and in like manner (by changing the sign of S),

$$\text{Mod.}^2 \text{ Denominator} = k^2(S + S_0)^2 \{1 + \tan^2 k_1 \tanh^2 k_2\};$$

and hence

$$\text{Mod.}^2 B = \left(\frac{S - S_0}{S + S_0} \right)^2, \quad . \quad . \quad . \quad . \quad . \quad (39)$$

where

$$S = \cos \theta(\sigma + \sigma')/\sigma. \quad . \quad . \quad . \quad . \quad (21)$$

If σ , the perforated area, is relatively great, it makes little difference what its actual value may be, but if σ is relatively small, as in the case of strong resonance, it is otherwise.

It would be preferable to suppose S fixed at S_0 and to calculate the effect of a variation of k with h given. The resulting expressions are, however, rather complicated, and it is evident without calculation that the reflexion will be very sensitive to changes of wave-length when there is high resonance as a consequence of small dissipation and accurate tuning. The spectrum of the reflected light [in the corresponding optical circumstances] would then show a *narrow* black band.

XXII. *The Quantum Theory of Electric Discharge.*
By Dr. D. N. MALLIK and Prof. A. B. DAS, M.Sc.*

1. **O**N the quantum theory, the energy of a source of radiation varies in a discontinuous manner by equal quanta. If this is to be accepted, since electrons give out radiation when their velocity is changed, the quantum theory would seem to suggest that these changes are effected by a series of jerks, that is in other words the changes involved are due to impulses only. Moreover, since a material atom is known to be made up of corpuscles and positive ions, having configurations satisfying conditions of dynamical stability, under their mutual attractions and repulsions, it is necessary to admit that they are revolving doublets or higher complexities. In view of their rapid movements and collisions, such systems must be admitted to have complex nutational motions as well. It stands to reason therefore, that the energy of such a system cannot be a continuously varying quantity. It must either remain constant or, if it changes at all, it must change by finite

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amounts, in view of the fact that the changes are effected by (generalized) impulses. In any case a variation of energy in a discontinuous manner is consistent with many well-known phenomena.

In the present paper we have attempted to interpret the behaviour of electric discharge in a vacuum-tube in terms of this theory.

2. In a vacuum-tube, as we know, the discharge is non-luminous or silent unless the difference of potential is greater than a certain amount, so that there is a minimum difference of potential for the passage of a spark.

Now assuming the formula

$$E_0 = \sum \frac{ch}{\lambda} \frac{1}{\frac{Nch}{e^R \lambda} - 1},$$

where c = velocity of light,
 λ = wave-length,
 h = Planck's constant,
 N = Avogadro's number,
 $R = \frac{pv}{T}$ for a gas,

and E_0 is the energy required for electronic oscillation of wave-length $\Sigma\lambda$, we observe that this quantity has a certain constant value, depending only on the nature of the gas. In other words, so long as a gas retains its specific properties, so as to yield a characteristic spectrum, the total energy required for luminosity has a constant value. That is, the energy corresponding to the ionization of the gas when attended with luminosity has a value which is constant and independent of the pressure of the luminous gas, so long at any rate as the gas continues to yield its characteristic spectrum.

3. If, then, E is the energy of a corpuscle which is in a condition to ionize, γE will be the energy that will be transferred to the ionized gas, where γ is a proper fraction. And for luminosity, γE must be equal to E_0 . It seems reasonable to suppose, therefore, that the minimum energy of the ionizing corpuscle must be E_0 . And if we further admit that E is proportional to spark-potential or at any rate of the form $a + bV$, where V is the spark potential, the minimum spark potential V_0 must be proportional to E_0 .

or of the form $a + bE_0$. The minimum spark potential must be then independent of the pressure. This has been found to be the case.

4. Now, it is well known that as the pressure of the gas decreases, a dark space is formed next to the cathode called the Crookes dark space. And it is obvious that until the energy of the ionizing corpuscle attains to the minimum value required, it will not be in a condition to ionize. This explains the formation of the Crookes dark space, and further suggests that the difference of potential between the cathode and the negative glow, called the cathode-fall of potential, must be approximately equal to the minimum spark potential and independent of pressure. This is also in accordance with known experimental results.

As the electric intensity in the Crookes dark space is very high, it will follow that the breadth of this space will be very small, increasing as the electric intensity decreases with pressure. It is seen further that the cathode-fall of potential must be proportional to E_0 , or at any rate be of the form $a_1 + b_1E_0$.

