275. A New Method of Finding the Condition That the General Conicoid Should Be One of Revolution and of Finding the Equation of Its Axis

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Source: *The Mathematical Gazette*, Vol. 4, No. 73 (Jul., 1908), pp. 319-320

Published by: [Mathematical Association](#)

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Accessed: 15-12-2015 09:10 UTC

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275. [L₂ 4. a.] A new method of finding the condition that the general conicoid should be one of revolution and of finding the equation of its axis.

In Note 200 of the *Mathematical Gazette* (May 1906) I gave a new method of finding the condition that the general conicoid should be one of revolution and of finding the equation of its axis. The following method is shorter and certain exceptional cases are treated.

Let the conicoid be

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0. \dots\dots(2)$$

Then $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d$
 $+ (lx + my + nz + p)^2 = 0 \dots\dots(2)$

is the system of conicoids having contact with (1) round the curves of section by the planes

$$lx + my + nz + p = 0. \dots\dots\dots(3)$$

If the conicoid (1) be one of revolution and the plane of section be perpendicular to its axis, in which case l, m, n will be the proportional to the direction cosines of that axis, one of the conicoids having contact with (1) round the curve of section by that plane will be a sphere.

The conditions that (2) should be a sphere are

$$a + l^2 = b + m^2 = c + n^2, \quad f + mn = 0, \quad g + nl = 0, \quad h + lm = 0.$$

Thus if f, g, h are all non-zero

$$l^2 = -\frac{gh}{f}, \quad m^2 = -\frac{hf}{g}, \quad n^2 = -\frac{fg}{h},$$

and the required condition is

$$a - \frac{gh}{f} = b - \frac{hf}{g} = c - \frac{fg}{h};$$

but if one of them, say g , is zero, then $l=0$ or $n=0$, say $l=0$, then $h=0$, and we have the condition $(a-b)(a-c)=f^2$.

Finally if all three are zero, two of the quantities l, m, n must be zero, i.e. two of the quantities a, b, c must be equal, which is the required condition in this case.

Again the equation of the axis of revolution is

$$\frac{x - \frac{U}{D}}{l} = \frac{y - \frac{V}{D}}{m} = \frac{z - \frac{W}{D}}{n}.$$

But from the above work we have the following:—If f, g, h are all non-zero,

$$l : m : n = 1/f : 1/g : 1/h;$$

if two of them, g and h , are zero, $l=0, m=\sqrt{a-b}, n=\sqrt{a-c}$ (the signs before the roots being chosen so as to make $mn+f=0$); and if all three are zero, in the case where the two equal coefficients among a, b, c are b and c we have $l=\sqrt{b-a}, m=0, n=0$. Thus the equation of the axis of revolution is

$$\frac{x - \frac{U}{D}}{1/f} = \frac{y - \frac{V}{D}}{1/g} = \frac{z - \frac{W}{D}}{1/h},$$

where f, g, h are all non-zero.

$$\frac{x - \frac{U}{D}}{0} = \frac{y - \frac{V}{D}}{\sqrt{a-b}} = \frac{z - \frac{W}{D}}{\sqrt{a-c}},$$

where $g=0$ and $h=0$, the sign given to the radicals being such as to make $\sqrt{a-b} \cdot \sqrt{a-c}$ have the opposite sign to f ;

$$\text{and} \quad \frac{x - \frac{U}{D}}{1} = \frac{y - \frac{V}{D}}{0} = \frac{z - \frac{W}{D}}{0},$$

where $f=0$, $g=0$, $h=0$ and the two equal quantities among a , b , c are b and c .

H. L. TRACHTENBERG.

ANSWER TO QUERY.

[52, p. 295.] General solution of $x^2 + y = a$,i.
 $y^2 + x = b$ii.

Let $c^6 - 4ac^4 + 4bc^2 - 1 = 0$;

whence c can be found in terms of a and b .

Substituting from i. in ii.,

$$4c^2(a - x^2)^2 + 4c^2x = 4c^2b = 1 + 4ac^4 - c^6;$$

whence $2c \left\{ x^2 - \left(a - \frac{c^2}{2} \right) \right\} = \pm (2c^2x - 1)$. W. H. LAVERTY.

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