6. Some Mathematical Researches. By H. Fox Talbot, Esq.

7. On the Small Oscillations of Heterogeneous Interpenetrating Systems of Particles, which Act upon each other according to Laws dependent only on their Mutual Distances. By A. R. Catton, M.A., Fellow of St John's College, Cambridge.

8. Note on a celebrated Geometrical Problem. By Professor Tait.

The following problem, originally proposed by Fermat to Torricelli, *To find the point the sum of whose distances from three given points is the least possible*, seems to have given considerable trouble to the older mathematicians, and even in modern times (see *Gregory's Examples*, p. 126) to have been solved in a very tedious manner. Simpler solutions have since been given (e.g., *Cambridge and Dublin Mathematical Journal*, viii. p. 92), but none, to my knowledge, so direct as that indicated by Quaternions. The object of this note is to show the simplicity of the quaternion method.

If α, β be the vectors of two of the given points, the origin being the third, and if ρ be the vector of the required point, we must have (by the conditions of the problem)

$$T\rho + T(\alpha - \rho) + T(\beta - \rho) \text{ a minimum.}$$

$$\text{Hence } S[U\rho - U(\alpha - \rho) - U(\beta - \rho)] d\rho = 0,$$

for all values of $Ud\rho$. Hence the versor sum in square brackets must vanish identically. The immediate interpretation is, that *lines parallel to $\rho, \rho - \alpha, \rho - \beta$, form an equilateral triangle*. The required point is therefore in the same plane as the three given points; and their distances, two and two, subtend equal angles at it, which is the well-known solution.

Equally simple is the quaternion solution of the same problem if more points than three be given. Let their vectors, to any