5. Again, the energy of a corpuscle when it collides with a molecule of the gas is evidently proportional to Xel , where X is the electric intensity and l the mean free path. Hence $Xel = E$. Moreover, it can be shown that if α is the coefficient which determines the rate of increase in the number of corpuscles, then, when α is equal to zero, Xel is constant. We find now that this constant value must be $= E_0$. Since, further, $l \propto \frac{1}{p}$ where p is the pressure, we find that when α is zero, $\frac{X}{p} = \text{const.}$, a result which was obtained otherwise in a previous paper (Phil. Mag. Oct. 1912).

6. Returning now to the Crookes dark space, we observe that it is limited by the point at which, through the action of the electric force, the energy $= E_0$, and, accordingly, this also marks the beginning of the negative glow. As the corpuscle moves through the negative glow, its energy decreases on account of collision, according to the law given (as J. J. Thomson has shown) by $E^2 = E_0^2 - 2\beta x$ or $E = E_0 \left(1 - \frac{\beta}{E_0^2} x\right)$ nearly, where x is the (small) distance

traversed and β is a function of the masses of the colliding ions. It will happen, therefore, that if X is the electric force, $E = E_0 - \frac{\beta x}{E_0^2} + \int X dx$, so that ionization will no longer be possible, when $\int X dx < \frac{\beta}{E_0^2} x$. This will then mark the beginning of the next or Faraday dark space.

7. Now, the energy at the extremity of the negative glow is evidently of the form $E_0 - kx$, where k may be taken to be constant as a first approximation. It is, moreover, known that x increases as pressure decreases. Assuming that $x \propto \left(\frac{A}{p} + B\right)$, this energy may be written equal to $E_0 - \frac{k_1}{p} - k_2$. If further, X_1 is the average electric intensity through the Faraday dark space and d is the length of this space, we must have $E_0 - \frac{k_1}{p} - k_2 + X_1 d = E_0$.

We have moreover $X_1 \propto \frac{1}{p}$ and $d = a' + \frac{b'}{p}$, approximately, during a certain range of pressure. We have thus, on this theory, a means of coordinating the various quantities associated with the cathode dark space, the negative glow, and the Faraday dark space.

8. Experiments show, however, that the linear law both as regards the negative glow and the Faraday dark space holds only for a limited range.

Obs. In both these experiments, the distance between the electrodes was 14.6 cm.

Experiment I.—Voltage in the primary about 11 volts.

(Curve I.)

Pressure in mm.	Scale-reading corresponding to the cathode terminal.	Scale-reading corre- sponding to the beginning of the positive column.	Length of the dark space.
27	14.9	14.7	.2 cm.
15	14.9	14.15	.75 „
5	14.9	13.48	1.42 „
3	14.9	13.25	1.65 „
1.5	14.9	11.75	3.15 „

After this stage, the cathode glow spreads up to the positive column and gradually overlaps it, practically the whole tube being illuminated.

Experiment II.—Voltage in the primary a little above 12 volts. (Curve II.)

(Illumination stronger than in the previous case.)

35	14.9	14.65	.25 cm.
17	14.9	14.3	.6 „
7	14.9	13.85	1.05 „
5	14.9	13.4	1.5 „
2	14.9	13.45	

As the pressure is further reduced, the cathode glow appears to extend up to the positive column and overlap it.

9. Again, with an induction coil, the line of demarcation between the Faraday dark space and the positive column is never well defined nor steady. This is in accordance with theory, for owing to a rapid variation in the potential of the induction coil, the energy of the field must vary in a rapid manner.

10. The peculiar behaviour of the striæ indicates that they are similar in character to the Faraday dark space and are of similar origin. On the Quantum theory, accordingly, they are regions over which the energy of discharge is less than E_0 . At very low pressure, before the disappearance of the striæ, both the Faraday dark space and the striæ continue to retain their breadth, showing that the corresponding change of energy is negligible.

XXIII. *Latent Heat and Surface Energy.*

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

IN a recent number of the *Phil. Mag.* (vol. xxxviii. p. 240), there appeared an interesting paper by D. L. Hammick on a relation between the surface tension (γ), the internal latent heat (L_i), the molecular diameter (σ), and the molecular volume (V). For argon, the data used were as follows :—

$$\gamma = 11.00 \text{ dynes/cm. at } T = 87^\circ \text{ abs., } L_i = 1284 \text{ calcs.,}$$

$$\sigma = 3.21 \times 10^{-8} \text{ cm., } d = M/V = 1.404,$$

whence

$$\frac{\gamma V}{\sigma} \cdot \frac{1}{J} = 226, \quad \frac{L_i}{6} = 214,$$

the agreement between these two numbers being considered good.

It does not appear to be generally known that the values of γ for liquid argon given by Baly and Donnan (*Journ. Chem. Soc.* lxxxi. p. 907 (1902)) are incorrect. The correct