Porismatic Equations

Author(s): Edward M. Langley

Source: *The Mathematical Gazette*, Vol. 1, No. 18 (Oct., 1899), pp. 273-276

Published by: [Mathematical Association](#)

Stable URL: <http://www.jstor.org/stable/3604522>

Accessed: 28-02-2016 02:52 UTC

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.

Mathematical Association is collaborating with JSTOR to digitize, preserve and extend access to *The Mathematical Gazette*.

<http://www.jstor.org>

THE MATHEMATICAL GAZETTE.

EDITED BY

W. J. GREENSTREET, M.A.

WITH THE CO-OPERATION OF

F. S. MACAULAY, M.A., D.Sc., AND PROF. H. W. LLOYD-TANNER, M.A., F.R.S.

LONDON :

GEORGE BELL & SONS, YORK ST., COVENT GARDEN,
AND BOMBAY.

A. 2. a.; L¹. 10. a.]

PORISMATIC EQUATIONS.

Read before the Mathematical Association on January 21st, 1899.

(CONTINUED FROM PAGE 257.)

10. Given the equations :

$$\left. \begin{aligned} u \cos \alpha \cos \beta + v \sin \alpha \sin \beta + w + u'(\sin \alpha + \sin \beta) \\ \quad + v'(\cos \alpha + \cos \beta) + w' \sin(\alpha + \beta) = 0, \\ u \cos \alpha \cos \gamma + \dots\dots\dots = 0, \\ u \cos \beta \cos \gamma + \dots\dots\dots = 0, \end{aligned} \right\} \dots\dots\dots (B)$$

let it be required to find the porismatic relation between the coefficients u, v, w, u', v', w' , it being supposed that α, β, γ are all unequal and that no two of them differ by a multiple of 2π .

If we put $\cos \alpha + i \sin \alpha = x,$
 $\cos \beta + i \sin \beta = y,$
 $\cos \gamma + i \sin \gamma = z,$

the first of the above equations (B) becomes

$$\begin{aligned} u(x+1/x)(y+1/y) - v(x-1/x)(y-1/y) + 4w \\ + 2(u'/i)(x-1/x+y-1/y) \\ + 2v'(x+1/x+y+1/y) + 2(w'/i)(xy-1/xy) = 0, \end{aligned}$$

or $a/xy + bxy + c + f(x+y) + g(1/x+1/y) + h(x/y+y/x) = 0,$

where $a = u - v + 2iw', \quad b = u - v - 2iw', \quad c = 4w,$
 $f = 2(v' - iu'), \quad g = 2(v' + iu'), \quad h = u + v.$

The porismatic relation (x.) therefore becomes

$$(u+v)^2 - \{(u-v)^2 + 4w'^2\} - 4w(u+v) + 4(u'^2 + v'^2) = 0,$$

or $w'^2 - u'^2 - v'^2 + wu + wv - uv = 0. \dots\dots\dots (xi.)$

M

11. If the three relations (ix.) are similarly transformed and the real and imaginary parts of each equated to zero, it is found that

$$\left. \begin{aligned} (u+v) \Sigma \cos \alpha - (u-v) \cos (\alpha + \beta + \gamma) \\ \quad - 2w' \sin (\alpha + \beta + \gamma) + 2v' = 0, \\ (u+v) \Sigma \sin \alpha - (u-v) \sin (\alpha + \beta + \gamma) \\ \quad + 2w' \cos (\alpha + \beta + \gamma) + 2u' = 0, \\ (u+v) \Sigma \cos (\beta + \gamma) + 2u' \sin (\alpha + \beta + \gamma) \\ \quad + 2v' \cos (\alpha + \beta + \gamma) - (u-v) = 0, \\ (u+v) \Sigma \sin (\beta + \gamma) - 2u' \cos (\alpha + \beta + \gamma) \\ \quad + 2v' \sin (\alpha + \beta + \gamma) - 2w' = 0, \\ 2u' \Sigma \sin \alpha + 2v' \Sigma \cos \alpha + (u-v) \Sigma \cos (\beta + \gamma) \\ \quad + 2w' \Sigma \sin (\beta + \gamma) + 4w - (u+v) = 0, \\ -2u' \Sigma \cos \alpha + 2v' \Sigma \sin \alpha + (u-v) \Sigma \sin (\beta + \gamma) \\ \quad - 2w' \Sigma \cos (\beta + \gamma) = 0. \end{aligned} \right\} \dots (\text{xii.})$$

It is however clear, *a priori*, that there are only two distinct relations between α, β, γ ; and in fact the last four can be deduced from the first two.

