quad p-q, \quad p+q, \quad p-2q, \quad p+2q.$$

But when α is not small, the convergence is slow, and a large number of terms will be required even for a moderately close approximation. The preceding table, due to Meissel, is condensed from Gray and Mathew's 'Bessel's Functions.' So far as π can be identified with 3, the values of α equal to 3, 6, 12, 18, 24 correspond to maximum deviations of phase (in both directions) equal to $\frac{1}{2}, 1, 2, 3, 4$ periods respectively. It appears that the largest value of $J_n(\alpha)$ occurs for a value of n somewhat less than α . Indeed, it is at once evident from (12) that frequencies in the neighbourhood of $p \pm q\alpha$ will be important elements.

VIII. *The Spark-Length of an Induction-Coil.*

By R. BEATTIE, B.Sc.*

IN an ordinary induction-coil the relationship of the secondary spark-length to the conditions which obtain at the primary break has, at one time or another, received a good deal of attention; and most of the facts which are likely to have any bearing on the action of the coil are well enough known. On one or two minor points, however, no very accurate or detailed data appear to exist. For instance, although Rijke† has studied the effect on the secondary spark-length of using different materials as the primary poles, and although he has indicated in a general way the influence of rate of separation of the primary poles, yet his experiments were incomplete in many respects. In making the experiments described in the present paper, the object was to extend Rijke's results, and to examine more fully how the length of the secondary spark depends upon the nature of the primary poles, the rate of break, and the electromotive force in the primary circuit (1) with no condenser across the break, (2) with condensers of various capacities across the break.

Apparatus.—The coil used for the purpose of the experiments was one by Apps, giving normally a 25 cm. spark.

Throughout the experiments the current made or broken

* Communicated by the Author.

† Rijke, Pogg. Ann. vol. xevii. p. 75.