ART. XX.—*A Pendulum Study*; by O. T. SHERMAN.

“LE mode d'installation du pendule, la nature du support sur lequel il est placé, en raison de l'élasticité de la matière dont il est composé ou de sa masse relativement peu considérable, peut exercer une influence sur le mouvement du plan de suspension. Cette influence est très-sensible pour un support en bois; . . . elle est encore appréciable entre un pilier en pierre . . . et le pilier très-massif sur lequel les observations ont été faites . . . à Genève.”—E. PLANTAMOUR.

The above describes pretty completely the only result which it is my fortune to draw from a series of pendulum observations taken during a northern journey. But, what is the mode of action of the disturbance? It affects the mean of the observations; can its correction be drawn from the observations themselves?

The stand with which the expedition was furnished was formed by three beams of about five inches square, and fastened by bolt and screw mutually at right angles. These are braced in each of the three coördinate planes. The pendulum case, of light wood, rested on a bracket and was secured to the upright by bolt and screw. The agate planes were fastened to a weighty, solid gun-metal casting, and it in turn by screws to the back of the case; the whole forming a support as stable as the material would allow. While studying the subject since returning, the case has been fastened to the brick wall of the clock room of the Observatory of Yale College. The connection is made through wood. The results show, if any, very little improvement over the portable stand. The pendulum itself is a Baily's bar made to beat in three-quarters of a second at Washington, D. C. It was customary in the field to measure the amplitude of the arc by making the cross wire of the observing theodolite coincide with the extremities of the arc, and noting the angle subtended. The observations were not frequent and no discrepancy was noticed. When the instrument again reached my hands it was provided with an arc divided into degree and ten-minute spaces. The scale was subjected to a critical study and the errors of the graduations tabulated. We then endeavored to trace the decrement of the arc by observations every five minutes. After making every correction it was found impossible to represent the observations by any smooth curve. The observations for the larger arcs might be fairly represented by a somewhat sinuous, the smaller arcs by a very jagged, line. Thoroughly dissatisfied with the action of this scale it was replaced by a net formed by a number of silk threads passed over two similar screws.

For better distinction the threads were divided into tallies—five black and five white. The interval between the threads was about seven-tenths of a millimeter. The observer then sitting behind the telescope made at each apparent contact of the knife edge with a thread, a signal on the chronograph at once representing the tally and thread which the knife edge seemed to touch. The result was so unforeseen as for some time to cause a doubt in the mind of the observer as to its correctness. He assured himself of its reality by obtaining fairly identical results again and again. Over some threads the knife edge passed with rapidity. On others it rested for an interval increasing with the decreasing rapidity of the movement. Many it touched, passed, retouched, repassed. A representation is annexed. The abscissæ being the interval in minutes since the commencement of observation. The ordinates, the number of threads from the center.

It is interesting to compare with the above paragraph a remark of the Coast Survey Observer, who notices that the discrepancies of the separate observations of the intervals at which his pendulum reached a given arc are several times larger than can well be attributed to errors of observation. Is it not quite probable that there was in his case an indeterminateness similar to that shown in the diagram?

It seemed likely that this was due to a walking of the pendulum, such as might effect a change in the absolute position of the bar during the time of swing. I therefore fixed on the bar a small bit of looking-glass in which was reflected a scale. At the beginning of the swing a thread was set so that when

covered by the cross-wire of the telescope it bisected the knife edge. The telescope was then directed to the mirror and focused so that a division of the reflected scale was visible on the cross-wire. The pendulum was then carefully set in motion and allowed to come to rest. The position of the reflected scale was first re-examined then the telescope re-directed toward the thread. Although a hundred and twenty observations were taken no change was detected.

The only other explanation which has suggested itself is that of small movements of the stand which are not synchronous with the swing of the pendulum. In one set of observations this resulted from the nature of the ground on which the stand rested; in another it varied with the varying tightness of the joints from the dryness of the wood; in a third which has lately been called to our attention it is apparently caused by the vibration of the clock on the same wall. It has been shown that the effect of the motion of the plane of suspension due to the elasticity of the support increased the time of vibration by a constant. If, however, there are other causes for the motion of this plane than the elasticity of the support, the variation of the time is represented by

$$\Delta T = \frac{1}{2g} \int_{\varphi=0}^{\varphi=\frac{\delta^2 s}{\delta t^2}} \frac{\cot \varphi}{V} \cdot \delta \varphi$$