The proof of the result (xi.) without imaginary quantities is rather long and intricate (Hobson's *Trigonometry*, pp. 87, 88). No mention is there made of the symmetrical relations (xii.).

12. In Nixon's *Trigonometry*, Chap. VIII., p. 163, No. 14, the following question is proposed :

"Show that the elimination of β between

$$x^2 \cos \alpha \cos \beta + x(\sin \alpha + \sin \beta) + 1 = 0,$$

and

$$x^2 \cos \beta \cos \gamma + x(\sin \beta + \sin \gamma) + 1 = 0,$$

produces

$$x^2 \cos \gamma \cos \alpha + x(\sin \gamma + \sin \alpha) + 1 = 0.$$

(*Ed. Times*, XLV., p. 123 ; *Math. Tripos*, '69 ;
John's, Camb. '81.)"

Here $u = x^2, v = 0, w = 1, u' = x, v' = 0, w' = 0$.

Hence (xi.) is identically satisfied, so that the third equation is derivable from the other two. Moreover, the symmetrical relations between the variables are

$$\Sigma \cos \alpha = \cos (\alpha + \beta + \gamma) = \Sigma \sin (\beta + \gamma) / 2x.$$

Similar questions in the same work are to be found on p. 155, No. 4 ; p. 166, No. 32 ; p. 167, No. 37 ; p. 361, No. 31 ; also in Hobson's *Trigonometry*, Chap. VI., Nos. 11, 12, 23, 32, 43, etc.

13. The result for four variables may be briefly registered.

Given the four equations :

$$\frac{a}{x_1 x_2} + b x_1 x_2 + c + f(x_1 + x_2) + g(1/x_1 + 1/x_2) + h(x_1/x_2 + x_2/x_1) = 0,$$

with similar equations involving (x_2, x_3) , (x_3, x_4) , (x_4, x_1) , the relation between the constants is

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0,$$

and the relation between the variables

$$\left. \begin{aligned} (x_1 + x_3 + f/b)(x_2 + x_4 + f/b) &= (f^2 - bc)/b^2, \\ (x_1 x_3 - h/b)(x_2 + x_4 + f/b) &= (x_2 x_4 - h/b)(x_1 + x_3 + f/b) \\ &= (bg - fh)/b^2, \\ (x_1 x_3 - h/b)(x_2 x_4 - h/b) &= (h^2 - ab)/b^2. \end{aligned} \right\}$$

14. Finally, the whole of the results of this note are true, *mutatis mutandis*, when a, b, c, f, g, h are no longer constants, but are symmetrical functions of the variables. R. F. DAVIS.

SOME CURIOSITIES IN DIVISION.

(CONTINUED FROM PAGE 208.)

Examples.

$$\begin{aligned} \text{(i.) } \frac{1}{7} &= \frac{14}{100-2} \\ &= \frac{14}{1-02} \\ &= \frac{14,28,56,12}{1-02} \\ &= \frac{14,28,56,12}{1-02} \\ &= \frac{14,28,56,12}{1-02} \\ \text{(ii.) } \frac{1}{17} &= \frac{6}{100+2} \\ &= \frac{6(100-2)}{10000-4} \\ &= \frac{588}{10000-4} \\ &= \cdot 0588 \{ 1 + \cdot 0004 + (\cdot 0004)^2 + \text{etc.} \} \\ &= \cdot 0588,2352,9408,7632,0528 \\ &\quad \quad \quad 3 \quad 15 \quad 60 \\ &= \cdot 0588235294117647. \\ &\quad \quad \quad M \quad 2 \end{aligned}$$