where $\frac{\delta^2 s}{\delta t^2}$ represents the variation of the velocity of the motion of the plane of suspension; V , the velocity of the pendulum due to its instantaneous position. Of this differential $\frac{\cot \varphi}{V}$ passes through its series of values with each swing. If $\frac{\delta^2 s}{\delta t^2}$ has the same period the integral is evidently a non-periodic function of the arc or a constant. If, however, $\frac{\delta^2 s}{\delta t^2}$ has a period, or series of periods, which are different from that of $\frac{\cot \varphi}{V}$, the value of the integral will depend on the phase with which $\frac{\delta^2 s}{\delta t^2}$ enters; and since in the course of the observation this phase will again occur with the commencement of the period of $\frac{\cot \varphi}{V}$, the function representing the disturbance of time of oscillation should be one possessing maxima and minima disposed according to some law. Since the phase of $\frac{\delta^2 s}{\delta t^2}$ at entrance reacts on

the time of swing, the limits of the integral will be irregular with respect to the portion of $\frac{\partial^2 s}{\partial t^2}$ included, and we should have a series of minor irregularities, themselves following some law, superposed on the curve produced by the varying phase at entrance. Again, since the value and relative period of $\frac{\partial^2 s}{\partial t^2}$ and V do not necessarily decrease in the same ratio with the continuance of the observation, the amplitude and period of the disturbance should continually increase or decrease. All of these are distinct from the traces of other errors of the pendulum. This curve is a record of the variation of the time of vibration produced by the motion of the plane of suspension, and therefore a definition of the steadiness of the stand. We deal, however, with means rather than the time of single vibration, and obtain a different defining curve for different limits of the second integral. But from whatever number of vibrations our means are derived we obtain the above peculiarities.

In practice there is also another variation from the fact that the transit of the cross-wire is not coincident with $\varphi=0$. Experimentally we can obtain but an approximation to the actual curve representing ΔT , but yet certainly a sufficient approximation. To this end the observer seated with his eye at the telescope has recorded on the chronograph the time of each transit in the same direction. Frequent short breaks are indulged in. They break the rhythm and better rather than injure the observation. Then reading the sheet we have taken

(e. g.) the mean of the interval bounded by the first and thousandth transit, the second, and thousandth and second, etc. Thus including in the second interval by far the greater part of the first interval, but depending on two different observations. This served to insure against accidental errors. These values are then carefully plotted, the number of transits from the beginning being used as the abscissæ. The curves thus derived fulfill our expectations as far as we can sharply criticise a statement so general.

Furthermore, curves from the same mounting preserve their general peculiarities for different observations. For different mountings the amplitude is greater with the less stability. In the preceding diagram we represent certain of these curves for the different places of observation. The sharp and quick irregularities are omitted on account of the smallness of the scale. In fact we have not now to deal with them. Since at the time of observation we supposed ourselves dealing with synchronous motion, the curves are not complete. They serve their purpose. The curve for Washington represents the least stability; that for Disco the greatest. At Disco we give three curves. The first represents the action of the stand when first set up, the parts being dry. The stand was exposed to the weather. Between the first and second series a heavy fog had swollen the parts. Between the second and third a heavy snow storm had still further swollen the various parts. For the first installation the amplitude of the curve is at least 0.0008 of a second; for the second at least 0.0003. For the third it is indeterminate. All refer to about the same amplitude of vibration. The curve for St. Johns represents fairly the increased effect of the disturbance with decreasing arc. The same is also shown in the curves for New Haven, and the second curve for Disco. It has not seemed necessary to correct the values here represented for arc.

We venture to think, then, that while an observer who finds himself compelled to work with a mounting other than he would wish can not consider that his time is increased by a constant, yet he is in a position to detect and define the effect of the stand movements at the moment of observation. This would seem the first step toward eliminating the effect. That such an elimination can be made—at least a practical elimination—we hope to show later. We can offer as yet no experimental data of its completeness.

It is to be observed that the curves derived from this cause are distinct from those produced by the form of the knife edge. It is easily shown that for any smooth, rounding, and practically, for any slightly waving form of the edge, the time of vibration is represented by

$$t = \sqrt{\frac{l}{g}} \sqrt{\frac{1}{1 + P\alpha^2}} \cos^{-1} \pm \frac{0\alpha^2 + 2(1 + P\alpha^2)\varphi}{\sqrt{0^2\alpha^4 + 4(1 + P\alpha^2)\alpha^2}}$$

where P and 0 are constants depending on the form of the knife edge and varying inclination of the instantaneous axis to the face of the pendulum. Such a variation, as has already been recognized (India Survey, Pendulum Operations; Coast Survey Report, 1876), is detected by a non-periodic acceleration or retardation of rate.